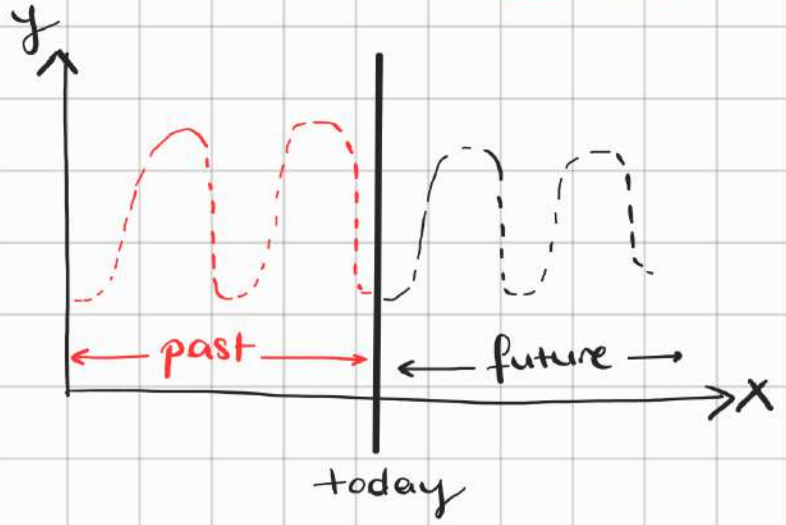
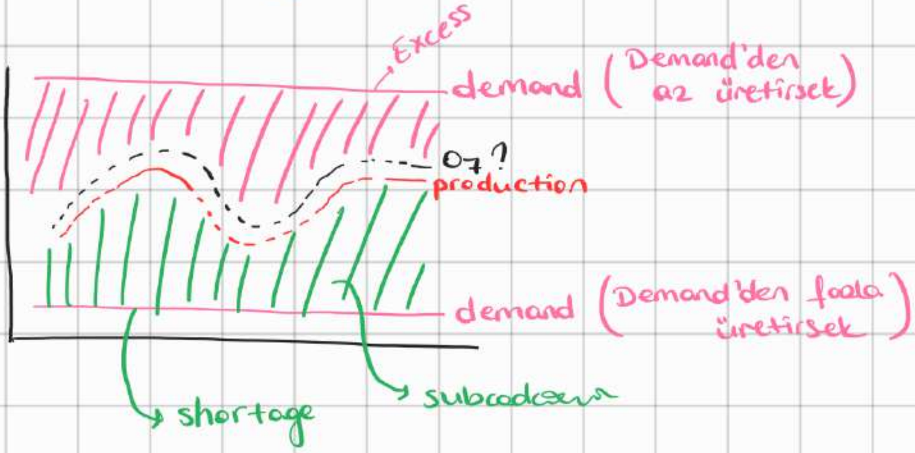
IE 222

Forecasting

$\boxed{\text{Amacı}}$: Minimize total error!



Ass Planning



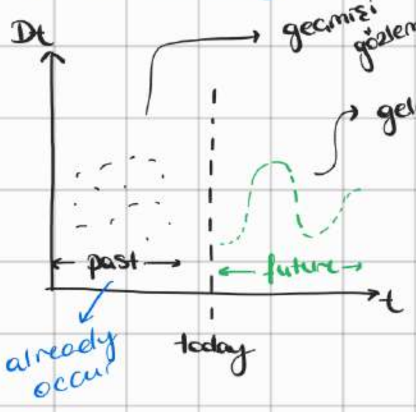
Inventory Management

Demand bilinirken

Demand bilinmezken

Forecasting

* understanding past.



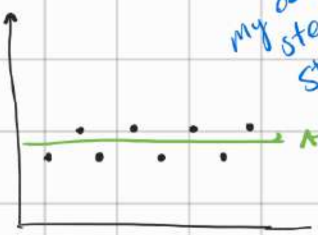
geçmişi gözlemleyip

geleceğe dair prediction yapmak

already occur

belirgin artış azalışlar var (Line halinde)

Stationary Model

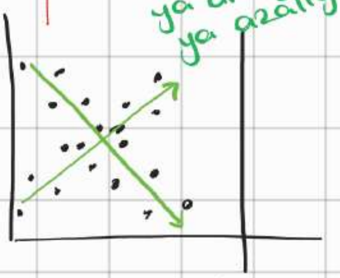


my observation standing on stationary position.

Moving average (MA)

Exponential Smoothing (ES)

Trend Model



ya artıyor ya azalıyor

Regression

Holt's Method

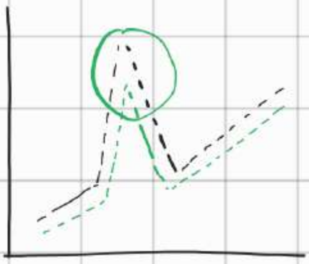
Seasonal Model

keskin artış ya da azalışlar var

farklı yıllarda benzer artış azalışlar olmuş.

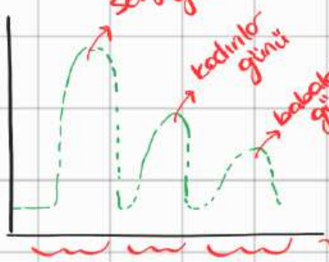
Seasonal Decomposition

Winter's Methods



⇒ Seasonal ve Trend'in karışımı

Seasonal

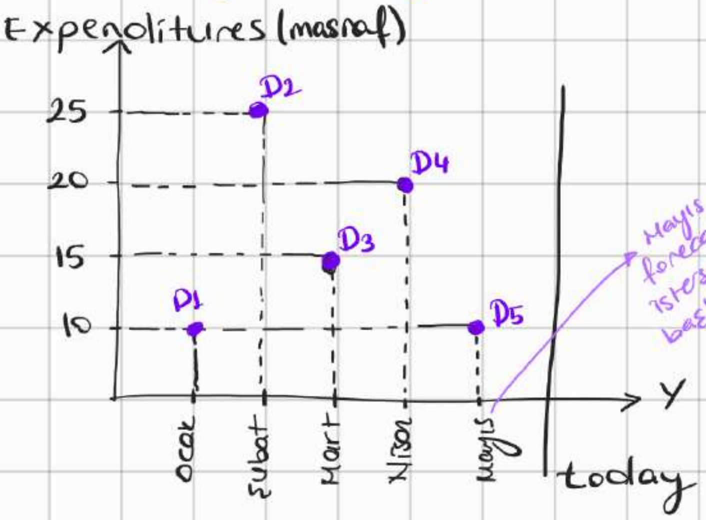


Çiçek production

→ 6. periyot için prediction yapıcaksan
ilk 5 periyodun average'ını almak demek! } MA(5) diye sorar ki 5 tane veri kullanabil.

(MA)
① Moving Average (N = num. of observations)

Expenditures (masraf)



$$F_t = \frac{D_{t-1} + D_{t-2} + \dots}{N}$$

starting with previous observation

* MA(3) ⇒ 3 observation kullanarak forecasting yap demek.

↳ $\frac{a + b + c}{3}$ (1 2 3)

$(N=2) \to F_6 = \frac{10+20}{2} = 15 \to \frac{D_5 + D_4}{2}$

$(N=3) \to F_6 = \frac{10+20+15}{3} = 15 \to \frac{D_5 + D_4 + D_3}{3}$

$(N=4) \to F_6 = \frac{10+20+15+25}{4} = 17.5 \to \dots\dots$

Sondan geriye geliyorsun toplarken

4 data kullanarak forecasting yap demek!

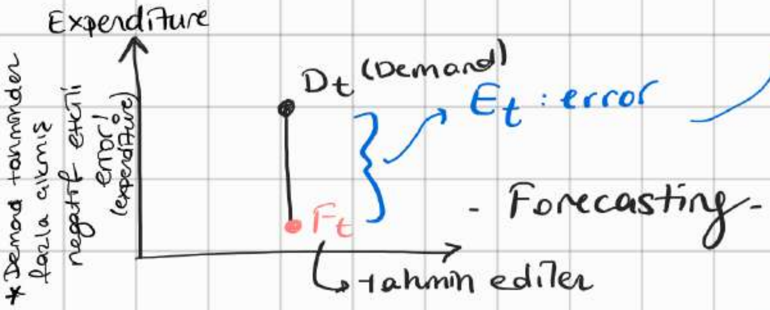
↳ Son 4 datayı kullanarak 6.'yı tahmin ettik!

Obj Func:

$$\boxed{Min\ Z = \sum \text{total Error}}$$

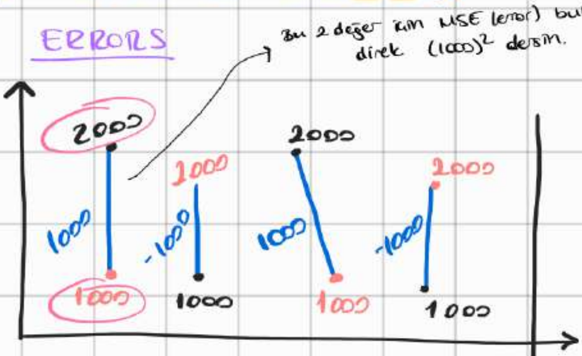
hangi N değerinin optimal olduğuna min error'u verene göre seçeriz.

$$\boxed{E_t = F_t - D_t}$$



- Forecasting -

ERRORS



$$MSE = \frac{(1000)^2 + (-1000)^2 + \dots}{4}$$

→ F-D arası errorların toplam average'ı oldu!

$$MAD = \frac{|1000| + |-1000| + \dots}{4}$$

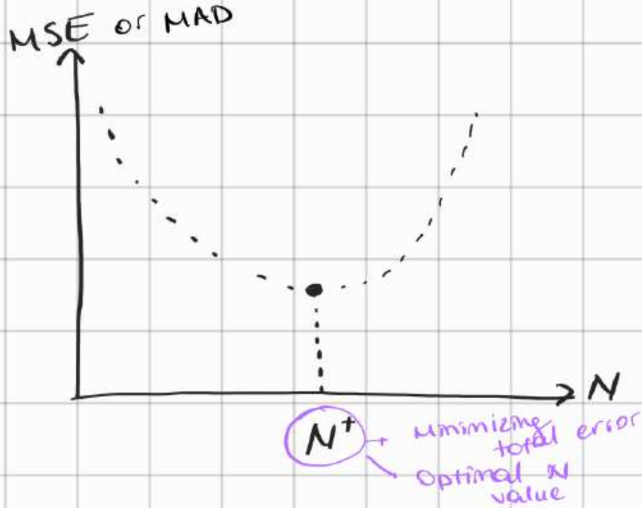
$$MAPE = \frac{|1000|}{2000} + \frac{|-1000|}{1000} + \dots$$

→ Demand

• MSE (mean square error)

• MAD (mean absolute error)

• MAPE (mean absolute percentage error)



Son 3 aya bakarak mı son 5 aya bakarak mı tahmin yapmak?

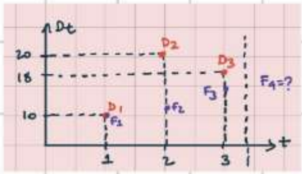
↳ son 3 aya bakmak ✓ (error miktarı daha düşük oluyor)

② Exponential Smoothing (ES(α))

*kaçıncı periyot için forecast istiyorsa

$$0 \leq \alpha \leq 1$$

$$F_t = \underbrace{D_{t-1}}_{\text{previous demand}} \cdot \alpha + \underbrace{F_{t-1}}_{\text{previous forecast}} \cdot (1-\alpha)$$



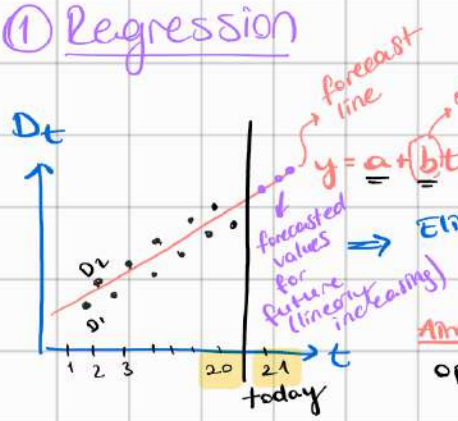
$F_4 = D_3 \cdot \alpha + F_3 \cdot (1-\alpha)$

$F_3 = D_2 \cdot \alpha + F_2 \cdot (1-\alpha)$

$10 = F_2 = \underset{10}{D_1} \cdot \underset{0.5}{\alpha} + \underset{10}{F_1} (\underset{0.5}{1-\alpha})$

$\boxed{F_1 = D_1}$ dir her zaman!

① Regression

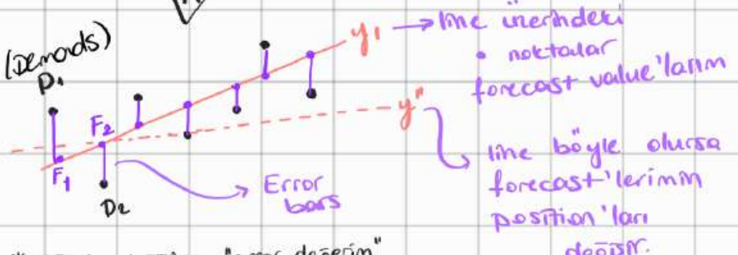


$y = a + bt$

⇒ Elimdeki hâlihazırdaki veriler

Aim is: what is my optimal a and b value?

⇓ büyütülmüş hâli

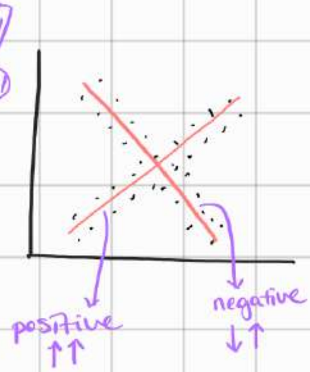


→ line üzerindeki noktalar forecast value'larım

line böyle olursa forecast'lerimin position'ları değişir.

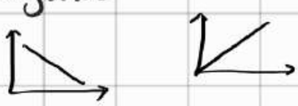
* Eğimin değişirse "error değerim" değişir otomatik olarak

a, b ve r benim için önemli.



$-1 \leq r \leq 1$

negative | positive



$0 \leq r^2 \leq 1$

* burdaki artış veya azalışlar hafif olur. (weak pred.)

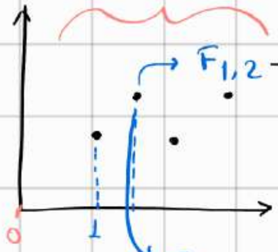
* burda artış azalışlar daha sivri ve ani olur. (strong)

* Moving average'da MA(7) hesaplamak için 8. periyottan başlar ve en son 7 periyodu alırız.

→ mixture of regression and Exponential Smoothing

(2) Holt's Method

⟹



* 0. yılda önümüzdeki 4 yıl için prediction yapıyor.

