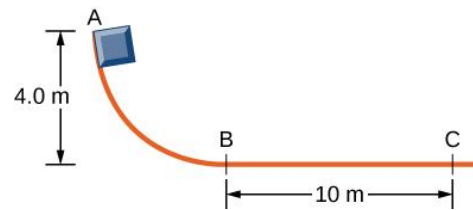


Izmir University of Economics

Name _____ Student No _____ Section _____

Each problem is worth 20 points. You have 90 minutes to complete the exam.

Problem 1. A block of mass 1 kg starts at rest at point A and slides to point C where it stops. Suppose that the track has no friction between points A and B. (a) Find the speed of the block at point B. (b) What is the work done by friction between the points B and C. (c) Determine the coefficient of kinetic friction for the track between the points B and C. ($g = 10 \text{ m/s}^2$)

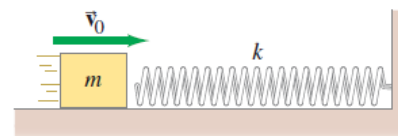


$$a) \quad mgh = \frac{1}{2}mv_B^2 \Rightarrow v_B = \sqrt{2gh} = \sqrt{2 \times 10 \text{ m/s}^2 \times 4.0 \text{ m}} \Rightarrow v_B = 8.9 \text{ m/s}$$

$$b) \quad W_{fr} = \Delta K_{BC} \Rightarrow W_{fr} = 0 - \frac{1}{2}mv_B^2 = -\frac{1}{2}(1 \text{ kg})(8.9 \text{ m/s})^2 = -40 \text{ J}$$

$$c) \quad W_{fr} = F_N \mu_k d_{BC} \Rightarrow 40 \text{ J} = (1 \text{ kg})(10 \text{ m/s}^2) \mu_k (10 \text{ m}) \Rightarrow \mu_k = 0.4$$

Problem 2. A block of mass 2 kg sliding along a rough horizontal surface is traveling at a speed 3 m/s when it strikes a massless spring head-on and compresses it a maximum distance 1 m. If the coefficient of kinetic friction between the block and the surface is 0.2, find the stiffness constant of the spring. ($g = 10 \text{ m/s}^2$)



$$E_{\text{initial}} = E_{\text{final}} \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}kx^2 + \mu_k mgx$$

$$k = \frac{mv_0^2}{x^2} - \frac{2\mu_k mg}{x} \Rightarrow k = \frac{(2 \text{ kg})(3 \text{ m/s})^2}{(1 \text{ m})^2} - \frac{2(0.2)(2 \text{ kg})(10 \text{ m/s}^2)}{(1 \text{ m})}$$

$$k = 10 \text{ N/m}$$

Problem 3. A child in a boat throws a 10 kg package out horizontally with a speed of 10 m/s. Supposing that the boat is at rest, determine the velocity of the boat just after the child throws the package. The masses of the child and the boat are respectively 25 kg and 50 kg.



$$P_{\text{initial}} = P_{\text{final}} \Rightarrow m_{bc} v_B + m_p v_p = m_{bc} v'_B + m_p v'_p$$

$$0 = (m_B + m_C) v'_B + m_p v'_p \Rightarrow v'_B = \frac{-m_p}{(m_B + m_C)} v_p$$

$$v'_B = \frac{-10 \text{ kg}}{75 \text{ kg}} (10 \text{ m/s}) = -1.3 \text{ m/s}$$

Problem 4. The platter of the hard drive of a computer rotates at 7200 revolutions per minute. (a) What is the angular velocity of the platter? (b) If the reading head of the drive is located 0.03 m from the rotation axis, what is the linear speed of the point on the platter just below it? (c) What is the centripetal acceleration of this point?

$$a) \quad f = \frac{7200 \text{ rev/min}}{60 \text{ s/min}} \Rightarrow f = 120 \text{ rev/s} = 120 \text{ Hz}$$

$$\omega = 2\pi f = 754 \text{ rad/s}$$

$$b) \quad v = R\omega = (0.03 \text{ m})(754 \text{ rad/s}) \Rightarrow v = 22.6 \text{ m/s}$$

$$c) \quad a_R = v^2/R = \frac{(22.6 \text{ m/s})^2}{0.03 \text{ m}} \Rightarrow a_R = 17025 \text{ m/s}^2$$

