İZMİR UNIVERSITY OF ECONOMICS

IZMIR UNIVERSITY OF ECONOMICS
Faculty of Engineering

Term	**: 2024-2025 Fall**
Course ID	**: EEE 301 / EEE 309**
Exam	**: Midterm Exam**
Date	**: 12 Nov 2024**
Duration	**: 90 min**
Instructor	**: Prof. Aydin Akan, Assoc. Prof. Mehmet Turkan, Asst. Prof. Faezeh Yeganli**

Full Name : SOLUTION KEY

Student ID : ....

Classroom : .... **Section** : ....

Information on exam rules

Electronic devices such as laptops, mobile phones, and smartwatches are generally prohibited in the examination room. However, exceptions can be made for individuals with special needs, provided they have valid medical documentation. Requests for exceptions must be submitted with prior written approval from the academic advisor, and they should include details on the necessary measures to maintain the integrity and security of the examination.

Please refrain from engaging in cheating or any other prohibited activities during the examination. Suspected cheating may result in a score of zero on your exam, and any students found cheating may face disciplinary actions in accordance with law #2547. This includes actions such as using unauthorized electronic devices, communicating with classmates, exchanging exam or formula sheets, or using unauthorized written materials during the exam, all of which qualify as attempted cheating.

This is a closed-book and closed-notes exam. Calculators, laptops, mobile phones are not permitted.

Declaration

I affirm that the activities and assessments completed as part of this examination are entirely my own work and comply with all relevant rules regarding copyright, plagiarism, and cheating. I acknowledge that if there is any question regarding the authenticity of any portion of my assessment, I may be subject to oral examination. The signatory of evidence records may also be contacted, or a disciplinary process may be initiated as per law #2547.

Signature of Student:

Question	1	2	3	4	5	6	7	8
Score	/ 20	/ 20	/ 30	/ 30	-	-	-	-
Total	/ 100							

Convolution Integral:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Convolution Sum:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Useful identities (CT):

$$Acos(\omega t + \theta) = \frac{A}{2}\left(e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right)$$
$$Asin(\omega t + \theta) = \frac{A}{2j}\left(e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}\right)$$
$$Ae^{j(\omega t+\theta)} = Acos(\omega t + \theta) + jAsin(\omega t + \theta)$$

Useful identities (DT):

$$Acos(\omega n + \theta) = \frac{A}{2}\left(e^{j(\omega n+\theta)} + e^{-j(\omega n+\theta)}\right)$$
$$Asin(\omega n + \theta) = \frac{A}{2j}\left(e^{j(\omega n+\theta)} - e^{-j(\omega n+\theta)}\right)$$
$$Ae^{j(\omega n+\theta)} = Acos(\omega n + \theta) + jAsin(\omega n + \theta)$$

Finite Sum Formula:

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & \text{if } \alpha = 1 \\ \dfrac{1-\alpha^N}{1-\alpha} & \text{if } \alpha \neq 1 \end{cases}$$

Infinite Sum Formula:

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad \text{for } |\alpha| < 1$$

Q1 (LO1). Determine whether the following signals are **periodic**. If they are periodic, calculate their **fundamental period** (N_0 or T_0), **fundamental frequency** f_0 and **angular frequency** ω_0.

a) $x[n] = 5 + \cos\left(\frac{3\pi}{4}n + \frac{\pi}{2}\right) + cos\left(\frac{\pi}{3}n\right)$

b) $x(t) = j^t + e^{j\frac{4\pi}{5}t}$

a) 5 is constant. It does not effect the periodicity.

$\omega_1 = \frac{3\pi}{4}$, $N_1 = \frac{2\pi}{3\pi/4} = \frac{8}{3} \times 3 = 8$ samples.

$\omega_2 = \frac{\pi}{3}$, $N_2 = \frac{2\pi}{\pi/3} = 6$ samples.

$LCM(8, 6) = 2.2.2.3 = 24$ samples.

$8 = 2^3$, $6 = 2.3$

$x[n]$ is periodic with $N_0 = 24$ samples.

$f_0 = \frac{1}{24}$ Hz.