$F_{1,2}$ → 2. periyoddan, 1. periyotta yapılan forecast'le göreli değeri

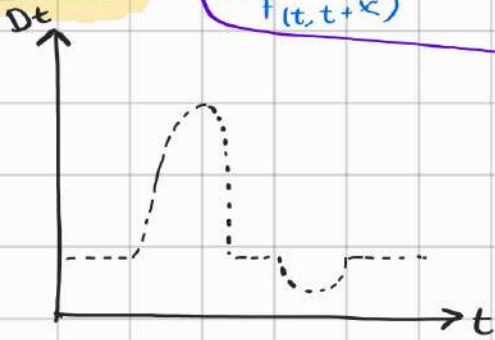
forecasting'i yaptığım periyod ← $F_{0,2}$ → kaçıncı periyot için forecast istediğim

S_1 ve G_1 değerleri bulunurken "S_0 ve G_0" soruda bize verilir.

Öre
$$F_{1,12} = S_1 + (12-1) \cdot G_1$$
$$F_{2,12} = S_2 + (12-2) \cdot G_2$$

$$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$$
$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$
$$F_{t,t+\tau} = S_t + \tau G_t$$

* Seasonal

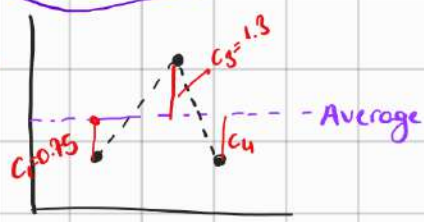


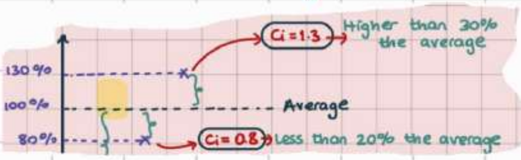
C_i = Seasonal factors

$C = 1$ → on average

$C < 1$ → Lower than average

$C > 1$ → higher " "





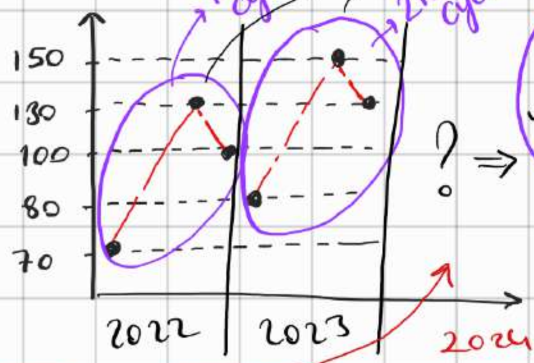
C → Average'a olan uzaklık

* Seasonal Model ;

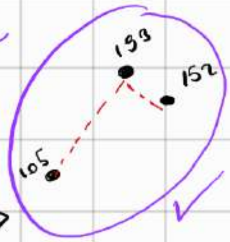
- Dirty
  - Additive
  - Multiplicative
- Seasonal Decomposition
- Winter's Method

* (1) Cycle length : how many data sets my system repeating itself

←



⟹



(2) Seasonal Factors (C_i)

⟹ Multiplicative

	D_t	Aver.	C_t
2022	70	100	0.7
	130	100	1.3
	100	100	1.0
2023	80	120	0.66
	150	120	1.25
	130	120	1.08
2024?	~105	150	0.68
	~193	150	1.27
	~152	150	1.04

$C_t = \frac{D_t}{Aver.}$

- Final c_i -

0.68 → $\frac{0.7 + 0.66}{2}$

1.275

1.04

ortalama alarak bulunuyor

→ Final C_t'lerim

2025 olsaydı bunun altında ona da aynılarını yazacaktık!

forecast

given that

çarp

Forecast Demand = Average × C_i

tersten gidiyor

⟹ Additive

C_t = (D_t - Aver)

C_t
-30
+30
0
-40
+30
+10

Final (ortalaması)

-35

+30

+5

ortalama alarak bulunuyor

(3) - (1) = C

115	→ 150 - 35
180	→ 150 + 30
155	→ 150 + 5

Forecasted demand = Average + C_{final}

tersten gidiyor

Holt's Method Exp e

Demands

4200
4300
4000

$$" S_t = a \cdot D_1 + (1-a)(S_{t-1} + G_{t-1}) "$$

$$" G_t = \beta \cdot (S_t - S_{t-1}) + (1-\beta) \cdot G_{t-1} "$$

$$" F_{t,t+\tau} = S_t + \tau \cdot G_t "$$

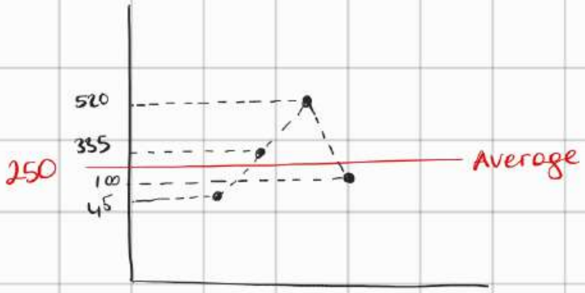
$$S_1 = 0,05 \times 4200 + 0.95 \times (3920 + 180)$$

$$S_2 = 0,05 \times 4000 + 0.95 \,(\quad + G_1)$$

$$F_{4,8} = S_4 + \tau \cdot G_4$$

$$\boxed{\tau = 8 - 4 = 4}$$

ÖR: Dirty Method (Sf.12)



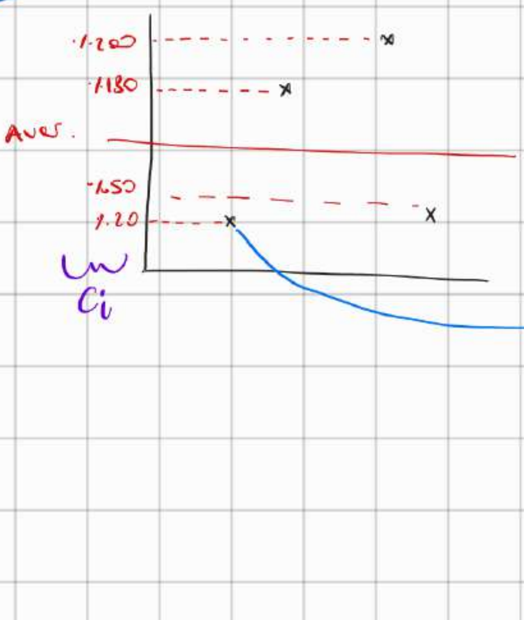
* %100 250 ise
? 45
%18

* %100 250
? 335
%134

* %100 250
? 520

* %100 250
? 100
%40

ÖR: (Sf 17-18)



$\frac{2600}{4} = 650$

4 → quarterlarla çalıştığım için → yıllık aver. buldum.

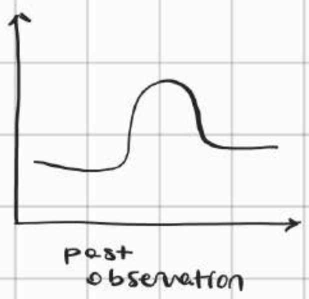
Actual value:

$650 \times 0.2 = 130$ → Ci değeriyle çarptım

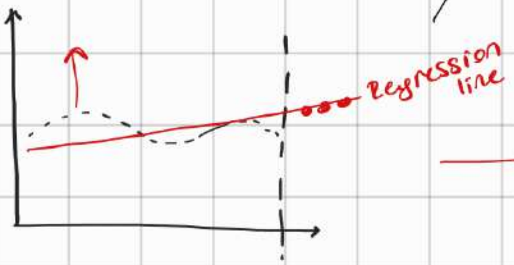
* Additive → Daha küçük ve az datam varsa elimde.

* Multicipilative → Daha büyük ve fazla verim varsa elimde

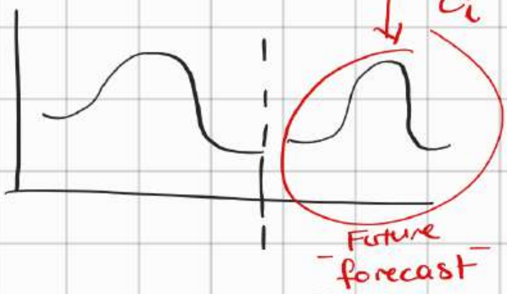
② Seasonal Decomposition



+ find C_i ⇒



regression line'dan yardım aldık



- Dirty Example -

18/03/2024 (5. hafta)

cycle'larda 3 data var!

1- identify (cycle length) = 3

2- Seasonal factors? → + C_i'leri bul

ÖR

Soruya additive ile yaklaşırsam;

Additive'le bulduğum C_i değerlerim

	Period	Demand	Average	C_i	Additive	Final C_i
1st cycle	1	100	$\frac{100+150+500}{3}$	$100/250 = \boxed{0.4}$	$100-250 = -150$	0.41 $\left(\frac{0.4+0.42}{2}\right)$
	2	150		$150/250 = \boxed{0.6}$	$150-250 = -100$	0.58 $\left(\frac{0.6+0.56}{2}\right)$
	3	500	= 250	$500/250 = \boxed{2}$	$500-250 = 250$	2.12 $\left(\frac{2+2.24}{2}\right)$
2nd cycle	4	150	$\frac{150+200+800}{3}$	$150/350 = \boxed{0.42}$		
	5	200		$200/350 = \boxed{0.56}$		
	6	800	= 350	$800/350 = \boxed{2.24}$		

last 6 months data

yüzde olarak

Average'dan 250 br üstte

Average'dan 100 br altta

birim olarak

Additive'le bulduğunda da final C_i'ye geçiş aynı yine "average" ile.

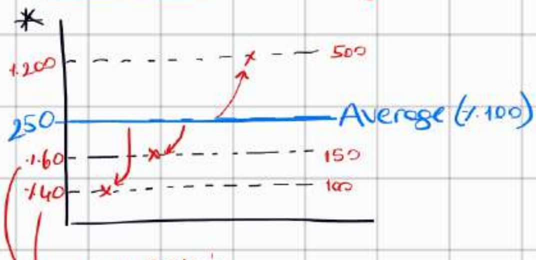
uzun vadede demand'lerimin position'ları böyle olur!

*

2 cycle var
3 data'da bir tekrar ediyor. → $\boxed{\text{Cycle length} = 3}$

C_i açıklaması; (1.st cycle için)

*



C_i değerleri

Multicipilative ya da additive kullanılabilir.

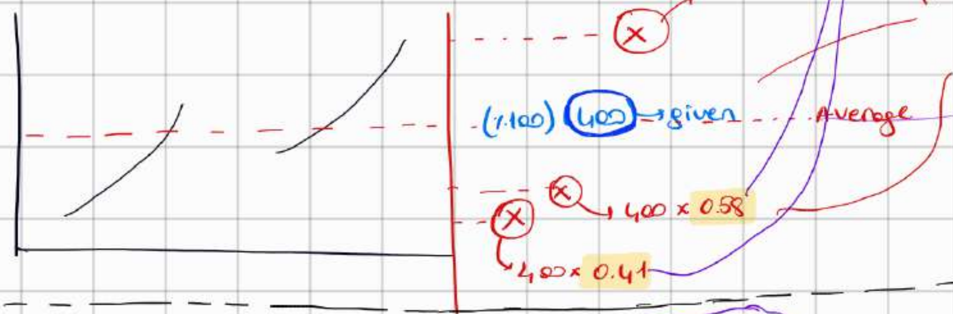
Final c_i'leri kullanarak forecast yapıcaz ε → period 7, 8, ... 'i arıyoruz.

c_i'lar ile çarptık

Prediction Part

* Dirty Method'da bu average line'ın değeri hazır veriliyor.

* Ayrıca bu soruda Multicipilative approach ile yaklaştık soruya.



- Seasonal Decomposition Exp -

Cycle length "3" old. için

Demand / CMA

* İkisi de kendi cycle'larında 2. periyottalar o yüzden hep yeptiğimiz şeyi yaptık. (ortalamayı aldık)

Period	Demand	MA (3) (Average'larım)	CMA (3)	C_i	Final C_i
1	100		—	—	
2	150		250 (2.)	150/250 = 0.6	0.57 (Period 1'de old için)
3	500		268 (3.)	500/268 = 1.9	0.605
4	150	$= \frac{500+150+100}{3} = 250$	283 (4.)	150/283 = 0.57	
5	200	$= \frac{150+500+150}{3} = 268$	350 (5.)	200/350 = 0.61	1.900 (Period 3'te old için)
6	800	$= \frac{200+150+500}{3} = 283$	—	—	
7		$= \frac{800+200+150}{3} = 350$			

4. adımdan başladım. Çünkü öncesinde min. 3 step olmalı.

1, 2, 3. adımların ortası 2.

4, 2 (?)

6, 5, 4'ün verilerini kullanıyor

* Bu adım için hala 3 geri gidebiliyorum

Bu adımda ben seasonal olan grafiğimi düz hale getirdim

Deseasonalized Demand

100 / 0.57 = 175

150 / 0.605 = 247

500 / 1.900 = 263

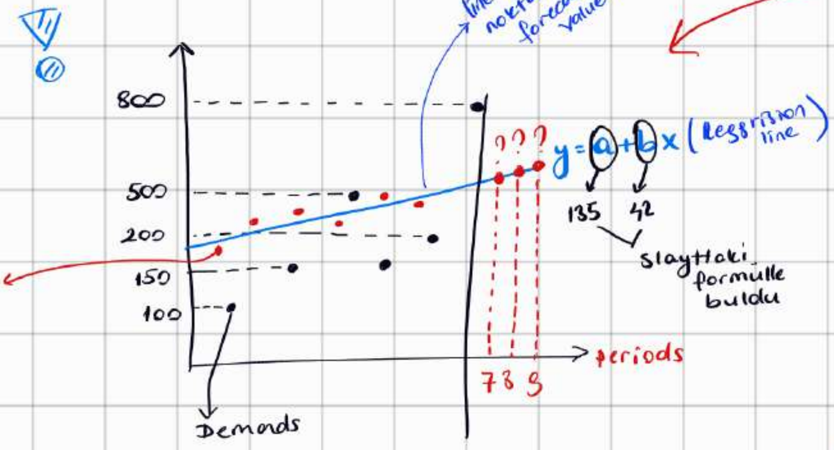
150 / 0.57 = 264

200 / 0.605 = 330

800 / 1.90 = 430

red points

(II)



7. → 135 + 42.(7) = 429

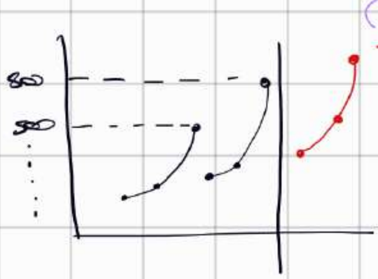
8. → 135 + 42.(8) = 471

9 → 135 + 42.(9) = 513

Predictions (by using the regression model)

Düz hale getirdiğimi tekrardan seasonal (dalgalı) hale getirdim.