Problem 5. A particle is located at $\mathbf{r} = (3 \mathbf{i} + 2 \mathbf{j} + 4 \mathbf{k})$ m. A force $\mathbf{F} = (2 \mathbf{i} + 3 \mathbf{j})$ N acts on it. Calculate the torque about the origin.

$$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \vec{\tau} = (3\hat{i} + 2\hat{j} + 4\hat{k}) \times (2\hat{i} + 3\hat{j}) \text{ Nm}$$

$$\vec{\tau} = \underbrace{6(\hat{i} \times \hat{i})}_0 + \underbrace{9(\hat{i} \times \hat{j})}_{\hat{k}} + \underbrace{4(\hat{j} \times \hat{i})}_{-\hat{k}} + \underbrace{6(\hat{j} \times \hat{j})}_0 + \underbrace{8(\hat{k} \times \hat{i})}_{\hat{j}} + \underbrace{12(\hat{k} \times \hat{j})}_{-\hat{i}} \text{ Nm}$$

$$\vec{\tau} = (-12\hat{i} + 8\hat{j} + 5\hat{k}) \text{ Nm}$$

- 1) Calculate the acceleration of gravity, at (a) 6400 m, and (b) 6400 km, above the Earth's surface. ($M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$; $R_{\text{earth}} = 6.38 \times 10^6 \text{ m}$) ($G = 6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$)

$$g_{\text{planet}} = G \cdot \frac{M_{\text{planet}}}{r^2}$$

a) $r = R_{\text{Earth}} + 6400 \text{ m}$

$$r = 6.38 \times 10^6 + 6400 \text{ m}$$

$$M_{\text{planet}} = M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$g_{\text{planet}} = \left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(5.98 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 + 6400 \text{ m})^2}$$

$$g_{\text{planet}} = 9.78 \text{ m/s}^2$$

b) $r = R_{\text{Earth}} + 6400 \text{ km}$

$$r = 6.38 \times 10^6 \text{ m} + 6.4 \times 10^6 \text{ m} = 12.78 \times 10^6 \text{ m}$$

$$g = G \frac{M_{\text{Earth}}}{r^2}$$

$$g = \left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(5.98 \times 10^{24} \text{ kg})}{(12.78 \times 10^6 \text{ m})^2}$$

$$g = 2.44 \text{ m/s}^2$$

- 2) A 265 kg load is lifted 23.0 m vertically with an acceleration $a = 0.150 g$ by a single cable. Determine (a) the tension in the cable; (b) the net work done on the load; (c) the work done by the cable on the load; (d) the work done by gravity on the load; (e) the final speed of the load assuming it started from rest.



a) $\sum F_y = F_T - mg = m \cdot a \rightarrow 0.150g$

$$F_T - mg = 0.150mg \Rightarrow F_T = 1.150mg = 1.150 (265 \text{ kg}) (9.80 \text{ m/s}^2)$$

$$F_T = 2.99 \times 10^3 \text{ N}$$

b) $W_{\text{net}} = F_{\text{net}} \cdot d \cdot \cos 0^\circ = (0.150 mg) \cdot d = (0.150)(265 \text{ kg})(9.80 \text{ m/s}^2) \cdot (23 \text{ m})$

$$W_{\text{net}} = 8.96 \times 10^3 \text{ J}$$

c) $W_{\text{cable}} = F_T \cdot d \cdot \cos 0^\circ = (1.150 mg) \cdot d = (1.150)(265 \text{ kg})(9.80 \text{ m/s}^2) \cdot (23 \text{ m})$

$$W_{\text{cable}} = 6.87 \times 10^4 \text{ J}$$

d) $W_g = mgd \cos 180^\circ = -(265 \text{ kg})(9.80 \text{ m/s}^2) \cdot (23 \text{ m}) = -5.97 \times 10^4 \text{ J}$

e) $W_{\text{net}} = K_2 - K_1 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \Rightarrow v_2 = \sqrt{\frac{2W_{\text{net}}}{m}} = \sqrt{\frac{2 \cdot (8.96 \times 10^3 \text{ J})}{265 \text{ kg}}}$

$$v_1 = 0$$

$$v_2 = 8.22 \text{ m/s}$$

- 3) A 0.144 kg baseball moving 28.0 m/s strikes a stationary 5.25 kg brick resting on small rollers so it moves without significant friction. After hitting the brick, the baseball bounces straight back, and the brick moves forward at 1.10 m/s . (a) What is the baseball's speed after the collision? (b) Find the total kinetic energy before and after the collision.

