PHYS100 Midterm Exam 08/11/2018

Izmir University of Economics

Name ________________ **Student No** ______________ **Section** _________

Choose 4 out of 7 questions to be graded. Please, indicate below which questions will be graded. If not, the very first 4 questions you answered will be considered for grading.

Problem 1. The position of an object is given by x(t) = 2 t^2 + 5 t where x is in meters and t is in seconds. (a) What is the average speed between t = 0 s and t = 3 s? (b) What is the instantaneous velocity at t = 5 s? (c) What is the instantaneous acceleration at t = 5 s?

a) $t=0 \quad x(0)=0$

$t=3 \quad x(3) = 2.9 + 5.3$

$= 18 + 15 = 33m$

$\bar{v} = \frac{33-0}{3-0} = 11 m/s$

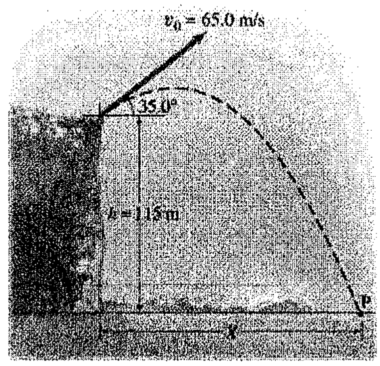
b) $v = \frac{dx}{dt}$ $\quad v(t) = 4t + 5$

$v(5) = 20 + 5 = 25 m/s$

c) $a = \frac{dv}{dt}$ $\quad a(t) = 4$

$a(5) = 4 m/s^2$

Problem 2. A projectile is shot from the edge of a cliff 115 m above the ground level with an initial speed of 65 m/s at an angle of 35° as shown in the figure on right. (a) Determine the time taken by the projectile to hit point P at the ground level, and (b) determine the distance X of point P from the base of the vertical cliff.



$y = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2$

$0 = y_0 + v_0 \sin\theta \, t - \frac{1}{2} g t^2$

$$t = \frac{-v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta - 4(-1/2) g y_0}}{2(-1/2) g}$$

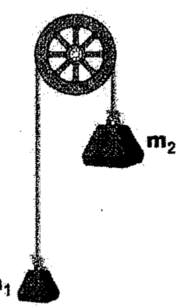
$$t = \frac{-65 \sin 35 \pm \sqrt{65^2 \sin^2 35 - 4(-1/2) 9.80 \cdot 115}}{2(-1/2) 9.80}$$

$t = 9.962 \, s$

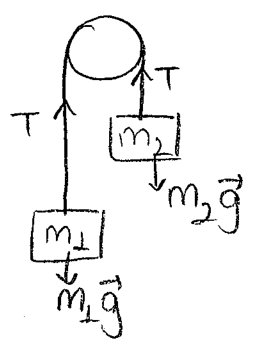
b) $\Delta x = v_x \cdot t = (v_0 \cos 35) \cdot t = (65 \cos 35) \cdot 9.962$

$= 530{,}42 \, m$

Problem 3. A system of two objects having masses m_1 and m_2 are suspended over a pulley by a cable, where $m_2 > m_1$ as shown. Assume that the cable and the pulley have no mass and we ignore the friction. (a) Draw a free body diagram for both objects. (b) Find the acceleration of the system, and (c) the tension T in the cord.



a)