Update Forecast (Seasonality)



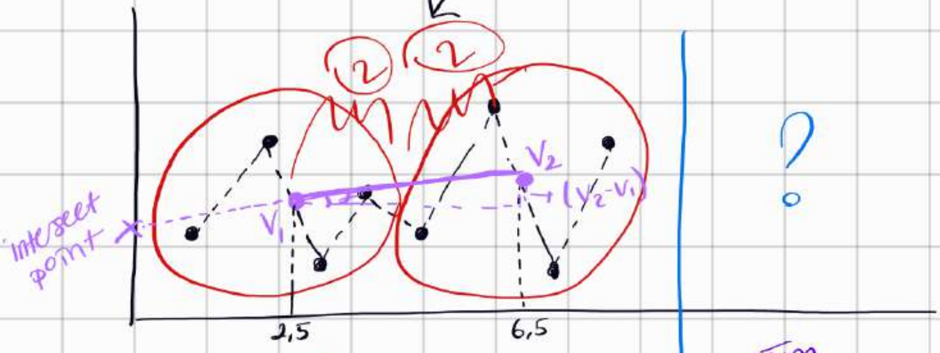
429 x 0.57 = 245

471 x 0.605 = 285

513 x 1.9 = 975

- Winter's Method Exp -

Period	Obs. Demand	Period	Obs. Demand
1st Q ('01)	72	1st Q ('02)	83
2nd Q ('01)	107	2nd Q ('02)	121
3rd Q ('01)	55	3rd Q ('02)	63
4th Q ('01)	88	4th Q ('02)	100



$V_1 = (D_{-2N+1} + D_{-2N+2} + \ldots + D_{-N})/N$

$V_2 = (D_{-N+1} + D_{-N+2} + \ldots + D_0)/N$

Bildiğimiz ortalama

$V_1 = \frac{1}{4} \times (72 + 107 + 55 + 88) = 80.5$

$V_2 = \frac{1}{4} \times (83 + 121 + 63 + 100) = 91.75$

cycle length = 4

C_i

$a = G_0 = \frac{V_2 - V_1}{N \; (6{,}5 - 2{,}5) \; (2+2)}$ → eğim

$ax + b$

$a \to G_0$, $b \to S_0$

V_i = Average values of cycles

V_2 = 2. cycle'ın average'ı

$b = S_0 =$ Denklemde "x" yerine "0" koyup bulduk

intersect point

$$C_t = \frac{D_t}{\left[V_i - \left(\left(\frac{(n+1)}{2}\right) - j\right) \times G_0\right]}$$

hangi cycle'da ise onun average'ı

number of the data in cycle

C_i değerini bulmak isted. noktanın cycle'da kaçıncı lokasyonda (kaçıncı data)

Demand	C_i
72	C_1
107	C_2
55	C_3
88	C_4
83	C_5
⋮	

$C_1 = \frac{72}{\left[80.5 - \left(\frac{5}{2} - 1\right) \times 2.80\right]} = 0.944$

$C_3 = \frac{55}{\left[80.5 - \left(\frac{5}{2} - 3\right) \times 2.80\right]}$

$C_5 = \frac{83}{\left[91.75 - \left(\frac{5}{2} - 1\right) \times 2.80\right]} = 0.948$

Final ci

$\frac{0.944 + 0.948}{2} = 0.946$

final ci değerleri

Son part: Making a Forecast

Winter's Method'la forecast hesaplama ⇒ Holt'de bu

$F_{t,t+\tau} = S_t + \tau G_t \; c_{t+\tau-n}$

$F_{0,1stQ} = S_0 + 1 * G_0 \; c_{1stQ} =$

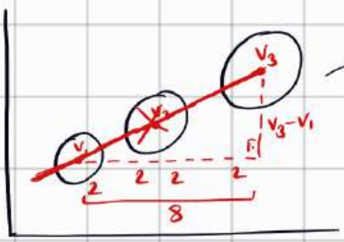
$95.97 + 1 * 2.813 \; * .944 = 93.25 \to (94)$

1-0

$F_{0,2ndQ} = 95.97 + 2 * 2.813 \; * 1.343 = 136.44$

2-0

ÖR:



→ 3 cycle var

→ cycle length = 4

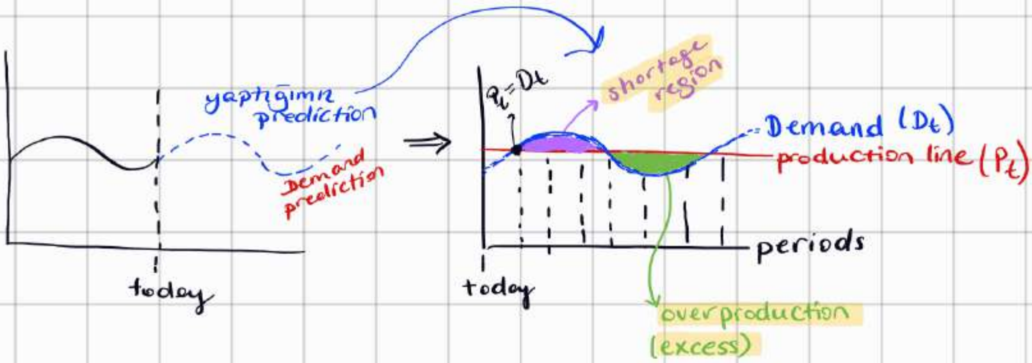
$G_0 = \frac{V_3 - V_1}{8}$

25/03/2024 (week 6-8 slaytı)

* subcontractor = taşeron
* inventory = depo
* idle production = boş üretim

- AGGREGATE PLANNING -

→ How can we satisfy the demand!



eksik kalan üretimimi tamamlayabilirim.

Shortage region; $(D > P)$

→ Subcontractor (S_t)

→ Over-time production (O_t)

→ Hiring worker (H_t) (işe alım)

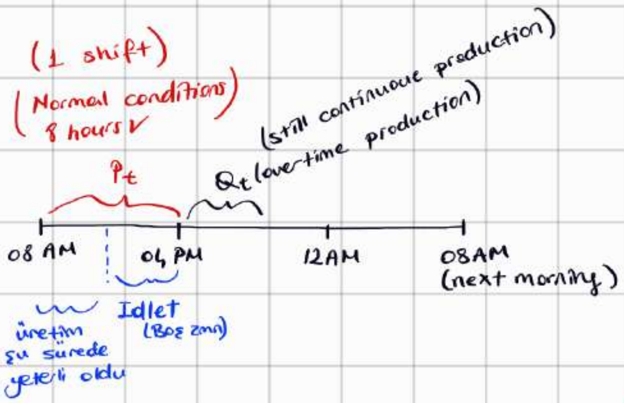
→ I_{t-1} kullanılabilir (önceki inventory)

Over production; $(P > D)$

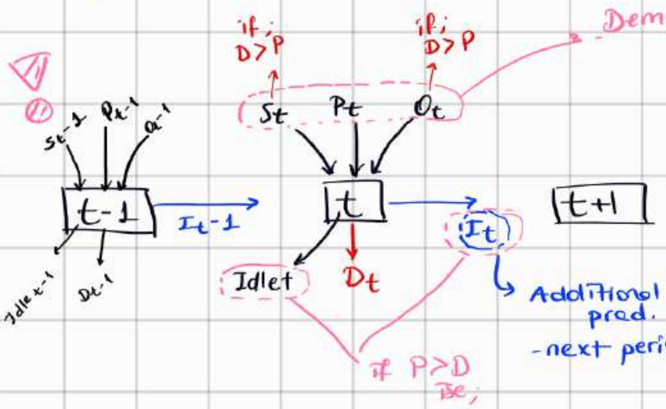
→ Inventory (I_t) (Depo)

→ Idle Production ($Idle_t$) az çalışıp prod. azaltmak

→ Firing worker (F_t)



(I)



Demand'i karşılama seçenekleri → Bu kısım D_t'den az veya çoksa

1. Eqution

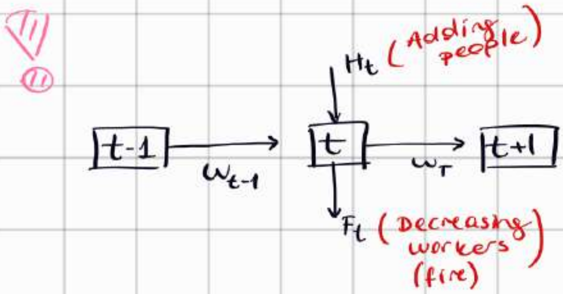
önceki periyottan gelen

$$\boxed{I_{t-1} + P_t + S_t + O_t = D_t + I_t + Idle_t}$$

if doesn't enough şeklinde $I \to P \to S \to$ _

Mass Balance

(II)



2. Equation

$$\boxed{W_{t-1} + H_t = W_T + F_t}$$

(A) Sets t = Periods → $(1 \ldots T)$

(B) Parameters D_t = Demand (Bize hazır verilen bilgiler)

(C) Variables $\left\{ \begin{array}{l} P_t, I_t, S_t, O_t, \ldots \\ W_T, H_T \ldots \end{array} \right.$ (Değişken)

(D) Obj Func. → $\text{Min} \sum_{t=1}^{T} \left((P_t \times C_p) + (I_t \times C_I) + (S_t \times C_S) \right) \ldots$ (cost'u min. etmeye çalışıyoruz)

(E) Constraints

① $I_{t-1} + P_t + S_t + O_t = D_t + I_t + Idle_t \quad , \forall_t \in T$

② $W_{t-1} + H_t - F_t = W_t \quad , \forall_t \in T$

③ All variables ≥ 0

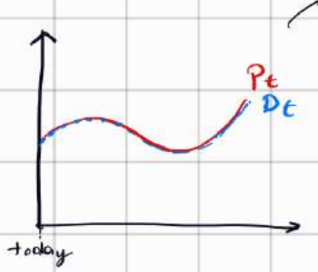
④ $W_t \times \Theta = P_t + O_T - Idle_t$ (bir işçinin ürettiği adet vs...) (fix değer)

→ D_t değerine bağlı olarak bunlardan biri pozitif değer alır, diğerleri "0" alır. (cost↓ için)

Chase Strategy → Daha basit işletmeler

→ P_t is chasing my demand! (following)

Demandi hep karşılıyorum



no I_t
no S_t
yes H_t
yes F_t

$* \; I_{t-1} + P_t + S_t + O_t = D_t + Idle_t + I_t$ (eşit)

o zaman bunlara hesh. değer vermeye gerek yok!

$\rightarrow W_t \times \Theta = P_t + O_t - Idle_t$

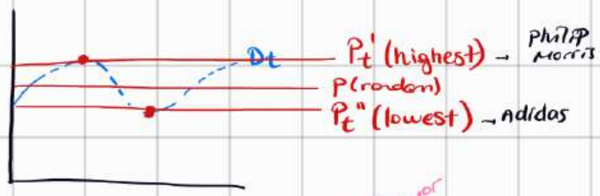
P'ye bağlı ↑, ↓ — değişken

↳ $* \; W_{t-1} + (H_t - F_t) = W_t$ → değişken

*Önceki stepteki herhangi bir değer olabilir

↳ bunlar etkiler! (Duruma göre hiring, duruma göre firing!)

Level Strategy → Fix üretim



$* \; I_{t-1} + P_t + S_t + O_t = D_t + Idle_t + I_t$ (artıp azalıyor)

↳ P_t' için: P_t sbt ve highest'ta, D_t değişken } I_t değişir, duruma göre (üretim fazla ise)

↳ P_t'' için: P_t sbt ve min'da. D_t değişiyor ama P_t'den ya hep fazla ya eşit. } S_t değişken

P_t'	P_t''
yes I_t	yes S_t
no H_t	no H_t
no F_t	no F_t

↳ P_t'ler sbt old. için W'de değişiklik meydana gelmiyor, o yüzden

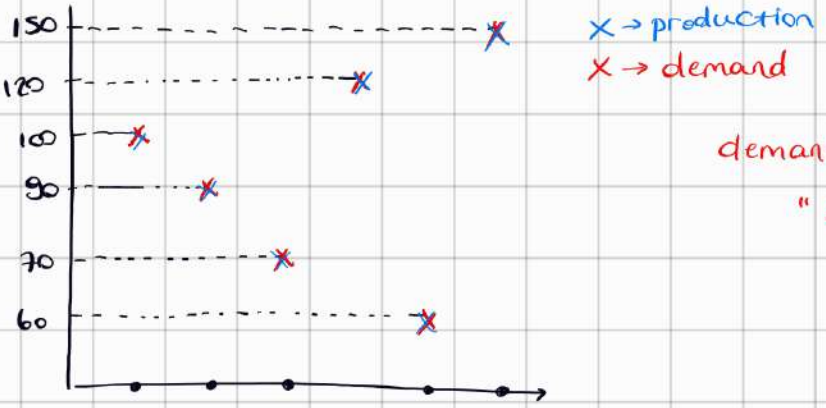
* O_t ve $Idle_t$ görmezden gelmiyor, sadece cost hesaplama kısmında lazımlar

Hybrid Strategy

① Chase Strategy

→ results of forecasting

Demand
100
90
70
120
60
150



demand = production "same"

$\Theta = 10$ pc/worker-month

$W_0 = 5$ people vardı elimde

$* \; W_T \times \Theta = P_t$ (10, 100)

↳ 1st period I need 10 workers

$100/\Theta = 100/10 = 10$

Prod.	I_t	S_t	W_t	H_t	F_t
100	0	0	10	5	–
90	0	0	9	–	1
70	0	0	7	–	2
120	0	0	12	5	–
60	0	0	6	–	6
150	0	0	15	9	–
+	+	+	+ 59	+ 19	+ 9

5 işçi daha aldım 10 yaptım

1 işçiyi çıkarttım 9 yaptım

her bir sütun toplanır ve kendi unit costlarıyla çarpılır. ⇒ Total cost bulunur.

Denklemle ihtiyacım olan sayıyı buldum

* Bütün sütunları topla ve total cost'a kat!

② Level Strategy

→ Backordering is allowed → demand'ı karşılayamadığın noktalar

→ " is not allowed

Başta "5" vardı 6 olması için "1" işe alım yaptım.

I_1'e "0" verdim çünkü $I\uparrow$ $S\uparrow$ olur. İkisinin de artması cost'u arttırır.