a) A → baseball
B → brick

$$m_A \cdot U_A = m_A \cdot \underline{U_A'} + m_B \cdot U_B'$$

$$U_A' = \frac{m_A U_A - m_B U_B'}{m_A}$$

$$U_A' = \frac{(0.144 \text{ kg})(28 \text{ m/s}) - (5.25 \text{ kg})(1.10 \text{ m/s})}{0.144 \text{ kg}} = -12.10 \text{ m/s}$$

↳ So the baseball's speed in the reverse direction is 12.1 m/s

b) $K_{\text{before}} = \frac{1}{2} m_A U_A^2 = \frac{1}{2} (0.144 \text{ kg}) \cdot (28 \text{ m/s})^2 = 56.4 \text{ J}$

$$K_{\text{after}} = \frac{1}{2} m_A U_A'^2 + \frac{1}{2} m_B U_B'^2 = \frac{1}{2} (0.144 \text{ kg}) (12.10 \text{ m/s})^2 + \frac{1}{2} (5.25 \text{ kg}) (1.10 \text{ m/s})^2$$

$$K_{\text{after}} = 13.7 \text{ J}$$

- 4) A cooling fan is turned off when it is running at 850 rev/min . It turns 1350 revolutions before it comes to a stop. (a) What was the fan's angular acceleration, assumed constant? (b) How long did it take the fan to come to a complete stop?

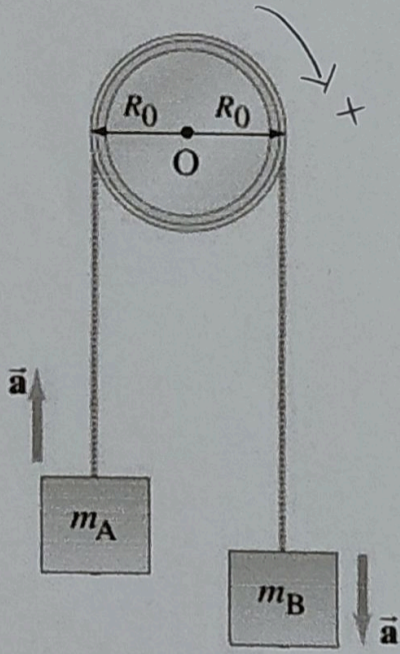
a) $\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{0 - (850 \text{ rev/min})^2}{2 \cdot (1350)} = \left(-267.6 \frac{\text{rev}}{\text{min}^2} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right)^2$

$$\alpha = -0.47 \frac{\text{rad}}{\text{s}^2}$$

b) The time to come to a stop can be found from $\theta = \frac{1}{2} (\omega_0 + \omega) \cdot t$

$$t = \frac{2\theta}{\omega_0 + \omega} = \frac{2 \cdot (1350 \text{ rev})}{850 \text{ rev/min}} \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = 190 \text{ s}$$

5)



An Atwood machine consists of two masses m_A and m_B , which are connected by an inelastic cord of negligible mass that passes over a pulley. If the pulley has radius R_0 and moment of inertia I about its axle, determine the acceleration as a function of the masses m_A , m_B , I and R_0 .

* The pulley has angular momentum $I\omega$, where $\omega = \frac{v}{R_0}$ and v is the velocity of m_A and m_B at any instant.

The angular momentum of m_A is $R_0 m_A v$ and that of m_B is $R_0 m_B v$.

The total angular momentum is;

$$L = R_0 m_A v + R_0 m_B v + I \omega = \frac{v}{R_0} \left[(m_A + m_B) R_0^2 + I \right]$$

The external torque on the system, calculated about the axis O , is

$$\tau = m_B g R_0 - m_A g R_0 = g R_0 (m_B - m_A)$$

$$\tau = \frac{dL}{dt} \Rightarrow g R_0 (m_B - m_A) = (m_A + m_B) R_0 \underbrace{\frac{dv}{dt}}_a + \frac{I}{R_0} \underbrace{\frac{dv}{dt}}_a$$

$$a = \frac{(m_B - m_A) g}{(m_A + m_B) + \frac{I}{R_0^2}}$$

→ Or you can solve it using Newton's second law.

