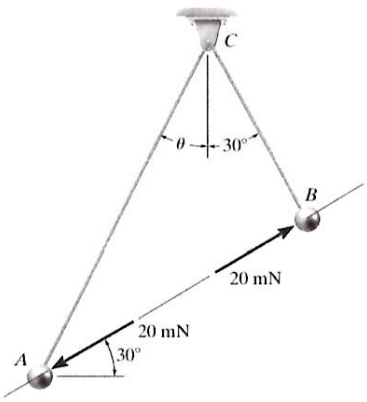
Statics for Engineers
ME 211 & ME 205
Fall 2023-2024
Midterm 1

14.11.2023

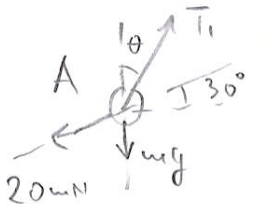
Name:
School Number:

	Q.1(25P)	Q.2(25P)	Q.3(20P)	Q.4(10P)	Q.5(20P)	TOTAL
POINTS						

1. **Two spheres** A and B have an equal mass and are electrostatically charged such **that the** repulsive force acting between them has magnitude of 20 mN and is directed along line AB. Determine the angle θ, the tension in cords AC and BC, **and the** mass of each sphere. Please draw the free-body diagrams of the masses **each (25 POINTS)**.



FBD of A:



(2)

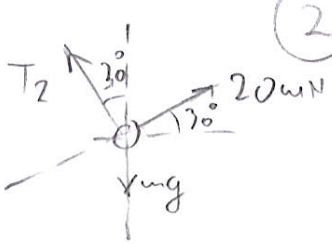
$\rightarrow \sum F_x = 0$ (2)

$T_1 \sin\theta - 20\cos 30^\circ = 0$ (1)

$+\uparrow \sum F_y = 0$ (2)

$T_1 \cos\theta - mg - 20\sin 30^\circ = 0$ (2)

FBD of B:



(2)

$+\rightarrow \sum F_x = 0 : 20\cos 30^\circ - T_2 \sin 30^\circ = 0$ (3) (2)

$+\uparrow \sum F_y = 0 : T_2 \cos 30^\circ + 20\sin 30^\circ - mg = 0$ (4) (2)

$\therefore$ FROM (3) WE HAVE $T_2 \sin 30^\circ = 20\cos 30^\circ$

$\boxed{T_2 = 34.64\ [mN]}$ (2)

$\therefore$ FROM (4), WE HAVE $34.64\cos 30^\circ + 10 - mg = 0 \rightarrow \boxed{40\ [mN] = mg}$ (2)

$$m = \frac{40 \times 10^{-3}\ [N]}{9.81} \simeq 4.077\ 10^{-3}\ kg \equiv 4.077\ g$$

$$\boxed{m = 4.077\ g} \quad (1)$$

FROM (1) AND (2), WE CAN WRITE

$$T_1 \sin\theta = 20 \cos 30^\circ$$
$$T_1 \cos\theta = 20 \sin 30^\circ + mg$$

(4)

$$T_1 \sin\theta = 17.321\ [mN]$$
$$T_1 \cos\theta = 10 + 40 + \text{See above } \boxed{mg \cong 410}\ [mN]$$

Take the ratio of the two:

$$\tan\theta = \frac{17.321}{50} \rightarrow \boxed{\theta = 19.1071^\circ} \quad (2)$$

(∴) use θ in (1)

$$T_1 \sin\theta = 17.32$$

$$T_1 = \frac{17.32}{\sin(19.1071)} = 52.919\ [mN]$$

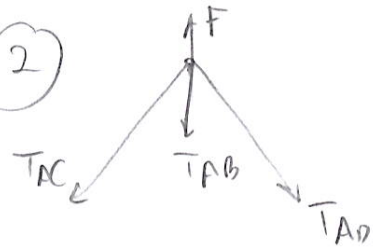
$$\boxed{T_1 = 52.919\ [mN]} \quad (2)$$