① P_t'' — Demand'in mini

Prod	I_t	S_t	W_t	H_t	F_t
60	0	40	6	1	—
60	0	30	6	—	—
60	0	10	6	—	—
60	0	60	6	—	—
60	0	—	6	—	—
60	0	90	6	—	—

② P_t' — Demand'in maxı

Prod	I_t	Cumulative	S_t	W_t	H_t	F_t
150	50	50	0	15	10	—
150	60	110	0	15	—	—
150	80	190	0	15	—	—
150	30	220	0	15	—	—
150	90	310	0	15	—	—
150	+ 310					

Prod, Demandlar

$$\underbrace{I_0}_{0} + 60 + \underbrace{S_1}_{40} = 100 + \underbrace{I_1}_{0}$$

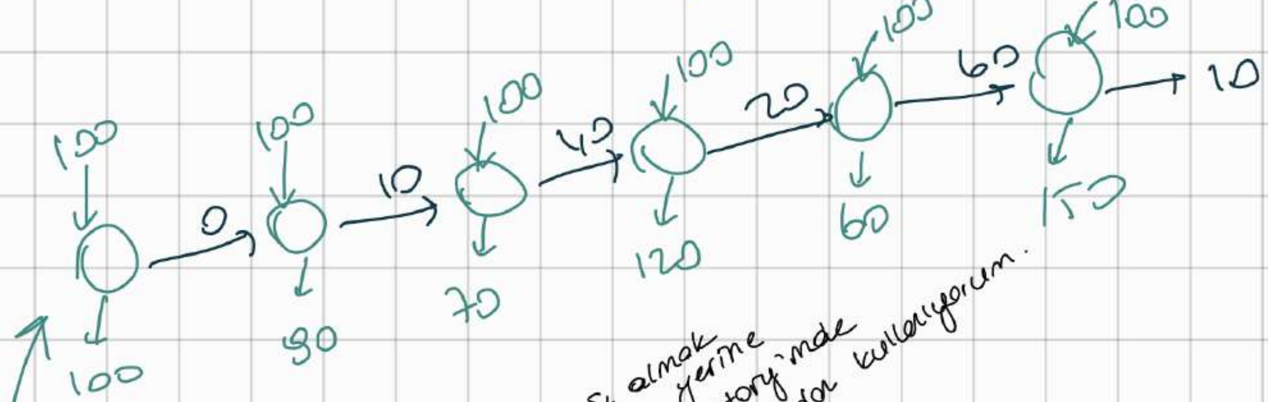
$$\underbrace{I_1}_{0} + 60 + \underbrace{S_2}_{30} = 90 + \underbrace{I_2}_{0}$$

$$\underbrace{W_1}_{6} \times 10 = \underbrace{P_1}_{60}$$

$$I_0 + 150 + S_1 = 100 + \underbrace{I_1}_{50}$$

$$\underbrace{W_1}_{15} \times 10 = 150$$

Demand	Cumulative Demand	Monthly Aver.
100	100	100 En büyüğü
90	190	95 → 190/2 = 95 (190 = 2 periyodun birikimi)
70	260	87 En küçüğü
120	380	95 (380 ÷ 4)
60	440	88 (440 ÷ 5) → 5 ay 88'er üretsem az-çok 440'ı karşılarım.
150	590	98



St almak yerine inventory'mde olandan kullanıyorum.

cumulative

	P_t (10\$)	I_t (1\$)	S_t (5\$)	W_t (2000\$)	H_t (1000\$)	F_t (1500\$)
(3)	100	–	–	10 (60)	5	–
	100	10	–	10	–	–
	100	40 (30+10)	–	10	–	–
	100	20 (40-20) harcadım	–	10	–	–
	100	60 (20+40)	–	10	–	–
	100	10 (60-50) kullandık	–	10	–	–
(4)	87	–	13	9	4	–
	87	–	3	9	–	–
	87	17	–	9	–	–
4-	87	–	16	9	–	–
	87	27	–	9	–	–
	87	–	36	9	–	–

Alttaki örnekte tren gibi çizdiğimiz şey aslında bu tablo!

Bu tablonun costu è

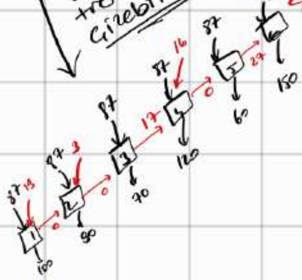
$= (140\times1) + (0\times5) + (60\times2000) + (5\times1000) + (600\times10)$

Bu tablonun costu è

$(44\times1) + (68\times5) + (54\times2000) + (4\times1000) + (522\times10)$

Bütün I_t, S_t'leri bulurken en baştaki demand tablosundan yararlandık. (100-90-70-...)

bu tabloları böyle tren şek. çizebiliriz



4. aşama için elimde $87 + 17 = 104$ adet üretilmiş var. (yukarıdaki cumulative'le aynı şey aslında)

120 → demand

$S_t = 120 - 104$
$= 16$

Bu 4 tablodan min. cost. olanı seçeriz.

Linear Regression

Month	Sales, y (000 units)	Advertising, x (000 \$)	xy	x^2	y^2
1	264	2.5			
2	116	1.3			
3	165	1.4			
4	101	1.0			
5	209	2.0			
Total					

$\bar{y} =$ $\qquad$ $\bar{x} =$

$a = \bar{y} - b\bar{x}$
$=$

$b = \dfrac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$
$=$

→ a ve b'nin formülle hesaplanışı

→ Soruya direk burdan girebilirsin!

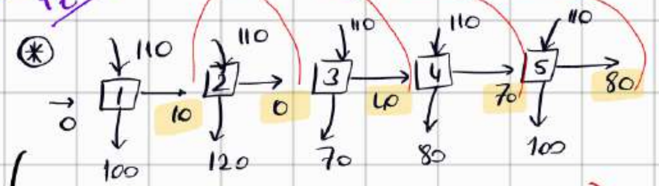
ÖR 6

	Demand	Cum. Demand	Monthly
1	100	100	100
2	120	220	110 → ($\frac{220}{2}$ → aylık üretirsem demand'i karşılarım)
3	70	290	97 (290/3) ay
4	80	370	93
5	100	470	94

Level

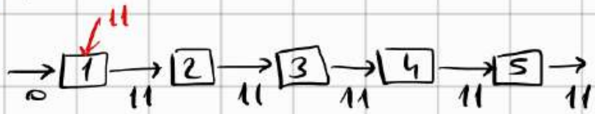
P_t''a fix.

* toplave inventory cost'la çarp mesela

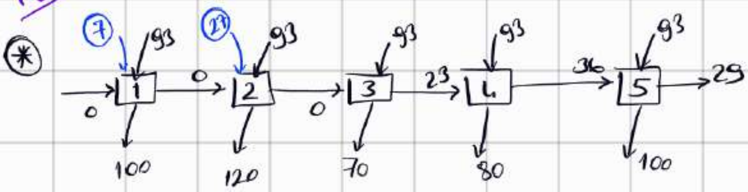


(her ay 110 ürettiği senaryoda)

for worker

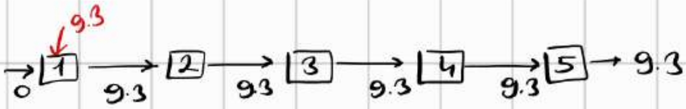


P_t'''a fix.



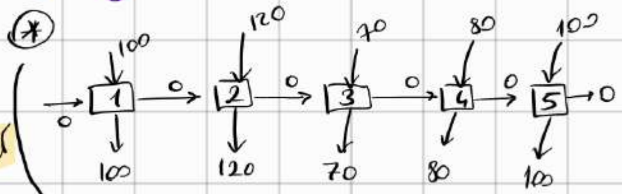
(her ay 93 ürettiği senaryoda)

for workers

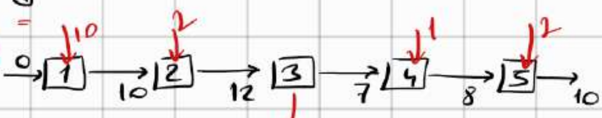


Chase olseydı:

yani demand = production



for worker



Hybrid: "Level" ve "Chase"in karışımı. Hoca bazı kısıtlar vereceği için bazı aylar üretim = demand olucak bazı aylar farklı.

$\boxed{\Theta = 10}$ → bir işçinin üretme kapasitesi

Cost Tablosu:

	I_t (2 \$)	S_t (10 \$)	P_t (5 \$)	W_t (1000 \$)	H_t (500 \$)	F_t (750 \$)
Chase →	0	0	470 × 5	47 × 1000	15 × 500	5 × 750
Level (max) →	(10 + 0 + 40 + 70 + 80) × 2 \$	0	110 × 5 × 5 = 2750	11 × 5 × 1000 = 55 000	11 × 500 (bir kez hiring yapıyor)	—
Level (min) →	(0 + 0 + 23 + 36 + 29) × 2 \$	(7 + 27) × 10 = 340	93 × 5 × 5 = 2325	9300 × 5 = 46 500	9.3 × 500 (bir kez hiring yapıyor)	—

Regrission (a, b bulma formülü) 8

$$y = a + bx$$

$$\boxed{b = \frac{S_{xy}}{S_{xx}}}$$

$$\boxed{a = \bar{D} - b(n+1)/2}$$

Demandlerin hepsinin ortalaması

$$* \; S_{xy} = \boxed{n \cdot \sum_{i=1}^{n} i \cdot D_i - \frac{n \cdot (n+1)}{2} \cdot \sum_{i=1}^{n} D_i}$$

$$* \; S_{xx} = \boxed{\frac{n^2 \cdot (n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4}}$$

We will apply regression analysis to the problem, treated in Examples 2.2 and 2.3, of predicting aircraft engine failures. Recall that the demand for aircraft engines during the last eight quarters was 200, 250, 175, 186, 225, 285, 305, 190. Suppose that we use the first five periods as a baseline in order to estimate our regression parameters. Then

$$\begin{aligned} S_{xy} &= 5[200 + (250)(2) + (175)(3) + (186)(4) + (225)(5)] \\ &\quad - [(5)(6)/2][200 + 250 + 175 + 186 + 225] \\ &= -70, \end{aligned}$$

$$S_{xx} = (25)(6)(11)/6 - (25)(36)/4 = 50.$$

Then

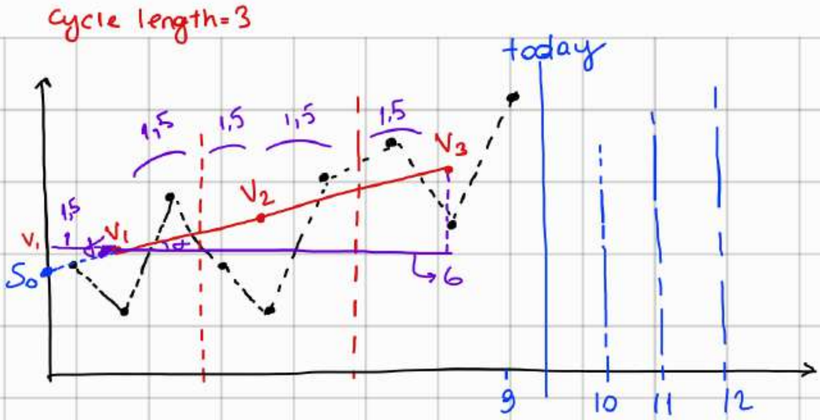
$$\begin{aligned} b &= S_{xy}/S_{xx} = -70/50 = -7/5, \\ a &= 207.2 - (-7/5)(3) = 211.4. \end{aligned}$$

It follows that the regression equation based on five periods of data is

$$\hat{D}_t = 211.4 - (7/5)t.$$ → it's the equation

$\hat{D}_t$ is the predicted value of demand at time t. We would use this regression equation to forecast from period 5 to any period beyond period 5. For example, the forecast made in period 5 for period 8 would be obtained by substituting $t = 8$ in the regression equation just given, which would result in the forecast $211.4 - (7/5)(8) = 200.2$.

Winter's Method Exp:



Demand	Average
100	
50	100
150	
125	
75	125
175	
200	
100	180
250	

$$G_0 = \frac{V_3 - V_1}{6} = \frac{183-100}{6} \approx \boxed{14} \text{ (tan}\alpha\text{)}$$

S_0 için → $\frac{V_1 - S_0}{1{,}5} = G_0 \approx 14$ → Bu da iç ters açıdan α çünkü (tanα)

$$\boxed{S_0 = 72}$$

Demand	→ C_i	
100	C_1 →	$100/100-\left(\frac{4}{2}-1\right)\times 14 = \bar{C}_1$
50	C_2 →	$50/100-\left(\frac{4}{2}-2\right)\times 14$
150		
125	C_4 →	$125/125-\left(\frac{4}{2}-1\right)\times 14 = \bar{C}_4$
⋮		
		$\bar{C}_7$

Final c_i

$\bar{C}_1 = \frac{C_1 + C_4 + C_7}{3}$

$\bar{C}_2 = \ldots$

$\bar{C}_3 = \ldots$

10, 11, 12'nci periyotlar için forecast yapıcaksan 9. periyotta yapıyorum gözlemimi;

- $F_{9,12} = (S_9 + 3\,G_9) \cdot \bar{C}_3$
- $F_{9,15} = (S_9 + 6\,G_9) \cdot \bar{C}_3$

⋮

Inv → 10 $
S → 25 $
H → 150 $
F → 250 $

Expβ * pc/worker-day = 2 (Bir çalışanın günde üretim kapasitesi)

# of working days	Cumulative (Level tam)	Demand (2 cycles)	Cum. Demand	Ratio
15	15	150	150	10.0
20	35	250	400	11.4 (max) (400/35)
25	60	250	650	10.8
30	90	300	950	10.6
20	110	150	1100	10.0 (min) (per day)
15	125	250	1350	10.8
20	145	150	1500	10.3
20	165	175	1675	10.2

Ratio → fixed prod. for 10.0 or 11.4
→ günlük ortalama üretim adetleri case'leri
öncekilerde 2 periyodu kapsadığı için "2" diyorduk ama burda 35 günü kapsıyor

Prod. (10)	Prod (11.4)	(10) Cum. Prod.	(11.4) Cum. Prod.	Worker	Hi.	Firing
10 × 15 = 150	11.4 × 15 = 171	150	171	150 / # of wor. days / 2		
10 × 20 = 200	11.4 × 20 = 228	350	399			
10 × 25 = 250	11.4 × 25 = 285	600	684			
300	342					
200	228					
150	171	1650	1881			
200	228					
200	228					

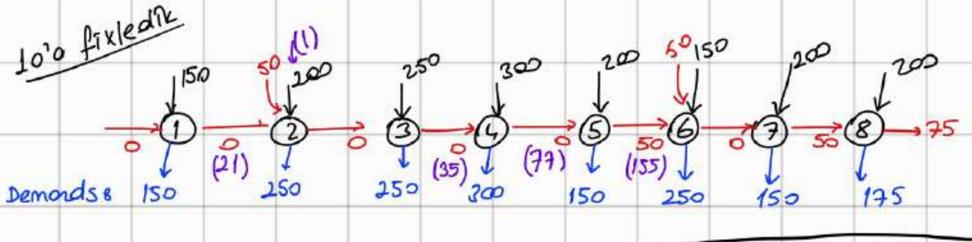
fix the prod. * (per day)

Baştaki demand

bir işçinin bir günde üretme kapasitesi

(toplam demand'dan fazla)
1881 > 1675

10'a fixledik

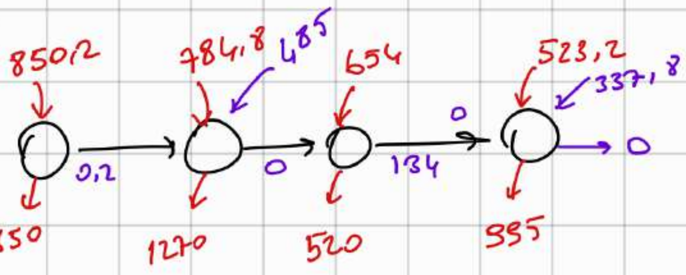


→ 11.4'e fixlediğimde olanlar

# of work days	→ Cum	Demand	Cum. Demand	Ratio
26	26	850k	850k	32,7 → min
24	50	1270k	2120k	42,4 → max
20	70	520k	2640k	37,7
16	86	995k	3635k	42,26

Prod (32,7) Prod (42,4)

32,7 × 26 = 850,2
32,7 × 24 = 784,8
32,7 × 20 = 654
32,7 × 16 = 523,2



Unanticipated Consequences= beklenmeyen sonuclar

(obj)
Min. total inventory cost

production > demand $\Rightarrow$ Inventory

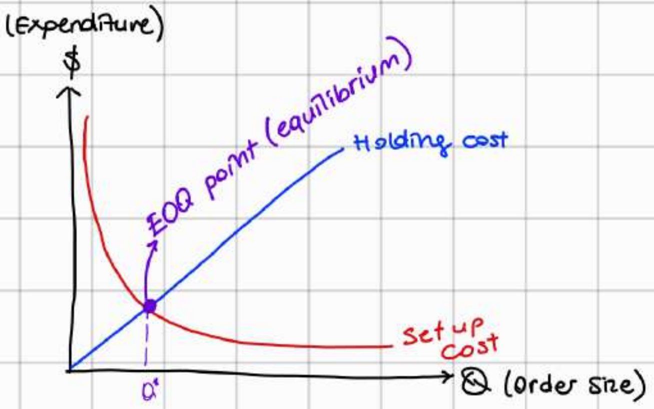
Inv. Management
- → Demand is known
- → Demand is unknown

→ Holding cost (h)
→ Setup cost (k) → fix cost
↳ supplier'dan bir şey istiyorsa bu direkt fix ödeniyor.