$\omega_0 = 2\pi . f_0 = \frac{\pi}{12}$ rad/sample.

b) $j^t = e^{j\frac{\pi}{2}t}$, $\omega_1 = \frac{\pi}{2}$ rad/sec.

$T_1 = \frac{2\pi}{\pi/2} = 4$ sec.

$e^{j\frac{4\pi}{5}t}$, $\omega_2 = \frac{4\pi}{5}$ rad/sec.

$T_2 = \frac{2\pi}{4\pi/5} = \frac{5}{2}$ sec.

$\frac{T_1}{T_2} = \frac{4}{5/2} = \frac{8}{5} \Rightarrow T_0 = 5T_1 = 8T_2 = 20$ sec.

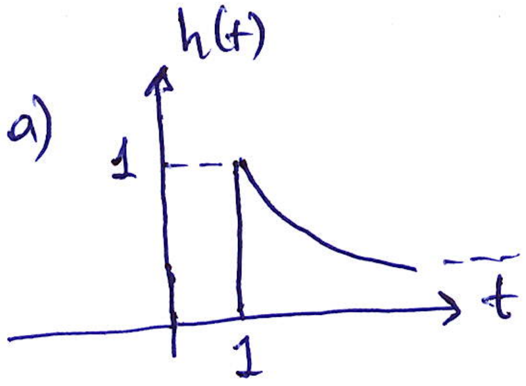
$x(t)$ is periodic with $T_0 = 20$ sec. $f_0 = \frac{1}{20}$ Hz.

$\omega_0 = 2\pi f_0 = \frac{\pi}{10}$ rad/sec.

Q2 (LO2). The impulse responses of two LTI systems are given below. Determine whether each system is **causal, stable** and **memoryless.**

a) $h(t) = e^{-(t-1)}u(t-1)$
b) $h[n] = u[n+1]$

a)



$h(t) = 0 \ , \ t < 0$. Causal.

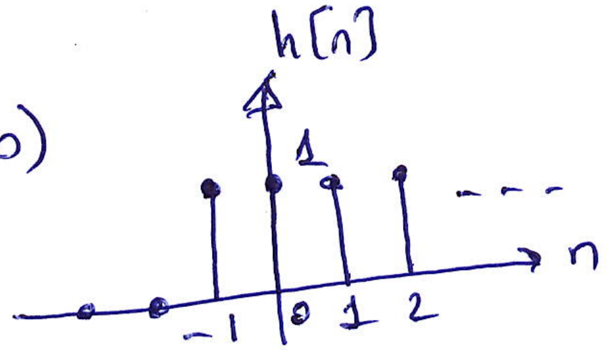
$h(t) \neq K\delta(t)$. With memory.

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{1}^{\infty} e^{-(t-1)}\,dt$$

$$= \frac{1}{-1} e^{-(t-1)} \Big|_{1}^{\infty} = 1 < \infty$$

Stable.

b)



$h[n] \neq 0 \ , \ n < 0$. Not causal.

$h[n] \neq K\delta[n]$. With memory.

$$\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{-1}^{\infty} 1 = \infty.$$

Not stable.

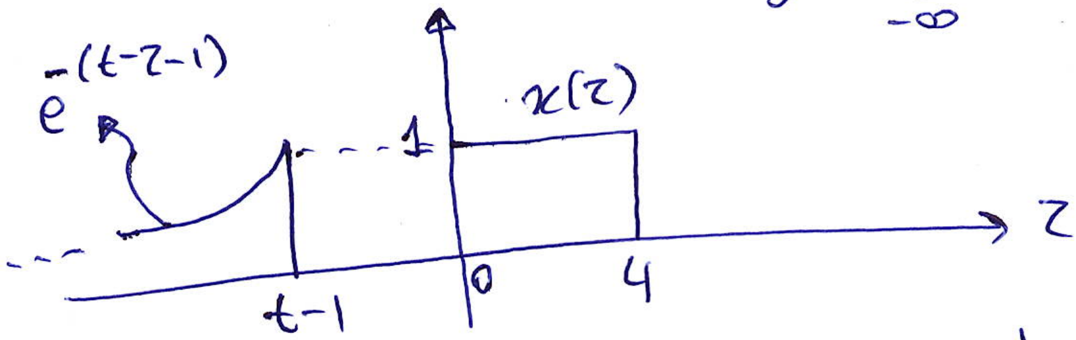
Q3 (LO5). LTI systems are given below. Calculate the output signals for the given inputs.

