# Statics for Engineers
# ME 211 & ME 205
# Fall 2023-2024
# Midterm 2

18.12.2023

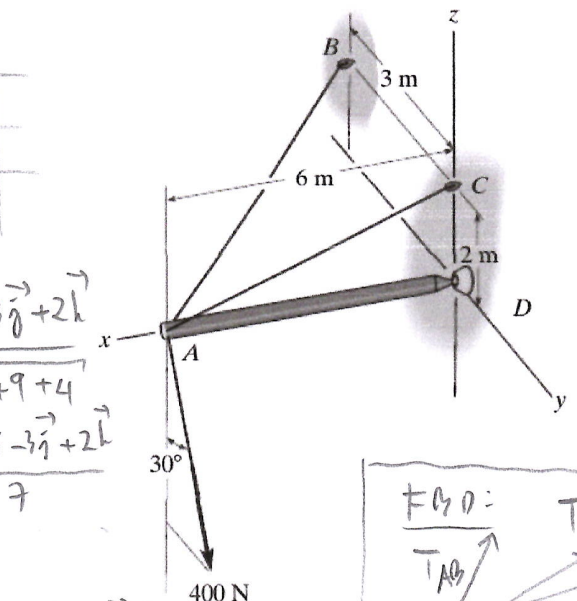
Name:
School Number:

| | Q.1(30P) | Q.2(20P) | Q.3(25P) | Q.4(25P) | TOTAL |
|---|---|---|---|---|---|
| POINTS | | | | | |

1. Determine the tension in each cable and the components of reaction at $D$ needed to support the load. **Draw the proper free body diagram of the structure and use Cartesian Vector Analysis during the solution. Hint: The applied force acts in two planes only (30 POINTS).**

COORDINATES:

| | X | Y | Z |
|---|---|---|---|
| A | 6 | 0 | 0 |
| B | 0 | -3 | 2 |
| C | 0 | 0 | 2 |




$\vec{F} = 400\sin 30\,\vec{j} - 400\cos 30\,\vec{k}$
$= 200\,\vec{j} - 346.41\,\vec{k}$

2

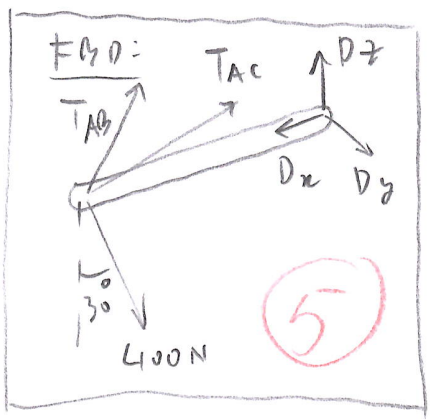
$\vec{u}_{AB} = \dfrac{\vec{r}_{AB}}{|r_{AB}|} = \dfrac{-6\vec{i}-3\vec{j}+2\vec{k}}{\sqrt{36+9+4}}$

$= \dfrac{-6\vec{i}-3\vec{j}+2\vec{k}}{7}$

2

$\vec{T}_{AB} = T_{AB}\,\vec{u}_{AB}$

$= |T_{AB}|\left(-\dfrac{6\vec{i}-3\vec{j}+2\vec{k}}{7}\right)$




5

$\vec{u}_{AC} = \dfrac{\vec{r}_{AC}}{|r_{AC}|} = \dfrac{-6\vec{i}-0\vec{j}+2\vec{k}}{\sqrt{36+4}}$

2

$\vec{T}_{AC} = |T_{AC}|\,\vec{u}_{AC}$

$= |T_{AC}|\left(\dfrac{-6\vec{i}+2\vec{k}}{6.324}\right)$

SUM FORCES TO ZERO : $\Sigma \vec{F} = 0$ : $\vec{T}_{AB} + \vec{T}_{AC} + \vec{F} + \vec{D}_x + \vec{D}_y + \vec{D}_z = 0$ (1)

(1) $T_{AB}\left(-\frac{6\vec{i}}{7} - \frac{3\vec{j}}{7} + \frac{2\vec{k}}{7}\right) + T_{AC}\left(\frac{-6\vec{i}}{6.324} + \frac{2\vec{k}}{6.324}\right) + (200\vec{j} - 346.41\vec{k}) + D_x\vec{i} + D_y\vec{j} + D_z\vec{k} = 0$

IN X: $\Sigma F_x = 0$ : (1) $-\frac{6T_{AB}}{7} - \frac{6T_{AC}}{6.324} + D_x = 0$ (EQ1)

IN Y: $\Sigma F_y = 0$ : (1) $-\frac{3T_{AB}}{7} + 200 + D_y = 0$ (EQ2)

IN Z: $\Sigma F_z = 0$ : (1) $\frac{2T_{AB}}{7} + \frac{2T_{AC}}{6.324} - 346.41 + D_z = 0$ (EQ3)

MORE UNKNOWNS THAN THE NUMBER OF EQUATIONS, SO WE HAVE USE MOMENT.