Demand is known

Minimizing invento

① EOQ (Economic Order Quantity) → this point is also minimizing total cost!



Set up cost için:

Demand	Q	#	SC
1000	10	100	100k
	100	10	10k
	1000	1	1k

→ supplier'ı 100 kere çağırmam gerekiyor

(Q = quantity)
10 pieces
⋮

Holding cost = $\frac{Q}{2} \cdot h$

Set up Cost = $\left(\frac{D}{Q}\right) \cdot k$ → her call'da "k" kadar ücret ödüyorum
↳ cycle sayısı (num. of call)

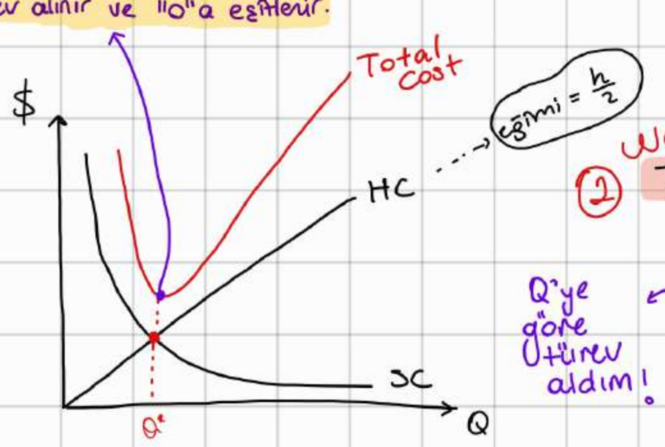
way-1 ①

$$\frac{Q}{2} \cdot h = \frac{D}{Q} \cdot k$$

$$Q^* = \sqrt{\frac{2kD}{h}}$$

for Equilibrium point

① most min cost için türev alınır ve "0"a eşitlenir.



way-2 ② Total Cost = HC + SC

$$\frac{\partial Tc}{\partial Q} = \left(\frac{Q}{2} \times h + \frac{D}{Q} \times k\right) = 0$$

Q'ye göre türev aldım!

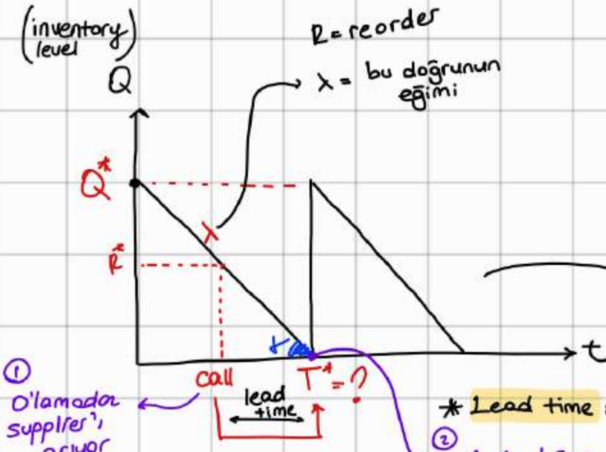
$$Q^* = \left(\frac{h}{2} - \frac{D}{Q^2} \times k\right) = 0$$

$$Q^* = \sqrt{\frac{2Dk}{h}}$$

① EOQ formula

→ optimal order quantity which is minimizing total cost!

② Reorder Eklenmiş Hâli



① Bütün demand'ler karşılanana kadar cycle'lar devam eder. (Supplier çağırmaya devam ediyorum günkü)

② Demand'i biliyorum.

$\frac{D}{Q}$ = cycle sayısı

* Lead time da açıdan bulunabilir. → siparişler gelene kadar tüketme devam ediliyor.

* call → next cycle yeni Q^* sipariş ediyorum. (supplierdan)

T^*'yi bulma yolları:

① $\boxed{\frac{Q^*}{T^*} = \lambda}$ ↳ consumption rate

② $\boxed{\frac{Q^*}{T^*} = \frac{R^*}{\text{lead time}}}$

(3) Quantity Discount Model

→ Discount model'larda h-p ilişkisi şeklinde verilir.

$k = 100\,\$$ (setup) (per order)

$h \cong \%10 \times (p)$ (holding)

$D = 1000$ pc (demand)

Q	P (unit price)
0-100	100 $
101-250	90 $
+250	80 $

(adet arttıkça discount oldu)

Q^* (Optimal Q)

$$Q_1^* = \sqrt{\frac{2 \times 100 \times 1000}{100 \times 0{,}1}} \cong 141 \rightarrow 100$$

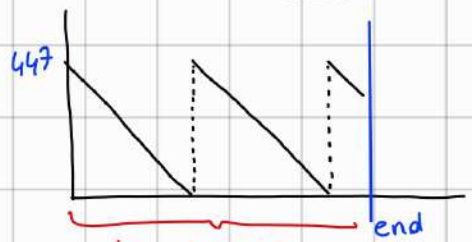
$$Q_2^* = \sqrt{\frac{2 \times 100 \times 1000}{90 \times 0{,}1}} \cong 149$$

$$Q_3^* = \sqrt{\frac{2 \times 100 \times 1000}{80 \times 0{,}1}} \cong 158 \rightarrow 250$$

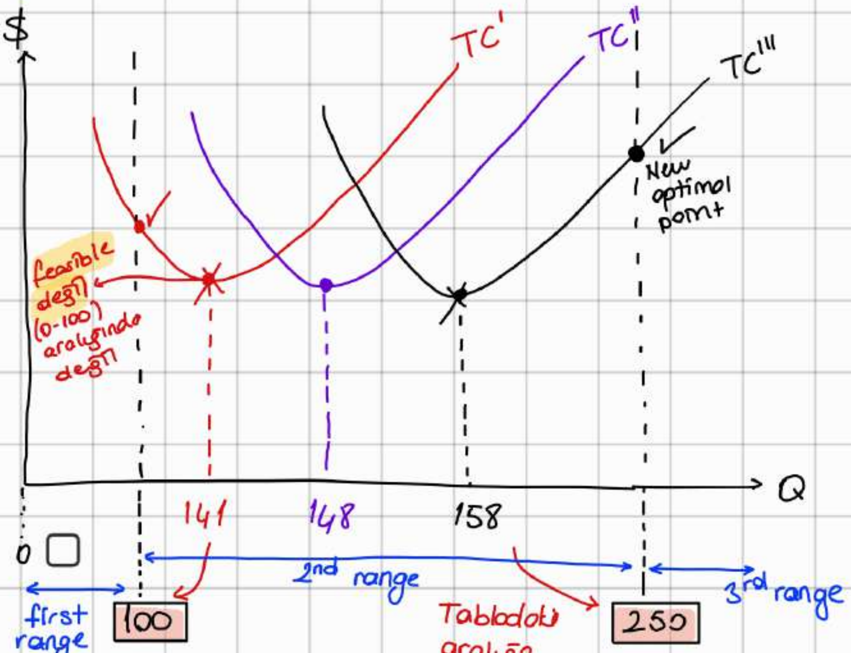
Eğer böyle olsaydı;

$h = 1\,\$$ (holding) (per piece)

$$Q^* = \sqrt{\frac{2 \times 100 \times 1000}{1}} \cong 447$$



demand 1000'i karşılamak için 2,... cycle gerekti



① Different price'ların different total cost'ları olur.

② 1. ve 3. Q^* kendi aralıklarında kalmadığı için update etmem gerekiyor.

$TC = HC + SC$

$= \frac{Q}{2} \times h + \frac{D}{Q} \times k$

$Q_1 = 100$ | $\frac{100}{2} \times (100 \times 0{,}1) + \frac{1000}{100} \times 100 = 1500$

$Q_2 = 149$ | $\frac{149}{2} \times (90 \times 0{,}1) + \frac{1000}{149} \times 100 = 1360$

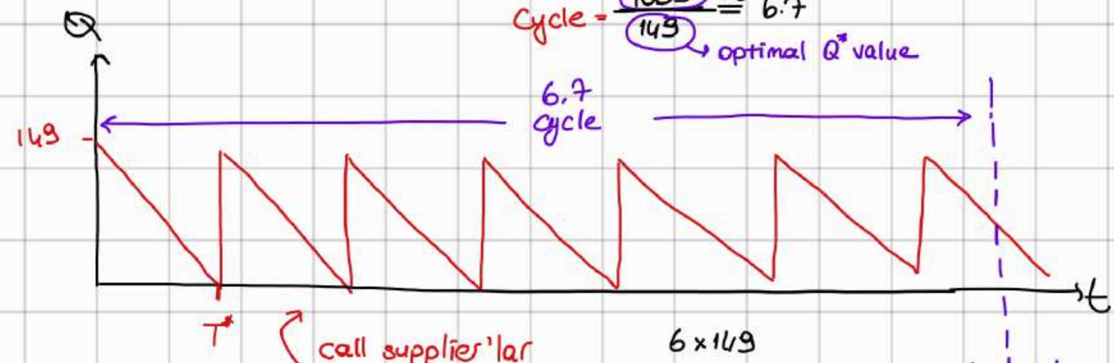
$Q_3 = 250$ | $\frac{250}{2} \times (80 \times 0{,}1) + \frac{1000}{250} \times 100 = 1400$

"min" olanı seçeriz! cost'u (y eksenindeki value'ları) bulduk

$$\text{Cycle} = \frac{1000}{149} \cong 6.7$$

1000 → demand

149 → optimal Q^* value



6 x 149 = 894 ürettim (6 cycle'da) ama demand'im 1000

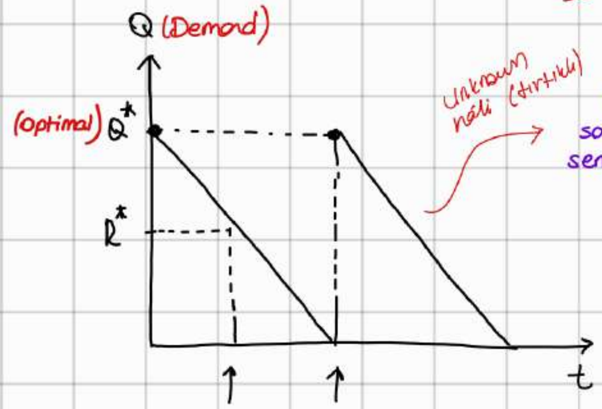
F / Ü 1000'lik demand'i karşılayabilmek için 6.7 cycle gerekiyor.

T^* = cycle length

t → demand'i karşıladığın süre

$$\boxed{T^* = t / \text{cycle sayısı}}$$

Demand is Unknown

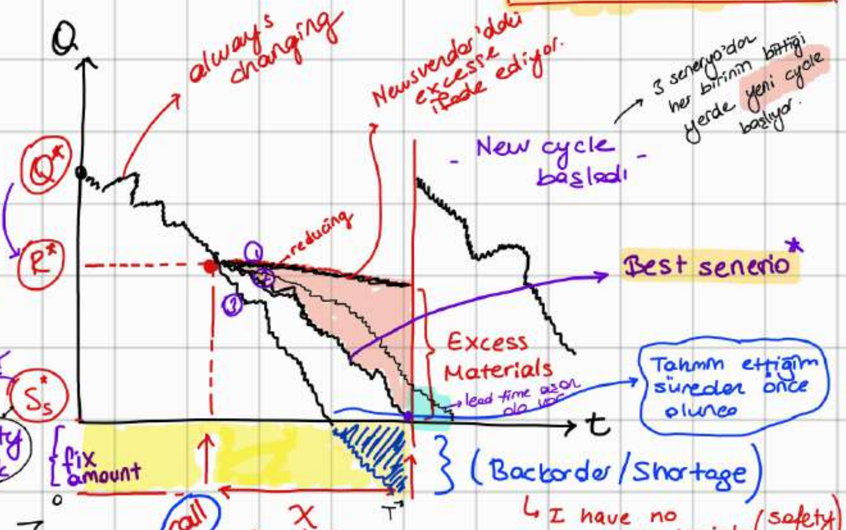


Demand is known senerio

unknown hali (titrekli)

* R^*'dan sonra 3 farklı seneryom olabilir!

S_s^* = safety stock



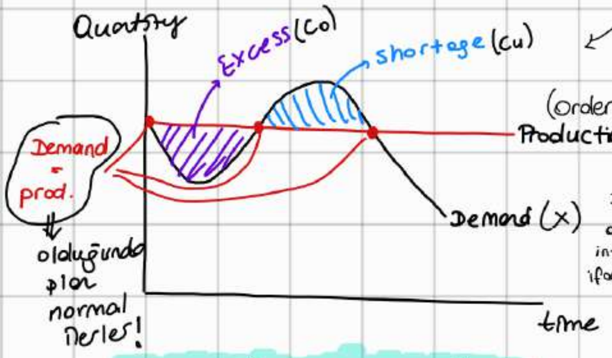
Shortage senerio'nun etkisini en aza indirgemek için

call the supply

↳ I have no materials (safety stock)

③ Cons. rate ↑ tahminden daha hızlı tüketiyor.

① Newsvendor Model (Single period)



Demand = prod. olduğunda plan normal derler!