İzmir University of Economics

Name _____

Student No _____

Please write section number and select day, time and lecturer of the course.

Section:	Day: <input type="checkbox"/> Monday	<input type="checkbox"/> Tuesday	Time: <input type="checkbox"/> 8:30-10:10	<input type="checkbox"/> 13:05-14:45
<input type="checkbox"/> Prof. Dr. Abbas Kenan Çiftçi	<input type="checkbox"/> Prof. Dr. Uğur Turnaklı		<input type="checkbox"/> Dr. Güzde Tektaş	
<input type="checkbox"/> Prof. Dr. Gürsoy Bozkurt Akgüç	<input type="checkbox"/> Doç. Dr. Göktuğ Karpat		<input type="checkbox"/> Öğr. Gör. Dr. Dilara Gül Kılıç	

Each question is 20 point.

1. A 4.5-kg object moving in two dimensions initially has a velocity $\vec{v}_1 = (10.0\hat{i} + 20.0\hat{j})$ m/s. A net force \vec{F} then acts on the object for 2.0 s, after which the object's velocity is $\vec{v}_2 = (15.0\hat{i} + 30.0\hat{j})$ m/s. Determine the work done by \vec{F} on the object.

$$W = \Delta K = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$v_2^2 = (15 \text{ m/s})^2 + (30 \text{ m/s})^2 = 1125 \text{ m}^2/\text{s}^2$$

$$v_1^2 = (10 \text{ m/s})^2 + (20 \text{ m/s})^2 = 500 \text{ m}^2/\text{s}^2$$

$$W = \frac{1}{2} \times (4.5 \text{ kg}) \times 1125 \frac{\text{m}^2}{\text{s}^2} - \frac{1}{2} \times (4.5 \text{ kg}) \times 500 \frac{\text{m}^2}{\text{s}^2}$$

$$W = 2531.25 - 1125 = 1406.25 \text{ J}$$

2. In the high jump, the kinetic energy of an athlete is transformed into gravitational potential energy without the aid of a pole. With what minimum speed must the athlete leave the ground in order to lift his center of mass 2.10 m and cross the bar with a speed of 0.70 m/s? ($g = 9.80 \text{ m/s}^2$)

$$\frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2$$

$$y_1 = 0 \quad v_2 = 0.70 \text{ m/s} \quad y_2 = 2.10 \text{ m}$$

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + m g y_2$$

$$v_1 = \sqrt{v_2^2 + 2 g y_2}$$

$$v_1 = \sqrt{(0.70)^2 + 2 \times 9.80 \times 2.10}$$

$$v_1 = 6.454 \text{ m/s}$$

3. A 0.280-kg croquet ball makes an elastic head-on collision with a second ball initially at rest. The second ball moves off with half the original speed of the first ball. What is the mass of the second ball? (Please select the direction of the first ball the positive direction.)

Let A represent the first ball, and B represent the second ball. ✓

$$v_A - v_B = -(v_A' - v_B') \quad v_B = 0 \quad v_B' = \frac{1}{2} v_A$$

$$v_A = -v_A' + \frac{v_A}{2} \quad v_A' = -\frac{v_A}{2}$$

$$p_{\text{initial}} = p_{\text{final}}$$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

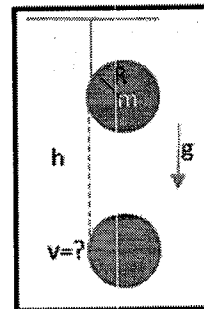
$$m_A v_A = -\frac{m_A v_A}{2} + \frac{m_B v_A}{2}$$

$$\frac{3}{2} m_A v_A = \frac{1}{2} m_B v_A$$

$$m_B = 3 m_A = 3 \times 0.280 = 0.84 \text{ kg}$$

4. A cylinder of Radius R and mass M with a wrapped rope around is left to fall down a distance h . As it falls it rotates and gravitational constant is g and $I = \frac{1}{2}MR^2$.