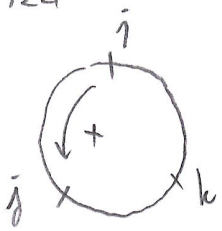
$\Sigma M_D = 0 \equiv \vec{r}_{DA} \times (\vec{F} + \vec{T}_{AB} + \vec{T}_{AC}) = 0$ (2)

$\vec{r}_{DA} = 6\vec{i}$ (1) $\rightarrow$ $6\vec{i} \times \left[200\vec{j} - 346.41\vec{k} + T_{AB}\left(\frac{-6\vec{i} - 3\vec{j} + 2\vec{k}}{7}\right) + T_{AC}\left(\frac{-6\vec{i} + 2\vec{k}}{6.324}\right)\right] = 0$ (3)

RESULTS IN $1200\vec{k} + 2078.4\vec{j} - \frac{18T_{AB}}{7}\vec{k} - \frac{12T_{AB}}{7}\vec{j} - \frac{12T_{AC}}{6.324}\vec{j} = 0$ (2)

FOR $\vec{i}$: (1) $2078.4 - \frac{12T_{AB}}{7} - \frac{12T_{AC}}{6.324} = 0$ (EQ4)

FOR $\vec{k}$: (1) $-\frac{18T_{AB}}{7} + 1200 = 0$ (EQ5)




FROM EQ5 $\rightarrow$ $T_{AB} = 466.67\,N$, FROM EQ4 $T_{AC} = 673.72\,N$ (1)

USING EQ1 : $\frac{6T_{AB}}{7} + \frac{6T_{AC}}{6.324} = D_x \rightarrow D_x = 1039.197\,N$ (1)

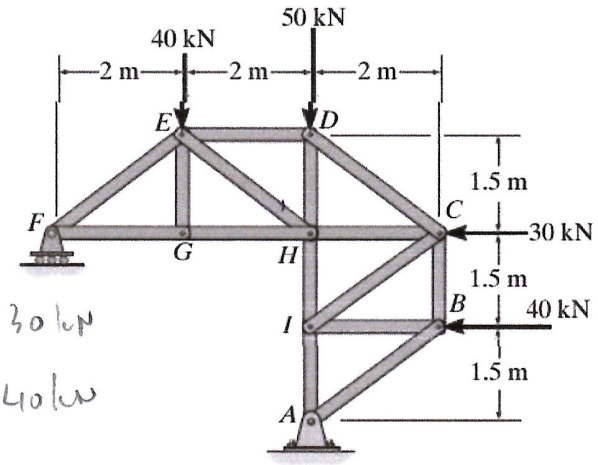
USING EQ2 $\frac{3T_{AB}}{7} - 200 = D_y \rightarrow D_y = -0.002 \simeq 0$ (1)

USING EQ3 $-\frac{2T_{AB}}{7} - \frac{2T_{AC}}{6.324} + 346.41 = D_z \rightarrow D_z = 0.01 \simeq 0$

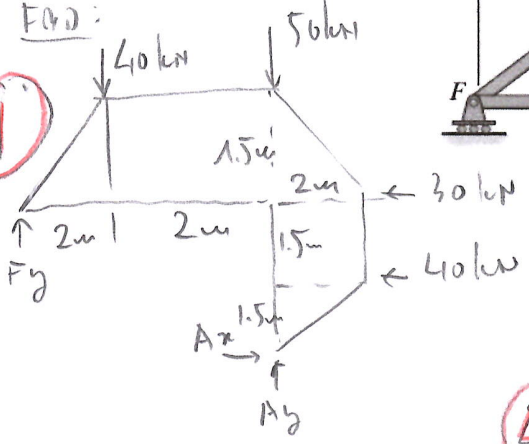
$D_y$ and $D_z$ ARE 0. (1)

2. **Determine the force in members *ED*, *EH*, and *GH* of the truss, and state if the members are in tension or compression. Draw the proper free body diagram(s) during your solution (20 POINTS).**

FIND THE REACTIONS AT F AND A.




FBD:




(1)

(4)

$\overset{+}{\circlearrowleft}\sum M_A = 0$: $F_y(4) - 40(2) - 30(3) - 40(1.5) = 0$

$4F_y = 80 + 90 + 60$

$F_y = 57.5$

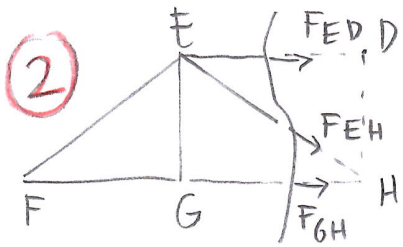
$\rightarrow \sum F_x = 0$: $A_x - 30 - 40 = 0$ : $A_x = 70$ (1)

$\uparrow \sum F_y = 0$: $F_y - 40 - 50 + A_y = 0$ : $A_y = 32.5$ (1)

($F_y = 57.5$)

$A_y + F_y = 90$ checks

CUT:




(2)

$\overset{+}{\circlearrowleft}\sum M_E = 0$: (3)

$F_y(2) - F_{GH}(1.5) = 0$

($F_y = 57.5$)