a) $h(t) = e^{-(t-1)}u(t-1)$ and $x(t) = u(t) - u(t-4)$

b) $h[n] = \delta[n+1]$ and $x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau$$

a)



Case 1. $t-1<0$, $t<1$, $y(t)=0$. No overlap.

Case 2. $0 \le t-1 < 4$, $1 \le t < 5$

$$y(t) = \int_0^{t-1} e^{-(t-\tau-1)}\,d\tau = e^{-t+1} \cdot e^{\tau}\Big|_0^{t-1}$$

$$= e^{-t+1}\left[e^{t-1} - 1\right] = 1 - e^{-t+1}$$

Case 3. $t-1 \ge 4$, $t \ge 5$

$$y(t) = \int_0^{4} e^{-(t-\tau+1)}\,d\tau = e^{-t+1} \cdot e^{\tau}\Big|_0^{4}$$

$$= e^{-t+1}\left[e^{4} - 1\right] = e^{-t+5} - e^{-t+1}$$

b) $x[n] * \delta[n+1] = x[n+1]$

$$y[n] = x[n+1] = \left(\frac{1}{3}\right)^{-(n+1)} u[-(n+1)-1]$$

$$y[n] = \left(\frac{1}{3}\right)^{-(n+1)} u[-n-2].$$

Q4 (LO3). Consider the continuous-time system: $y(t) = x(t-3) + x(3-t)$.

a) Is this system **memoryless?** Justify your answer.
b) Is this system **causal?** Justify your answer.
c) Is this system **BIBO stable?** Justify your answer.
d) Is this system **time-invariant?** Justify your answer.
e) Is this system **linear?** Justify your answer.
f) Is this system LTI? If so, find **the impulse response** $h(t)$ of the system. If not, justify your answer.

$y(t) = x(t-3) + x(-t+3)$

a) The system is with memory because output depends on the past and future values of the input.

E.x. $y(0) = x(-3) + x(+3)$

b) The system is not causal because output depends on the future values of the input.

c) Assume $|x(t)| \leq A < \infty$, $|y(t)| = |x(t-3) + x(3-t)|$

$|y(t)| \leq \underbrace{|x(t-3)|}_{\leq A} + \underbrace{|x(-t+3)|}_{\leq A} \leq 2A < \infty$. stable.

d) input | output

$x_1(t)$ → $y_1(t) = x_1(t-3) + x_1(-t+3)$

$x_2(t) = x_1(t-t_0)$ → $y_2(t) = x_2(t-3) + x_2(-t+3)$

Check if $y_2(t) \stackrel{?}{=} y_1(t-t_0)$

$x_1(t-3-t_0) + x_1(-t+3-t_0) \stackrel{?}{=} x_1(t-t_0-3) + x_1(-t+t_0+3)$

Not time-invariant (time-varying)

e) input

$x_1(t)$

$x_2(t)$

$x_3(t) = a x_1(t) + b x_2(t)$

output

$y_1(t) = x_1(t-3) + x_1(-t+3)$

$y_2(t) = x_2(t-3) + x_2(-t+3)$

$y_3(t) = x_3(t-3) + x_3(-t+3)$

Check if $y_3(t) \overset{?}{=} a y_1(t) + b y_2(t)$

$a x_1(t-3) + b x_2(t-3) + a x_1(-t+3) + b x_2(-t+3)$

$\overset{?}{=} a x_1(t-3) + a x_1(-t+3) + b x_2(t-3) + b x_2(-t+3)$

✓

Linear

f) System is not LTI. It is Linear but Not time-invariant.

$h(t) = \delta(t-3) + \delta(-t+3)$ [Not LTI]