- a) What is the linear acceleration of cylinder?
 b) What is the linear velocity at the end? (You may use either acceleration from part a) or energy conservation).



$$a) Mg - T = Ma$$

$$\tau = T \times R = I \times \alpha = I \times \frac{a}{R}$$

$$T \times R = \frac{1}{2}MR^2 \times \frac{a}{R} \quad T = \frac{1}{2}Ma$$

$$Mg - \frac{1}{2}Ma = Ma \quad Mg = \frac{3}{2}Ma$$

$$a = \frac{2}{3}g$$

$$b) \text{ from } v^2 = 2ah$$

$$v^2 = 2 \times \frac{2}{3}gh = \frac{4}{3}gh$$

$$v = \sqrt{\frac{4}{3}gh}$$

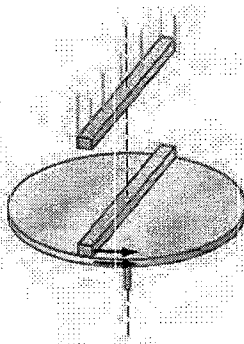
from energy conservation

$$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = \frac{1}{2} \times \frac{1}{2}MR^2 \times \frac{v^2}{R^2} + \frac{1}{2}Mv^2$$

$$Mgh = \frac{1}{4}Mv^2 + \frac{1}{2}Mv^2 = \frac{3}{4}Mv^2$$

$$v = \sqrt{\frac{4}{3}gh}$$

5. A uniform disk turns at 3.7 rev/s around a frictionless spindle. A nonrotating rod, of the same mass as the disk and length equal to the disk's diameter, is dropped onto the freely spinning disk, Figure. They then turn together around the spindle with their centers superposed. What is the angular frequency in rev/s of the combination?



$$I_{\text{disk}} = \frac{1}{2}MR^2, R: \text{radius of the disk, } M: \text{mass of the disk}$$

$$I_{\text{rod}} = \frac{1}{12}Ml^2, l: \text{length of the rod, } M: \text{mass of the rod}$$

$$\omega_1 = 3.7 \text{ rev/s} \quad I_{\text{rod}} = \frac{1}{12}M(2R)^2$$

$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{rod}}}$$

$$\omega_2 = \omega_1 \left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{1}{12}M(2R)^2} \right]$$

$$\omega_2 = \frac{3}{5}\omega_1 = \frac{3}{5} \times 3.7 \text{ rev/s} = 2.22 \text{ rev/s}$$

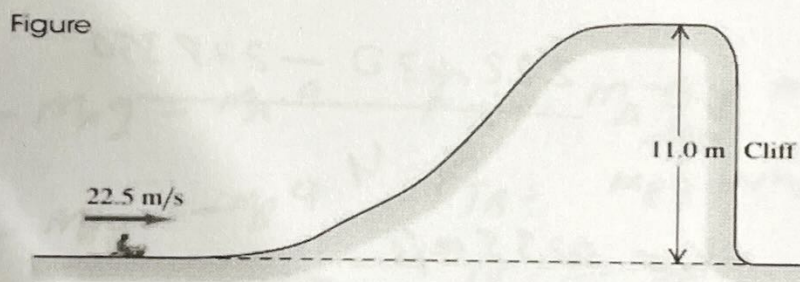
Phys 100 General Physics I

Final exam, Fall 2021.

Name:

Section:

1. A sled with rider having a combined mass of 130kg travels over a perfectly smooth icy hill (figure). How far does the sled land from the foot of the cliff? ($g=9.81\text{m/s}^2$)



Energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mgh$$

$$v'^2 = v^2 - 2gh = 22.5^2 - 2 \cdot 9.8 \cdot 11$$

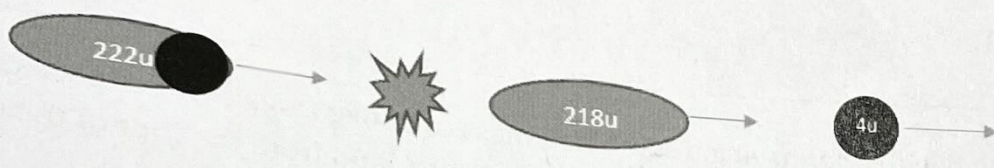
$$v' = 17.04 \text{ m/s}$$

$$\text{fall time } \frac{1}{2}gt^2 = h \Rightarrow t = \sqrt{\frac{2 \cdot 11}{9.8}} = 1.5 \text{ s}$$