Amaç = Yukarıdaki area'ların alanlarını bulmak

total cost function

$$\int_0^Q \underbrace{(Q-x)}_{\text{Excess kısmı için}} \cdot f_x \cdot d_x \cdot \underbrace{C_o}_{\text{Excess kısmı için unit cost (overage cost)}} + \int_Q^\infty \underbrace{(x-Q)}_{\text{shortage kısmı için}} \cdot f(x) \cdot d_x \cdot C_u = G(Q)$$

↳ shortage kısmı için unit cost (underage cost)

* Q'ye göre türev al! (min cost için) — burası için türev alınır = 0'a eşitler

Integralin türevini aldığımızda integralin içindeki ifade aynen çıkar.

$$= C_o \left((Q-x) \cdot f(x) \cdot dx \right) \Big|_0^Q + \left((x-Q) \cdot f(x) \cdot dx \right) \cdot C_u \Big|_Q^\infty = 0$$

$$= \left(Q \cdot f(x) \cdot dx - x \cdot f(x) \cdot dx \right) \quad \left(x \cdot f(x) \cdot dx - Q \cdot f(x) \cdot dx \right)$$

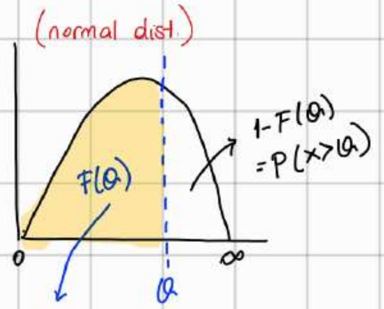
bunların türevi "0"

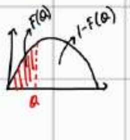
Türev al

$$f(x) \cdot dx \Big|_0^Q = F(Q) \qquad - f(x) \cdot dx \Big|_Q^\infty = 1 - F(Q)$$

$$F(Q) = P(x \le Q)$$

$$C_o \cdot F(Q) - C_u \cdot (1 - F(Q)) = 0$$





Demand unknown seneryoda Q^* bulabiliriz burdan

cdf

$$\boxed{F(Q^*) = \frac{C_u}{C_u + C_o}}$$

probability

↳ Cumulative distr. value of Q

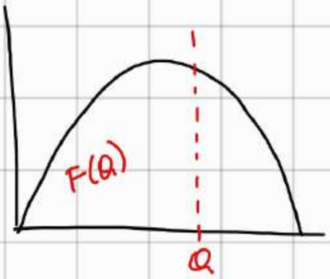
C_u, C_o → unit costlar

f(x) → prob. density func.

t-Dist da önemli

⇒ Normal Distribution

* if my demand is Normally distributed



$$F(Q) = P(x \le Q) \Rightarrow P\left(\overbrace{\frac{x-\mu}{\sigma} \le \underbrace{\frac{Q-\mu}{\sigma}}_{Z}}^{\text{Z-func. cinsinden}}\right)$$

Z-tablosu

$$F(Q) = \frac{C_u}{C_u + C_o} \rightarrow \frac{0.4}{0.4+0.1} = 0{,}9$$ → F(Q) bana grafiğin altındaki alan değerini veriyor.

$$Z = \frac{Q - \mu}{\sigma} = 1.28 \rightarrow \boxed{Q^* = \sigma.(z) + \mu}$$

ÖR: Optimal Q value? (Normal ise)

$C_u = 4$ \$/pc

$C_o = 3$ \$/pc

$$F(Q) = \frac{4}{4+3} = \frac{4}{7} = 0.6$$

→ z tablosunda olasılık değeri "0,6" olan z value'sunu bulduk

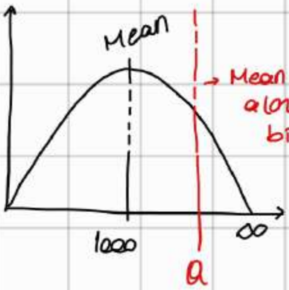
↳ $z = 0.25$

$X \sim N(1000, 2500)$ → $\boxed{x \sim N(1000, \sigma^2)}$ x, is normally distributed (1000: mean, σ^2: standart deviation)

$$P(x \le Q^*) \Rightarrow P\left(\frac{x-\mu}{\sigma} \le \frac{Q^*-\mu}{\sigma}\right) = 0.25$$

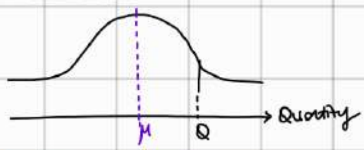
↳ $z = 0.25 \longrightarrow Q^* = (0{,}25) \times 50 + 1000$

$= 1012.5$ pc



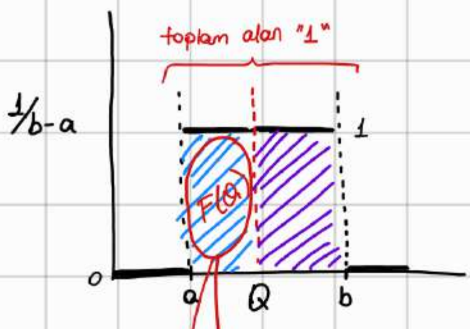
→ Mean'in solunun alanı 0,5 ama biz F(Q) = 0,6 bulduk. O zaman mean'in sağında kalıyormuş demek ki Q.

Probabilistic Demand



if normally distributed → $\boxed{\text{Q - Mean = Safety Stock}}$

⇒ Uniform Distribution (if demand is uniform distributed)



$$(b-a) \times \frac{1}{(b-a)} = 1$$

$$F(Q) = (Q-a)\frac{1}{(b-a)}$$ üstten 0,9 old. biliyoruz

$$F(Q) \times (b-a) + a = Q^*$$

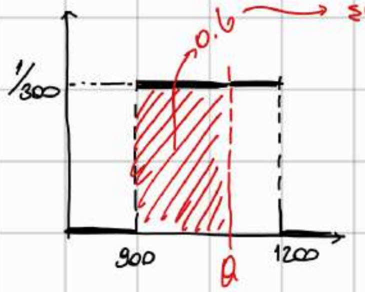
→ $P(x \le Q) = F(Q)$

→ olasılık değerini bilirsin mesela z tablosundan "Q" ya geçersin.

$$0.6 = (Q^* - a) \times \frac{1}{b-a}$$ → a-Q* arası alan yaptık aslında

ÖR: $x \sim U(900, 1200)$

Optimal Q?



→ şu değeri üstteki sorudan aldık direkt

ya da $F(Q) = \frac{C_u}{C_u + C_o}$ → burdan da bulabilirsin F(Q)'yu!

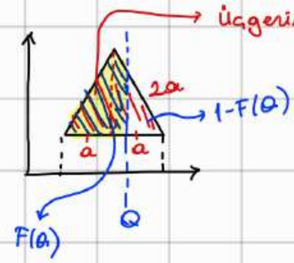
dikdörtgende alan hesaplaması yaptık

$$0{,}6 = \underbrace{(Q^* - 900) \times \frac{1}{300}}$$

$$180 = Q - 900$$

$$Q^* = 1080 \text{ pc}$$

⇒ IE Distribution



!! F(Q)'yu biliyorsak Q'yu da bulabiliriz.

ÖR: $\mu = 5000$, $\sigma = 2000$

- INFORMS (The Institute for Operations Research and Management Science) will hold its annual meeting in Washington D.C. in 2008. Six months before the meeting begins, INFORMS must decide how many rooms should be reserved at the conference hotel.
    - At this time, rooms can be reserved at a cost of $50 per room.
    - It is estimated that the demand for rooms is normally distributed with mean 5000 and standard deviation 2000.
    - If the number of rooms required exceeds the number of rooms reserved, extra rooms will have to be found at neighboring hotels at a cost of $80 per room.
    - The inconvenience of staying at another hotel is estimated at $10.
- How many rooms should be reserved to minimize the expected cost?

underage - overage "2 senaryo"

X = # of participants

Q = # of the room that I reserved

total cost at this case

$$(x < Q) = \overbrace{50}^{C_o} Q$$

$$(x > Q) = 50Q + \underbrace{(x-Q)\,80}_{\text{addition rooms}} + (x-Q).10$$

$$= 90x - \underbrace{40}_{C_u} Q$$

$$F(Q) = \frac{C_u}{C_u + C_o} = \frac{40}{90} = \frac{4}{9} \cong 0.44$$

$Z = -0.14$

$$P(x \le Q) = P\left(\frac{x - \mu}{\sigma} \le \frac{Q^* - \mu}{\sigma}\right)$$

$$Q^* = \sigma z + \mu$$

$$= 2000(-0.14) + 5000$$

$$\boxed{Q^* = 4720}$$

ÖR:

Mrs. Kandell has been in the Christmas tree business for years. She keeps track of sales volume each year and has made a table of the demand for the Christmas trees and its probability (frequency histogram).

optimal Q value?

Demand, D	Probability f(D)
22	0.05
24	0.10
26	0.15
28	0.20
30	0.20
32	0.15
34	0.10
36	0.05

Solution:

Q – order quantity; Q^* - optimal

D – demand: random variable with probability density function $f(D)$

$F(D)$ – cumulative probability function: $F(D) = \Pr(\text{demand} \le D)$

c_o – cost per unit of positive inventory

c_u – cost per unit of unsatisfied demand

Economics marginal analysis: overage and underage costs are balanced

↳ Veriler belirgin ve ayrık "Discrete"

Demand D	Probability f(D)	Cum Probability F(D)
22	0.05	0.05
24	0.10	0.15
26	0.15	0.30
28	0.20	0.50
30	0.20	0.70
32	0.15	0.85
34	0.10	0.95
36	0.05	1.00

Mrs. Kandell estimates that if she buys more trees than she can sell, it costs about $40 for the tree and its disposal. If demand is higher than the number of trees she orders, she looses a profit of $40 per tree.

(underage - overage cost burdan bulunabilir)

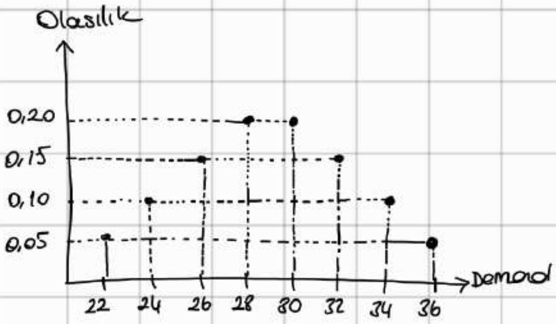
$$F(Q^*) = \frac{c_h}{c_h + c_s} = \frac{40}{40+40} = 0.50 \longrightarrow Q^* = 28$$

cum. propobility

D > ?

Normal ise → z kullanıcan

Uniform ise → uniform process



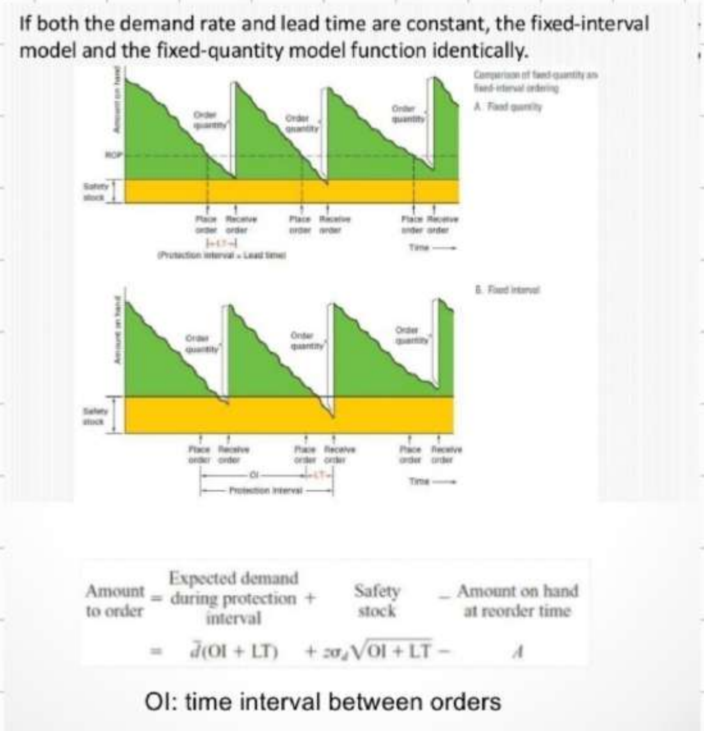
$$F(Q^*) = \frac{40}{40+40} = 0{,}50$$

Bu grafik continus ilerlemiyor, 22, 24, --- diye gider bir senaryo var

$Q^* = 28$ olur (0,50'yi arıyorum cum.'da o noktaya denk geldi)

0,50 olmasaydı cum.'de 0.45 - 0.60 olsaydı bu arada derdik ama optimal Q için 0,60'ın karşılığına yollardık. (büyüğe yuvarla)

"OR



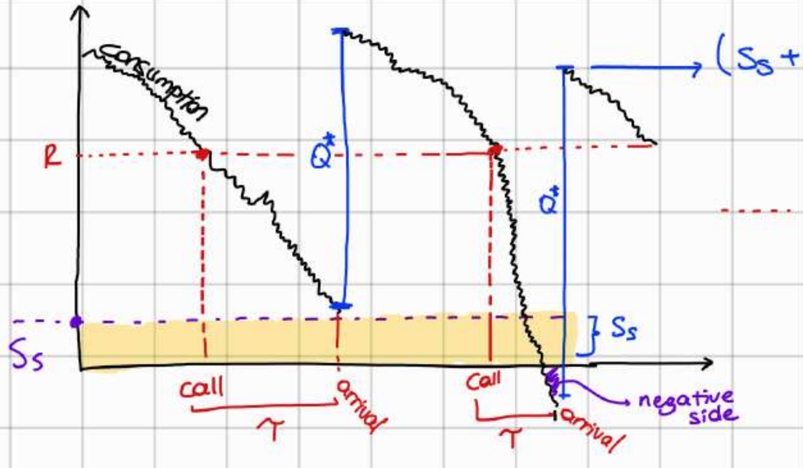
two seneryos → QR / Excess } model

① Q-R model Cont. the system

* (Inventory → periyodik kontrol edilir (Her cuma mesela))

we are searching these

② Q-R Model (Continuous System) → Continuously checking my system.

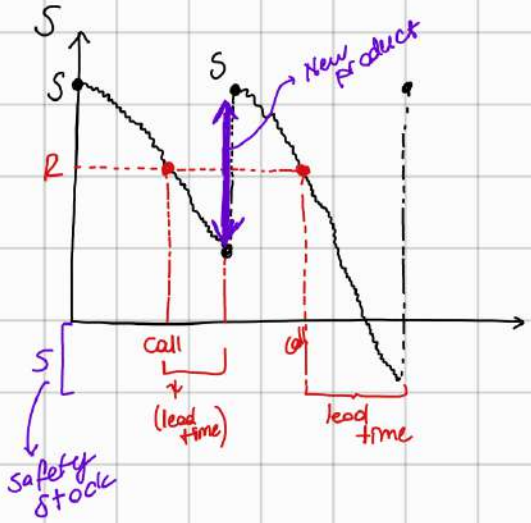


we are always checking our inventory R'ye gelindiği an supplier arıyor.

* Q → order volume is fix! → # of pieces

* At the end of the previous lead time I'm getting this Q unit. (fix)

S-s Model (periodic system)



S : Max inventory level

s : Safety stock

① Inventory her zaman değil bazı spesifik zamanlarda kontrol ediliyor.

Eğer kontrol ettiğin periyotta R'in altındaysan sipariş veriyorsun. Eğer R'in üstündeysen sipariş vermene gerek yok!

New product = | current position − S value |

At Q-R Model: we want to find both optimal Q, R values!