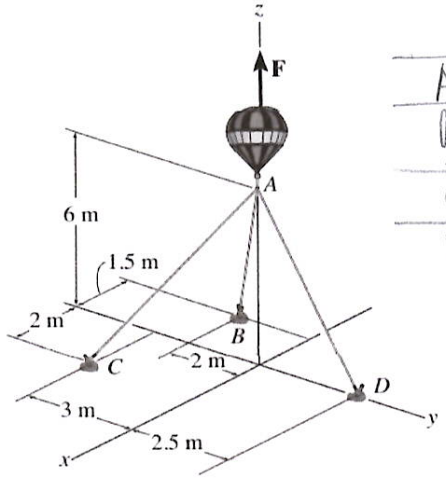
2. If the balloon is subjected to a net uplift force of $F = 750$ N, determine the tension developed in ropes AB, AC, AD. Please draw the free-body diagram of the particle (25 POINTS).

use Cartesian Vectors

FBD of the particle at A:

(2)





	X	Y	Z
A	0	0	6
B	-1.5	-2.0	0
C	2.0	-3.0	0
D	0	2.5	0

(3) $\vec{T}_{AC} = T_{AC}\,\vec{u}_{AC}$

$= T_{AC}\,\dfrac{(2\vec{i} - 3\vec{j} - 6\vec{k})}{\sqrt{49}} = T_{AC}\left(\dfrac{2\vec{i} - 3\vec{j} - 6\vec{k}}{7}\right)$

(3) $\vec{T}_{AB} = T_{AB}\,\vec{u}_{AB}$

$= T_{AB}\,\dfrac{-1.5\vec{i} - 2.0\vec{j} - 6\vec{k}}{\sqrt{1.5^2 + 2^2 + 6^2}}$

$= T_{AB}\left(\dfrac{-1.5\vec{i} - 2.0\vec{j} - 6\vec{k}}{6.5}\right)$

(3) $\vec{T}_{AD} = T_{AD}\,\dfrac{0\vec{i} + 2.5\vec{j} - 6\vec{k}}{\sqrt{2.5^2 + 6^2}}$

$= T_{AD}\left(\dfrac{2.5\vec{j} - 6\vec{k}}{6.5}\right)$

(1) $\vec{F} = 750\vec{k}$

EQUATION OF EQUILIBRIUM WRITES

(2) $\sum \vec{F} = 0 \;:\; \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{F} = 0$

$\left(\dfrac{2T_{AC}}{7}\vec{i} - \dfrac{3T_{AC}}{7}\vec{j} - \dfrac{6T_{AC}}{7}\vec{k}\right) +$

$\left(\dfrac{-1.5T_{AB}}{6.5}\vec{i} - \dfrac{2T_{AB}}{6.5}\vec{j} - \dfrac{6T_{AB}}{6.5}\vec{k}\right) +$

$\left(\dfrac{2.5T_{AD}}{6.5}\vec{j} - \dfrac{6T_{AD}}{6.5}\vec{k}\right) +$

$750\vec{k} = 0$

$\left(\dfrac{2T_{AC}}{7} - \dfrac{1.5T_{AB}}{6.5}\right)\vec{i} +$

$\left(-\dfrac{3T_{AC}}{7} - \dfrac{2T_{AB}}{6.5} + \dfrac{2.5T_{AD}}{6.5}\right)\vec{j} +$

$\left(-\dfrac{6T_{AC}}{7} - \dfrac{6T_{AB}}{6.5} - \dfrac{6T_{AD}}{6.5} + 750\right)$

$= 0$

EQUATE $\hat{i}, \hat{j}, \hat{k}$ COMPONENTS TO ZERO

$\hat{i}$: $\frac{2T_{AC}}{7} = \frac{1.5T_{AB}}{6.5}$ (1) (2)

$\hat{j}$: $-\frac{3T_{AC}}{7} - \frac{2T_{AB}}{6.5} + \frac{2.5T_{AD}}{6.5} = 0$ (2) (2)

$\hat{k}$: $\frac{6T_{AC}}{7} + \frac{6T_{AB}}{6.5} + \frac{6T_{AD}}{6.5} = 750$ (3) (2)

$\hat{i}$: $0.286T_{AC} = 0.231T_{AB}$ $\boxed{T_{AC} = 0.808T_{AB}}$ (2)