The distance from the foot of the hill

$$x = v \cdot t = 17.04 \cdot 1.5 = 25.5 \text{ m}$$

2. An atomic nucleus initially moving at 420m/s emits an alpha particle in the direction of its velocity, and the remaining nucleus slows to 350m/s. If the alpha particle has a mass of 4.0 u (unified atomic mass units) and the original nucleus has a mass of 222 u, what speed does the alpha particle have when it is emitted?

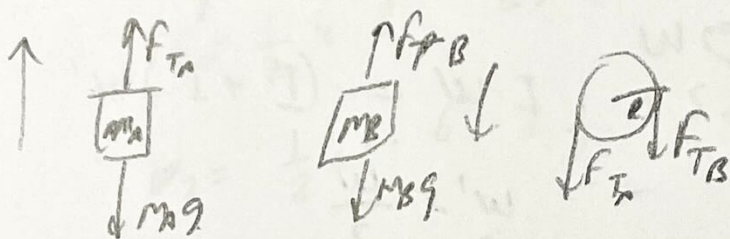
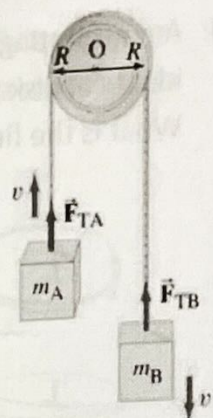


$$222 \cdot 420 = 218 \cdot 350 + 4 \cdot V_{\alpha}$$

$$V_{\alpha} = \frac{222 \cdot 420 - 218 \cdot 350}{4}$$

$$V_{\alpha} = 4235 \text{ m/s}$$

3. An Atwood's machine consists of two masses m_A and m_B which are connected by a massless inelastic cord that passes over a pulley, Figure. If the pulley has radius R and moment of inertia I about its axle, determine the acceleration of the masses m_A and m_B and compare to the situation in which the moment of inertia of the pulley is ignored. (Hint: The tensions F_{TA} and F_{TB} are not equal.)



$$F_{TA} - m_A g = m_A a \quad F_{TA} = m_A g + m_A a$$

$$F_{TB} - m_B g = -m_B a \quad F_{TB} = m_B g - m_B a$$

$$(F_{TB} - F_{TA}) R = I \alpha \quad \alpha = \frac{a}{R}$$

$$F_{TB} - F_{TA} = \frac{I a}{R^2}$$

$$m_B g - m_B a - m_A g - m_A a = \frac{I a}{R^2}$$

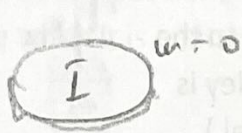
$$(m_B - m_A) g = \left(m_A + m_B + \frac{I}{R^2} \right) a$$

$$a = \frac{(m_B - m_A) g}{\left(m_A + m_B + \frac{I}{R^2} \right)}$$

In comparison

to $a = \frac{(m_B - m_A) g}{m_B + m_A}$ it is smaller

4. A nonrotating cylindrical disk of moment of inertia I is dropped onto an identical disk rotating at angular speed w . Assuming no external torques, What is the final common angular speed of the two disks?



$$L = I w$$



$w \rightarrow w'$

conservation of L

$$I \cdot w = (I + I) w'$$

$$w' = \frac{w}{2}$$

5. If it requires 5.0J of work to stretch a particular spring by 2.0 cm from its equilibrium length, how much more work will be required to stretch it an additional 4.0cm?

$$W_1 = \frac{1}{2} k x^2$$

$$5 = \frac{1}{2} k \cdot 2^2 \Rightarrow k = 2.5 \text{ N/cm}$$

$$W_2 = \frac{1}{2} \cdot k \cdot 6^2 = \frac{1}{2} \cdot 2.5 \cdot 36 = 45 \text{ J}$$

The difference is

$$\text{so } \Delta W = 45 - 5 = 40 \text{ J needed.}$$