Cost Minimization

- Expected Cost Function:

$$G(Q,R) = \underbrace{h\left(\frac{Q}{2} + R - \lambda\tau\right)}_{\text{holding cost}} + \underbrace{\frac{K\lambda}{Q}}_{\text{set-up cost}} + \underbrace{\frac{p\lambda n(R)}{Q}}_{\text{shortage cost}} + \lambda c \Longrightarrow \text{Total Cost func.}$$

türev alınca için cost önemi yok bu cost'un

h: unit holding cost; $\lambda\tau$: safety stock; K: unit set-up cost; λ: Demand aslında; p: penalty cost; Q: # of cycles

Önce Q'ya göre türev al!

- Partial Derivatives:

(1)

$$\frac{\partial G}{\partial Q} = \frac{h}{2} - \frac{K\lambda}{Q^2} - \frac{p\lambda n(R)}{Q^2} = 0 \Rightarrow Q = \sqrt{\frac{2\lambda[K + p\,n(R)]}{h}}$$

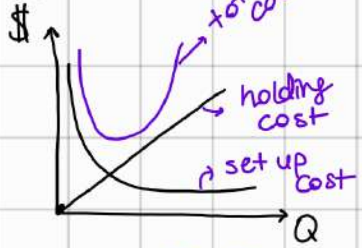
Sonra R'a göre türev al!

(2)

$$\frac{\partial G}{\partial R} = h + \frac{p\lambda}{Q} n'(R)$$

This is the first equation we will use to determine optimal values Q and R

Önceki konudan:



$n(R)$ = Expected # of pieces goes to the shortage stage
↳ (zor ama integralle bulunabilir o alan)

λ = consumption rate
= expected demand (given)

τ = lead time

λ / Q = # of calls (cycles)

p = penalty cost

Set up Cost = $\frac{D}{Q} \cdot k$
D/Q: Supplier kaç kere çağırılıyor (# of cycles)
k: Çağırılma başına ücret

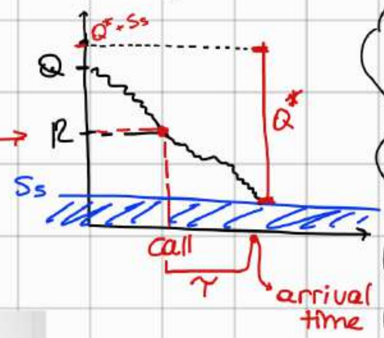
Holding Cost = $h \times Q/2$
h: unit başına
Q/2: ortalama inventory level

$$Q = \sqrt{\frac{2\lambda[k + p\,n(R)]}{h}}$$ → bunu bulabilmek için R'a ihtiyacımız var!

$$R = h + \frac{p\lambda}{Q} n'(R)$$ → bunu bulabilmek için Q'ya ihtiyacımız var!

Bu ilk adımı EOQ formülüyle bul!

Q^*	R^*
$Q_0 = EOQ$ →	R_1 (Q_0'ı kullanarak R_1'i bulduk)
Q_1	R_2^*
Q_2	⋮
⋮	



Mesela R=100 olsun
Ss=30

* Supplier arandıktan sonra (γ) 70 adet tüketim olması bekleniyor.

- Partial Derivatives:

(2) $$\frac{\partial G}{\partial R} = h + \frac{p\lambda}{Q} n'(R)$$

Note : $n(R) = \int_R^\infty (\underbrace{x}_{\text{Demand}} - R) f(x)\,dx$ → Shortage region'daki # of pieces

$$n'(R) = -(1 - F(R))$$

$$1 - F(R) = \frac{Qh}{p\lambda}$$

This is the second equation we will use to determine optimal values Q and R

Uniform Dist

h = holding cost , K = Set up cost , P = penalty cost (shortage)

Ör

Example

$\lambda = 800$

- A company purchases air filters at a rate of 800 per year
- $10 to place an order (set up)
- Unit cost is $25 per filter
- Inventory carry cost is $2/unit per year
- Shortage cost is $5
- Lead time is 2 weeks
- Assume demand during lead time follows a uniform distribution from 0 to 200
- Find (Q,R)

D = 800 pc/h	γ = 2 weeks
K = 10$	x ~ U(0,200) (uniformly dist.)
C = 25 $/pc	
h = 2 $/pc	
p = 5 $/pc	

Finding Q and R, iteratively

1. Compute Q = EOQ.
2. Substitute Q in to Equation (2) and compute R.
   $$1 - F(R) = \frac{Qh}{p\lambda}$$
3. Use R to compute n(R) in Equation (1).
   $$n(R) = \int_R^\infty (x - R) f(x)\,dx$$
4. Solve for Q in Equation (1).
   $$Q = \sqrt{\frac{2\lambda[K + p\,n(R)]}{h}}$$
5. Go back to Step 2, continue until convergence.

– Iteration Stepleri Bunlar –

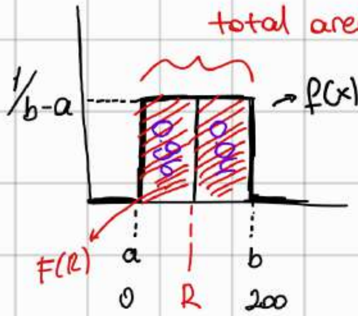
⇒ Bu stepler izlenerek Q ve R bulunacak!

$$Q_0 = \sqrt{\frac{2 \cdot \overset{10}{K} \cdot \overset{800 \cdot \lambda}{D}}{\underset{2}{h}}} = 89.44 \quad \text{(EOQ formulation)}$$

$$\hookrightarrow 1 - F(R) = \frac{Q \cdot h}{p \cdot D} = \underbrace{\left(\frac{\cancel{89.44}^{\,93.76 \text{ (yeni iter.)}} \times 2}{5 \times 800} \right)}_{0.04} \quad \text{(step 2)}$$

$$\hookrightarrow F(R) = 0.96$$

total area = 1



$X \sim U(0,200)$

$$\begin{matrix} 1 & & 200 \times h \\ & \times & \\ 0.96 & & (R-0) \times h \end{matrix}$$

(Bütün dikdörtgenin alanı)

$$\boxed{R = 0.96 \times 200} = 192 \ (R_1) \rightarrow R \text{ buldum!}$$

(2. iteration'da yeni bir "R" bulunacak)

Aşağıda bulduğum 0.198'i yaz Q_1'i bul! $\rightarrow \boxed{Q_1 = 93.76}$

$$Q_1 = \sqrt{\frac{2 \times 800 \times (10 + 5 \times \overset{192}{n(R)})}{2}}$$

$\rightarrow$ expected num. of pieces goes to shortage

(Q bul! → step 4 uygulayarak)

$$n(R) = \int_{R=192}^{\infty} (x-R) \cdot f(x) \, dx \qquad f(x) = \frac{1}{b-a} \qquad U(0,200)$$

$$= \int_R^{200} (x-R) \frac{1}{200} \cdot dx = \frac{1}{200} \left(\frac{x^2}{2} - Rx \Bigg|_{x=R}^{x=200} \right) = \frac{1}{200} \left(\frac{200^2}{2} - 200R - \frac{R^2}{2} + R^2 \right)$$

$$= 100 + \frac{R^2}{400} - R \rightarrow n(R) \text{ func.}$$

192 koy

* n(R) değerini bul!

$$n(R) = 0.198$$

↳ Bunu step 4'te denklemde yerine koyduğumda $Q_1 = 93.76$ çıktı

$$n(R_0) = \frac{(192)^2}{400} - 192 + 100 = .198$$

$$Q_1 = \sqrt{8000 + 4000(.198)} = 93.76$$

$$1 - F(R_1) = \frac{94}{2000} = .05$$

$$R_1 = (.95)(200) = 190$$

yeni buld. "R"

$Q_1 = 93.76$'yı kullanarak R'a geçme process'ine devam et!

1) $n(R) = \dfrac{R^2}{400} - \overset{190}{R} + 100$

2) Yeni $n(R)$ value bul

3) $Q_1 = \sqrt{8000 + 4000\, n(R)}$

4) $1 - F(R) = \dfrac{Q}{2000}$

$$\boxed{S_S = R - \lambda T}$$

Demand'in lead time'a göre revize edilmiş hâli

Soruda vereceği expected demand

Ss level (üst sınır)

→ Iteration 3 :

$$n(R_1) = \frac{190^2}{400} - 190 + 100 = .2197$$

Q'ler eşit

$$Q_2 = \sqrt{8000 + 4000(.2197)} = 94.228$$

$$(1 - F(R_2)) = \frac{94}{2000} = .05$$

$$R_2 = 190$$

Bi önceki iteration'da da 190'dı. 190 = 190 ✓

Stop!

$$n(R) = \frac{R^2}{400} - R + 100$$

$$Q_1 = \sqrt{8000 + 4000\, n(R)}$$

$$1 - F(R) = \frac{Q}{2000}$$

! R'lar eşit çıktığında iteration'lara devam etme !

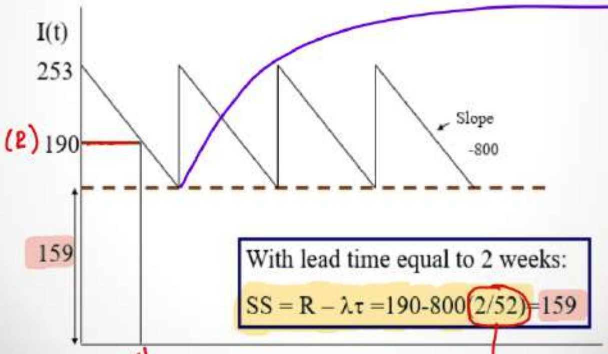
Çünkü bunun Q'ları da eşit çıkacak

⇒ Solution :

Q = 94,228 R = 190

Solution

- R didn't change => CONVERGENCE
- $(Q^*,R^*) = (94,190)$



→ safety stoğa çarptığı an siparişler geldi "Q^* kadar" (94,...)

yıllık baz aldığımız için mi?

ÖR : slayt 80

Example

- Demand is Normally distributed with mean of 40 per week and a weekly variance of 8
- The ordering cost is $50
- Lead time is two weeks
- Shortages cost an estimated $5 per unit short to expedite orders to appease customers
- The holding cost is $0.0225 per week
- Find (Q,R)

→ yine EOQ ile başka process'e

Demand → $N(40, 2\sqrt{2})$ per week

* Lead time is two weeks. During the lead time :

Mean demand $= 2 \cdot (40) = 80$

Variance is $= 2 \cdot (2\sqrt{2})^2 = 16$

! Soru per week cinsinden old. için her şey aylığa çevrilir.
Eğer demeseydi "annual"a çevirirdik.

Annual demand gerekseydi $= D = 40 \times (52 \text{ week})$

→ Normal Dist.

aylık ortalama demand

ÖR: $X \sim N(28, 8)$ → The distribution of monthly demand is normally distributed with mean and standart deviation.

Q^*, R^* ?

✓ $T = 14$ week (3.5 ay)

$h =$ %30 annual rate of interest diyor → $6\$ \times 0.3 = 1.8\$$

✓ $k = 15\$$

✓ $P = 10\$$ (unsatisfied demand cost)

$c = 6\$$ (unit cost)

① $EOQ = \sqrt{\dfrac{2 \cdot k \cdot D}{h}}$ — formüldeki demand "yıllık" cinsten

initial → $Q_0 = \sqrt{\dfrac{2 \cdot 15 \cdot (12 \times 28)}{1.8}} \cong 75$ pc

② R bul!

3. adımda bulduğun yeni Q'yu yaz ve yeni R bul! (R_2)

$$1 - F(R) = \frac{h \times Q}{p \cdot D} = \frac{1.8 \times 75}{10 \times (12 \times 28)}$$

$$F(R) = 0.96$$

$$P(x \leq R) = P\left(\frac{x-\mu}{\sigma} \leq \frac{R-\mu}{\sigma}\right) = 0.96$$

μ: $28 \times 3{,}5 = 98$

σ: 15 $(8 \times \sqrt{3.5})$

1.75 (z-tablosu)

↳ $R_1 = 124$

3) Q_1'i bul!

$$Q_1 = \sqrt{\frac{2 \times D \times (k + p \cdot n(R))}{h}}$$

Bunu bulmam lazım

$$n(R) = \underbrace{\int_{R=124}^{\infty} (x-R) \cdot f(x) \cdot dx}$$

→ Normal Dist. ise bu integrali kolaylaştırmak için direk "Lost func." cinsinden yazabiliriz.
↳ Sonra lost table'a bakılır.

$$n(R) = \boxed{\sigma \times \text{Lost function}\left(\overbrace{\frac{R-\mu}{\sigma}}^{z}\right)} = 0.24 \ (n(R))$$

σ (st. dev.)

(kolaylık olsun diye)

tablodaki iç değer: 0.03

↳ Burdan R_2'ye git.

R_1 ve R_2 aynı mı bak!

⚠ O zmn Normal varsa:

→ R'ı bulmak için "z Table"

→ Q'yu bulmak için "Lost Table" kullanılır.

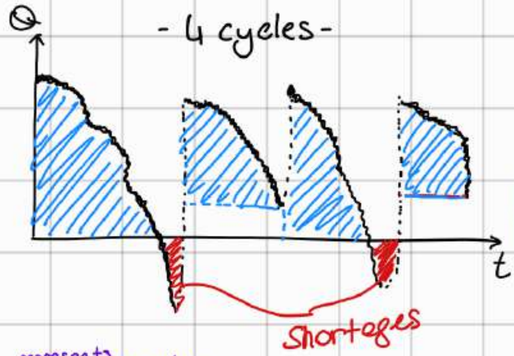
⚠ Normal ve Uniform'daki tek fark n(R) hesaplanışı.

Service Level

↳ Type I (α)

↳ Type II (β)

Consider how many of my cycles in the shortage area and how many of them are in satisfied area



$$\boxed{\alpha = \frac{2}{4}} \text{ (cycles)}$$

shortage cycle

$$\boxed{\beta = \frac{\Sigma(\text{blue})}{\Sigma(\text{blue}) + \Sigma(\text{red})}}$$

satisfied demand

unsatisfied demands

(products) (satisfied demands)

demand satisfaction ratio

(Q, R)

Q	R
$Q = E.OQ \rightarrow$	R_1
Q	

$\sqrt{\dots n(R)}$

(Q formülü)

represents excess value

$$1 - F(R) = \frac{h \cdot Q}{p \cdot D}$$

$\underline{P(x < R)}$

$P(x > R) \Rightarrow$ shortage (1 - satisfied demands)

$\boxed{1-\alpha} = 1 - F(R)$

n(R) kullanılıp Q bulunabilir.

Mesele SS ise

SS (ratio)

* $n(R) \sim (\beta)$ → releated with pieces
* $F(R) \sim (\alpha)$ → releated with whole picture (long-term observation)

where is the satisfaction?

* $\underbrace{F(R)}$ represents no stock-out region.
* $P(x \leq R)$ → for this cycle there is no stock out.

$$F(R) = \alpha$$

↳ Q, R bulurken F(R) olan formülde yazarsın direk uğraşmazsın.

Örneğin Stock out %90 of the order cycles.