$\hat{j}$: $\frac{3T_{AC}}{7} + \frac{2T_{AB}}{6.5} = \frac{2.5T_{AD}}{6.5}$

$0.346T_{AB} + 0.308T_{AB} = 0.385T_{AD}$

$\boxed{0.654T_{AB} = 0.385T_{AD}}$ (2)

$\boxed{T_{AD} = 1.699T_{AB}}$

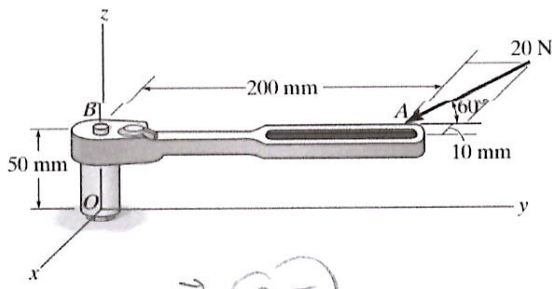
$\hat{k}$: WRITE IN TERMS OF T_{AB}:

$0.693T_{AB} + 0.923T_{AB} + 1.568T_{AB} = 750 \rightarrow \boxed{T_{AB} = 235.55\ N}$

$\boxed{T_{AC} = 190.326\ [N]}$ $\boxed{T_{AD} = 400.2\ [N]}$ (1)

Please use Cartesian vector analysis and

3. **The 20-N** horizontal force acts on the handle of the socket wrench. Determine the **moment** of this force about point O. Specify the coordinate direction angles α, β, γ **of the** moment vector **(20 POINTS)**.



$\vec{F} = 20\sin 60\,\vec{i} - 20\cos 60\,\vec{j}$ (2.5)

$\vec{r} = -0.01\,\vec{i} + 0.2\,\vec{j} + 0.05\,\vec{k}$ (2.5)

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.01 & 0.2 & 0.05 \\ 17.32 & -10 & 0 \end{vmatrix} = +0.5\,\vec{i} - \vec{j}\,(-0.05 - 17.32) + \vec{k}\,(0.1 - 0.2\,17.32)$$

(10)

$$\vec{M} = 0.5\,\vec{i} + 0.866\,\vec{j} - 3.364\,\vec{k} = M_{Ox}\,\vec{i} + M_{Oy}\,\vec{j} + M_{Oz}\,\vec{k}$$

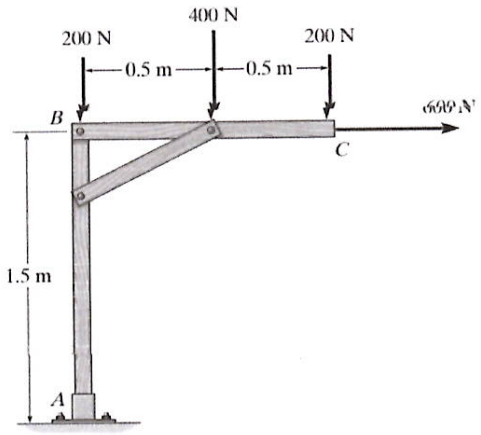
Magnitude of the Moment Vector

(2) $M_O = \sqrt{0.5^2 + 0.866^2 + 3.364^2} = 3.509$ [Nm]

$\cos\alpha = \frac{M_{Ox}}{M_O} = \frac{0.5}{3.509}$; $\cos\beta = \frac{0.866}{3.509}$; $\cos\gamma = \frac{-3.364}{3.509}$

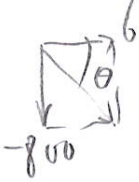
$\alpha = 81.8^\circ$ (1) $\quad \beta = 75.7$ (1) $\quad \gamma = 163.47$ (1)

4. **Replace** the loading on the frame by a single resultant force. Specify where its **line of** action intersects the vertical line along member AB, measured from A (10 **POINTS**).