b) $m_2 g - T = m_2 a$

$T - m_1 g = m_1 a$

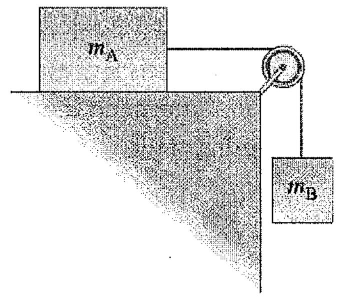
$(m_2 - m_1) g = (m_2 + m_1) a$

$$a = \frac{(m_2 - m_1) g}{m_2 + m_1}$$

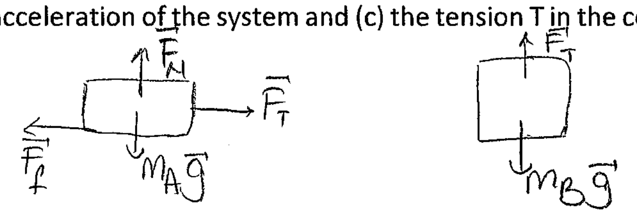
c) $m_2 g - m_2 a = T$

$$T = \frac{m_2 g m_1 + m_2^2 g - m_2^2 g + m_1 m_2 g}{m_1 + m_2} = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Problem 4. The figure shows a block of mass m_A on a rough horizontal surface, connected by a thin cord that passes over a pulley, to a second block of mass m_B, which hangs vertically. The coefficient of kinetic friction between the horizontal surface and mass m_A is given by μ_k. (a) Draw a free body diagram for both of the masses. (b) Find the acceleration of the system and (c) the tension T in the cord.



a)



b) $F_T - \mu m_A g = m_A a$

$m_B g - F_T = m_B a$

$m_B g - \mu m_A g = a(m_A + m_B)$

$$a = \frac{m_B g - \mu m_A g}{m_A + m_B}$$

c) $F_T = m_B g - m_B a = m_B g - \dfrac{m_B^2 g - \mu m_B m_A g}{m_A + m_B}$

$$F_T = \frac{m_A m_B g - \mu m_B m_A g}{m_A + m_B}$$

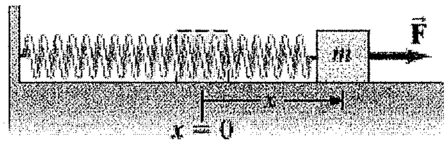
Problem 5. Suppose the space shuttle is in orbit 400 km from the Earth's surface, and it circles the Earth about every once 90 min. Find the centripetal acceleration of the space shuttle in its orbit. Express your answer in terms of g, the gravitational acceleration at the Earth's surface.

$T = 90 \text{ min} = 5400 \text{ s}$

$r = 6380 + 400 = 6.78 \, 10^6 \text{ m}$

$$a_R = \frac{4\pi^2}{(5400)^2}(6.78 \, 10^6) = 9.169 \frac{m}{s^2} = 0.935 g\text{'s}$$

Problem 6. A mass is attached to a spring (having the spring constant k) which is held stretched a distance x by a force of F and then released. The spring compresses, pulling the mass. Ignore the friction. Determine the speed of the mass m when the spring returns: (a) to its normal length (x = 0) and (b) to half its original extension x/2.

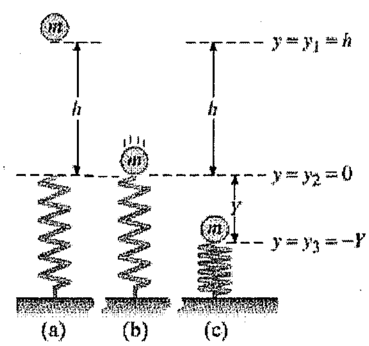


a) $\frac{1}{2}m v^2 = \frac{1}{2}kx^2 = \frac{1}{2}\frac{F}{x}x^2 \qquad v = \sqrt{\frac{Fx}{m}}$

b) $W = \int_{x/2}^{x} F\,dx = \int_{x/2}^{x} kx\,dx = \frac{1}{2}kx^2\Big|_{x/2}^{x} = \frac{3}{8}kx^2$

$\frac{1}{2}mv^2 = \frac{3}{8}kx^2 \qquad v = \sqrt{\frac{3Fx}{4m}}$

Problem 7. A ball of mass m, starting from rest, falls a vertical distance h before striking a vertical coiled spring, which it compresses an amount Y. Assume the spring has negligible mass, and ignore the air resistance. Measure all distances from the point where the ball first touches the uncompressed spring (y = 0 at this point). (a) Find the speed of the ball when it first touches the uncompressed spring (y = 0). (b) Determine the spring constant k.



a) $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$

$mgh = \frac{1}{2}mv_2^2 \qquad v_2 = \sqrt{2gh}$

b) $\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ky_2^2 = \frac{1}{2}mv_3^2 + mgy_3 + \frac{1}{2}ky_3^2$

$\frac{1}{2}mv_2^2 = -mgY + \frac{1}{2}k(-Y)^2$

$k = \frac{m}{Y^2}\left(v_2^2 + 2gY\right)$