$= \boxed{\alpha = 0.9}$

Örneğin 90 percent of filling gate or demand satisfaction.

↳ $\beta = 0.90$

$$n(R) = 1 - \beta$$

→ Bunu yapabilirsem lost function kullanmama gerek kalmaz hiç!

↳ Formüllerde n(R) olan yerlere yazarsın direk.

Agg. Planning Soruları

ÖR:

1) Mr. Meadows Cookie Company makes a variety of chocolate chip cookies in the plant in Albion, Michigan. Based on orders received and forecasts of buying habits, it is estimated that the demand for the next four months is 850, 1,270, 520, ~~and~~ 995, expressed in thousands of cookies. During a 46-day period when there were 120 workers, the company produced 1.7 million cookies.

Assume that the number of workdays over the four months ~~are respectively~~ 37 26, 24, 20, and 16. There are currently 100 workers employed, and there is no starting inventory of cookies.

a. Find a plan that aims zero inventory with hiring and lay offs to meet demand over the next four months?

b. Assume that c_I = 10 cents per cookie per month, c_H = \$100, and c_F = \$200. Evaluate the cost of the plan derived in part (a).

c. Formulate as a linear program. Be sure to define all variables and include the required constraints.

d. Solve for the optimal solution.

a) Let x be the number of cookies made by a worker in one day then

$46*120*x=1.7*10^6 \quad x=308$

W0	100	workers
K	308	per worker/day
hiring cost	100	per worker
firing cost	200	per worker
holding cost	0.1	per cookie per month

Month	Working Days	Demand	Worker Required	Rounded	# hired	# fired	cum. Net demand	monthly prod.	cum. Prod	ending inventory
1	26	850000	106.1438561	107	7		850000	856856	856856	6856
2	24	1270000	171.8073593	172	65		2120000	1271424	2128280	8280
3	20	520000	84.41558442	85		87	2640000	523600	2651880	11880
4	16	995000	201.9074675	202	117		3635000	995456	3647336	12336
				Total units	189	87				39352

b) Total # hires = 7 + 65 + 117 = 189

Total # fires = 87

Cost of this plan = (100)(189) + (200)(87) + (0.1)(39,5352) = \$40,235.2

c) Decision Variables

H_t: number of workers hired at the beginning of month t

F_t: " " " fired " " " " " "

W_t: " " " available at the beginning of month t.

I_t: inventory left over at the end of month t.

P_t: number of cookies produced at month t. $t=1,2,3,4$

Parameters

C_I = Inventory holding cost for per cookie per month = \$0.10

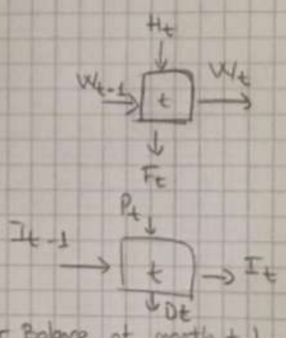
C_H = Hiring cost of a worker = \$100

C_F = Firing " " " " = \$200

wd_t: working day number at month t

D_t: demand for cookies at month t.

Model



$$\min \sum_{t=1}^{4} (H_t * C_H + F_t * C_F + I_t * C_I)$$

subject to

$W_{t-1} + H_t = W_t + F_t \quad \forall t$ (Worker Balance at month t)

$308 * wd_t * W_t = P_t \quad \forall t$ (Total cookies produced)

$P_t + I_{t-1} = D_t + I_t \quad \forall t$ (Inventory balance equation)

$W_0 = 100$ (Number of workers at the beginning)

$I_0 = 0$ (No cookies on hand at the beginning)

$W_t, I_t, P_t \geq 0 \quad \forall t$ (Sign / Type Restrictions)

$H_t, F_t \in Z^+$ (" / " ")

d) According to Gams result (you can try using OPL)

Total # hires = 7 + 63 + 100 = 170

Total # fires = 80

Total Inventory= 6856+3496+47896+4216=62464

Total Cost = (100)(170) + (200)(80) + (0.1)(62464) = \$39,246.4

OR

2) Harold Grey owns a small farm in the Salinas Valley that grows apricots. The apricots are dried on the premises and sold to a number of large supermarket chains. Based on past experience and committed contracts, he estimates that sales over the next five years in thousands of packages will be as follows:

Year	Forecasted Demand (thousands of packages)
1	300
2	120
3	200
4	110
5	135

b) According to Gams result (you can try using OPL)

Total # hires = 7 + 1 = 8

Total # fires = 5+1=6

Total Inventory= 105000

Total Cost = $739,200

Assume that each worker stays on the job for at least one year, and that Grey currently has three workers on the payroll. He estimates that he will have 20,000 packages on hand at the end of the current year. Assume that, on the average, each worker is paid $25,000 per year and is responsible for producing 30,000 packages. Inventory costs have been estimated to be 4 cents per package per year, and shortages are not allowed. Based on the effort of interviewing and training new workers, Farmer Grey estimates that it costs $500 for each worker hired. Severance pay amounts to $1,000 per worker.

a. Formulate this as a linear program.

b. Solve the problem and round-off the solution and determine the cost of the resulting plan.

a)

1) Decision Variables

H_t: number of workers hired at the beginning of year t.

F_t: " " " fired " " " " " ".

W_t: " " " available " " " " " ".

I_t: inventory left over at the end of year t.

P_t: number of packages produced at year t t=1,2,3,4,5

Parameters

c_I: inventory holding cost for per cookie per year = $0.04

c_H: hiring cost of a worker = $500

c_F: firing cost of a worker = $1000

D_t: Demand for cookies at year t.

wp: salary of a worker = $25,000 per year.

2) Model

$$\min \; z = \sum_t \left(c_H{}^* H_t + c_F{}^* F_t + c_I{}^* I_t + wp^* W_t\right)$$

subject to

$W_{t-1} + H_t = W_t + F_t + W_t$ (Worker Balance in year t)

$30000^* W_t = P_t \quad \forall t$ (Total production in year t)

$P_t + I_{t-1} = D_t + I_t \quad \forall t$ (Inventory Balance in year t)

$W_0 = 3$ (Beginning worker number)

$I_0 = 20000$ (Beginning inventory)

$W_t, I_t, P_t \geq 0$, $H_t, F_t \in \mathbb{Z}^+$ (Sign/Type Restrictions)

ÖR 8

3) The Paris Paint Company is in the process of planning labor force requi production levels for the next four quarters. The marketing department has provide with the following forecasts of demand for Paris Paint over the next year:

Quarter	Demand Forecast (in thousands of gallons)
1	380
2	630
3	220
4	160

Assume that there are currently 280 employees with the company. Employees are hired for at least one full quarter. Hiring costs amount to $1,200 per employee and firing costs are $2,500 per employee. Inventory costs are $1 per gallon per quarter. It is estimated that one worker produces 1,000 gallons of paint each quarter.

Assume that Paris currently has 80,000 gallons of paint in inventory and would like to end the year with an inventory of at least 20,000 gallons.

a. Determine the minimum constant workforce plan for Paris Paint and the cost of the plan. Assume that stock-outs are not allowed.

b. If Paris were able to back-order excess demand at a cost of $2 per gallon per quarter, determine a minimum constant workforce plan that holds less inventory than the plan you found in part (*a*), but incurs stock-outs in quarter 2. Determine the cost of the new plan.

c. Formulate this as a linear program. Assume that stock-outs are not allowed.

d. Solve the linear program. Round the variables and determine the cost of the resulting plan.

3) a) Since there are 80,000 gallons of paint at beginning 380-80=300(in thousands) can be taken as net demand for the first quarter. Similarly, 20,000 gallons are needed asending inventory so demand for quarter 4 can be updated as 160+20=180(in thousands).

Quarter	Unit/Worker (000)	(000) Net Demand	Cum Net Dem.	Min. Work Force
1	1	300	300	300
2	1	630	930	465(930/2)
3	1	220	1150	384(1150/3)
4	1	180	1330	333(1330/4)

Bizim production kabul ettiğimiz sayı işçi olarak kabul etmiş ve production'ı fixlediği için worker sayısını da fixlemiş.

Hence the min. constant workforce is 465 workers.

The cost of the resulting plan is:

Quarter	Cum Production (000)	Cum Net Demand (000)	Ending Inventory (000)
1	465 +465	300	165
2	930 +465	930	0
3	1395 +465	1150	245
4	1860	1330	530
		Total	940

We must also add back in the 20,000 required to be on hand in the fourth quarter. Hence the total cost of this plan is:

$(1,200)\underbrace{(465 - 280)}_{\text{işe alınan sayısı}} + (\$1)(1000)(940 + 20) = \$1,182,000.$

This is (1)(1000) since production and demand are expressed in (000) of cans.

b) If we use the minimum number of workers required through period 3 of 1150/3 = 384, it will satisfy the conditions stated.

Quarter	Cum Prod	Cum Net Dem.	Ending Inv.
1	384	300	84
2	768	930	~~-162~~
3	1152	1150	2
4	1536	1330	206

206
84
20
+ 2
312

(1\$.1000) 2\$.(1000)

Total cost = (1,200)(384 - 280) + (1,000)(312) + $\underbrace{(2,000)(162)}_{\text{Excess demand için}}$ = \$760,800.

Better than policies in parts (a) or (b).

c)

1) Decision Variables

H_t : Number of workers hired at the beginning of quarter t.

F_t : " " " fired " " " " " "

W_t : " " " available " " " " " "

I_t : inventory left over at the end of quarter t.

P_t : number of gallons produced at quarter t   t=1,2,3,4

Parameters

c_I = \$1 per gallon per quarter

c_H = \$ 1200

c_F = \$ 2500

D_t : demand of quarter t

Model

min $z = \sum_t (c_H * H_t + c_F * F_t + c_I * I_t)$

subject to

$W_{t-1} + H_t = W_t + F_t \quad \forall t$ (Worker balance at quarter t)

$1000 * W_t = P_t \quad \forall t$ (Total production at quarter t)

$P_t + I_{t-1} = D_t + I_t \quad \forall t$ (Inventory balance equation)

$W_0 = 280$ (Number of workers at beginning)

Instead of writing (I_0 and I_5 update D_1 and D_4 as

$D_1 = 300000$ and $D_4 = 180000$

$W_t, I_t, P_t \geq 0 \quad H_t, F_t \in \mathbb{Z}^+ \quad \forall t$ (Sign/Type Restrictions)

Ö2 4

Quarter	Demand	Previous quarter's output	1425 units
1	1500	Beginning Inventory	100 units
2	1800	Backordering Cost	$120 per unit
3	1600	Inventory Holding Cost	$10 per quarter/unit
4	1200	Hiring Workers	$50 per unit
		Laying Off Workers	$90 per unit
		Production Cost	$20 per unit

`the following production plans is better:

a. Plan A–chase demand by hiring and layoffs

b. Plan B–level strategy and backordering

c. Plan C–level strategy and subcontracting (subcontracting cost is $60 per unit

alculate the total cost of each production plan.

- cumulative yapmamış -

4) a) **Chase Strategy by hiring and layoffs:**

işe alım

Since beginning inventory is 100, demand for first quarter can be taken as 1500-100= 1400.

işten çıkarma

Quarter	Demand	Production	Units Increase	Units Decrease
1	1400	1400	0	25
2	1800	1800	400	0
3	1600	1600	0	200
4	1200	1200	0	400
Total Units		6000	400	625

Total Cost = Production Cost + Increasing Cost + Decreasing Cost

Total Cost = 6000*20 + 400*50 + 625*90 = $196,250

b) Level strategy and backordering

6000/4=1500 min. production to satisfy total demand during four quarter.

1500 - 1425 = 75 işe alım

Quarter	Demand	Production	Inventory	Backorder	Units Increase	Units Decrease
1	1400	1500	100	0	75	0
2	1800	1500	0	200	0	0
3	1600	1500	0	100	0	0
4	1200	1500	0	0	0	0
Total Units	6000	6000	100	300	75	0

Total Cost = Production Cost + Inventory Cost + Backorder Cost + Increasing Cost

Total Cost = 6000*20 + 100*10 + 300*120 + 75*50 = $160,750

c) Level strategy and subcontracting (subcontracting cost is $60 per unit)

1425-1200

Quarter	Demand	Production	Inventory	Subcontracting	Units Increase	Units Decrease
1	1400	1200	0	200	0	225
2	1800	1200	0	600	0	0
3	1600	1200	0	400	0	0
4	1200	1200	0	0	0	0
Total Units		4800	0	1200	0	225

Total Cost = Production Cost + Subcontracting Cost + Decreasing Cost

Total Cost = 4800*20 + 1200*60 + 225*90 = $188,250

Based on cost of all the plans, Plan B is better.

ÖR 5 6

5) XYZ Company has the following aggregate demand requirements for the upcoming four quarters.

Quarter	Demand
Quarter 1	1750
Quarter 2	2050
Quarter 3	2750
Quarter 4	1450

Costs/Other Data
Previous quarter's output = 1,300 cases
Beginning inventory = 0 cases
Stockout cost = $150 per case
Inventory holding cost = $40 per case at end of quarter
Hiring employees = $40 per case
Terminating employees = $80 per case
Subcontracting cost = $60 per case
Unit cost on regular time = $30 per case
Overtime cost = $15 extra per case
Capacity on regular time = 1,800 cases per quarter

Develop an aggregate production plan using:

a. Chase Demand strategy

b. Level (constant workforce) strategy and backordering. Calculate the total cost of each production plan.

5) a) **Chase strategy:**

Previous quarter's output is 1300 and first quarter's production is 1750, so 1750-1300=450 will be the units increase in producion in first quarter.

yetersizlik durumları var (sub, backorder, overtime... hangisinin cost'u verildiyse onu alırsın!)

Quarter	Demand	Capacity	Regular Production	Overtime Production	Units Increase	Units Decrease
1	1750	1800	1750	0	450	0
2	2050	1800	1800	250	50	0
3	2750	1800	1800	950	0	0
4	1450	1800	1450	0	0	350
Total Units			6800	1200	500	350

Total Cost = Regular Time Production Cost + Overtime Production Cost + Increasing Cost + Decreasing Cost

Total Cost = 6800*30 + 1200*15 + 500*40 + 350*80 = $270,000

b) **Level (constant workforce) strategy and backordering:**

Previous quarter's output is 1300 and first quarter's output is 1800, so 1800 -1300 =500 will be the units increase in production in first quarter.

Backorder kabul etmeyi söylemiş soruda

Quarter	Demand	Capacity	Regular Production	Backorder	Inventory	Units Increase	Units Decrease
1	1750	1800	1800	0	50	500	0
2	2050	1800	1800	200	0	0	0
3	2750	1800	1800	950	0	0	0
4	1450	1800	1800	0	350	0	0
Total Units	8000		7200	1150	400	500	0

8000/4=2000 min production to satisfy total demand during four quarter.

Since regular production capacity = 1800 < 2000 and backorder must be used in this strategy regular production will be taken as 1800 and backorder will take place when needed.

Total Cost = Regular Time Production Cost + Backorder Cost + Inventory Holding Cost + Increasing Cost

Total Cost = 7200*30 + 1150*150 + 400*40 + 500*40 = $424,500