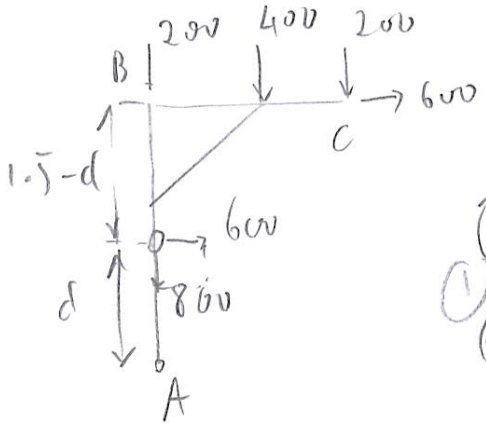
(1) $\overset{+}{\rightarrow} \Sigma F_{R_x} = 600$

$+\uparrow \Sigma F_{R_y} = -800$



$\tan\theta = \frac{800}{600} \rightarrow \theta \cong 53^\circ$ (2)

(2) $F_R = \sqrt{600^2 + 800^2} = 1000\ N$



(1) $\overset{+}{\circlearrowleft} \Sigma M_A^R = \overset{+}{\circlearrowleft} \Sigma M_A$ (1) (1) (1)

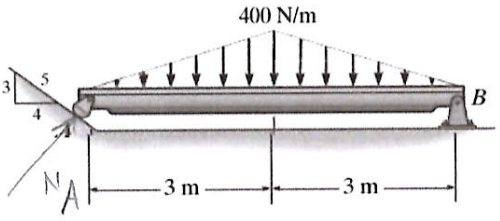
$600\,d = 400(0.5) + 200(1m) + 600(1.5)$

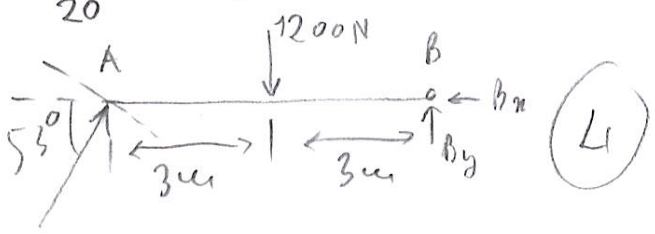
$600\,d = 200 + 200 + 900$

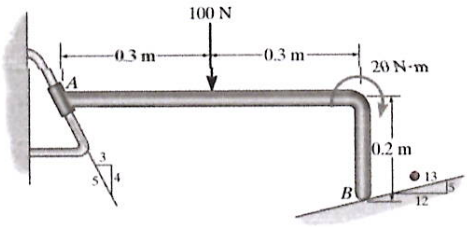
(1) $d = 2.167[m]$

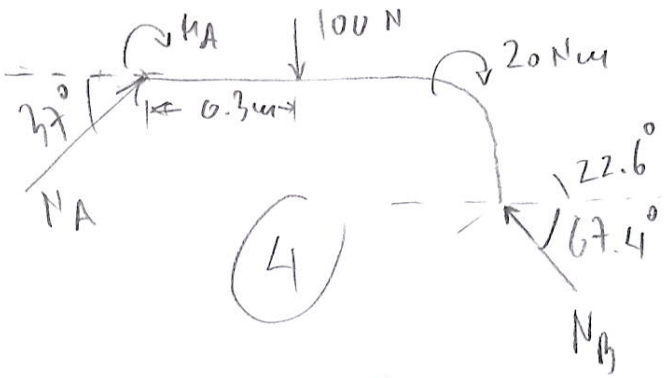
↑ INCLUDE GEOMETRICAL DETAILS AS WELL

5. **Draw** the free-body diagrams (FBD) for the following systems. If the mass of the systems are not given, then it is negligible (~~15~~ 20 **POINTS**).

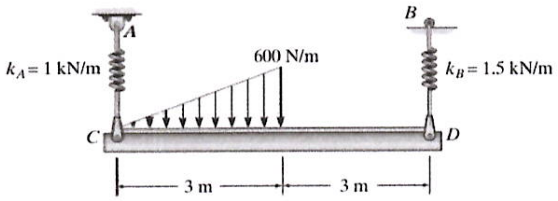


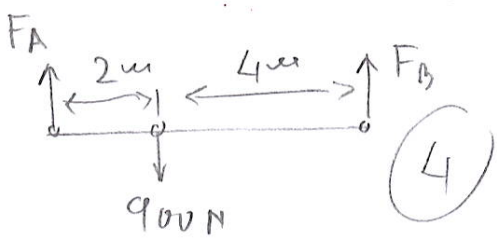






FBD of **rod** CD in horizontal position.





The cylinder hanging on rod AB in the below picture has a mass of 50 kg.