$F_{GH} = 76.66$ kN T

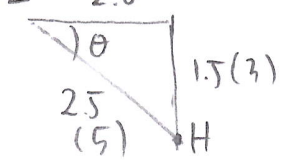
(1)

$\overset{+}{\circlearrowleft}\sum M_H = 0$:

$F_y(4) + F_{ED}(1.5) - 40(2) = 0$

$F_{ED} = -100$ kN C

(3)




3-4-5 TRIANGLE

$\sin\theta = \frac{3}{5}$

$\overset{+}{\uparrow}\sum F_y = 0$:

$F_y - F_{EH}\sin\theta - 40 = 0$ (2)

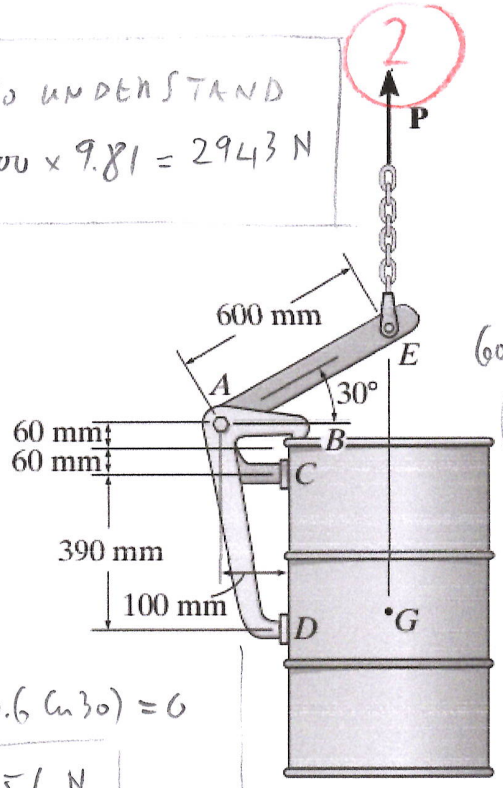
$57.5 - F_{EH}\left(\frac{3}{5}\right) - 40 = 0$

$\left(\frac{3}{5}\right)F_{EH} = 17.5 \rightarrow F_{EH} = 29.16$ kN T (2)

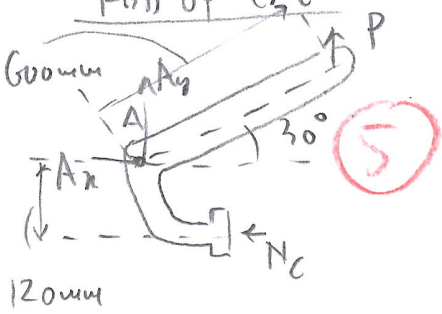
3. If the 300-kg drum has a center of mass at point $G$, determine the horizontal and vertical components of force acting at pin $A$ and the reactions on the smooth pads $C$ and $D$. **The grip at $B$ on member $DAB$ resists both horizontal and vertical components of force at the rim of the drum. Draw the necessary free body diagrams of the elements for the solution (25 POINTS).**

IT IS IMPORTANT TO UNDERSTAND THAT $P = mg = 300 \times 9.81 = 2943$ N

(2)




FBD of CAE




(5)

$\overset{+}{\circlearrowleft} \Sigma M_A = 0 : N_C(0.12) - 2943(0.6\cos 30) = 0$

(4)

$N_C = 12743.56$ N

$N_C \simeq 12.74$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0 : A_x - N_C = 0$

(2)

$A_x = 12.74$ kN

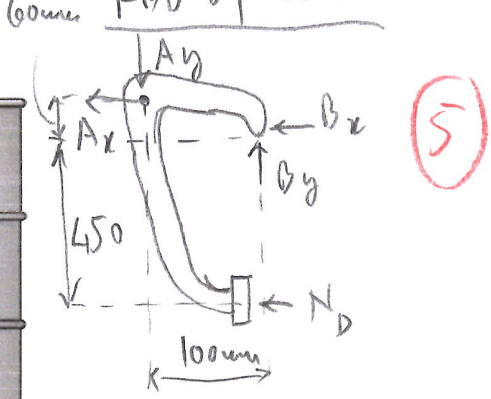
$\overset{+}{\uparrow} \Sigma F_y = 0 : P + A_y = 0$

(2) $2943 + A_y = 0$

$A_y = -2943$ N

POINTS DOWNWARD

FBD of BAD:




(5)

TO FIND $N_D$; LETS TAKE A MOMENT W.R.T B:

$\overset{+}{\circlearrowleft} \Sigma M_B = 0 : -A_y(0.1) - A_x(0.06) + N_D(0.45) = 0$

$A_y = -2943$

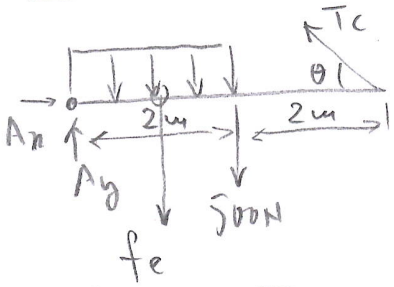
$2943(0.1) - 12743.56(0.06) + N_D(0.45) = 0$

$N_D = 1645.13$ N

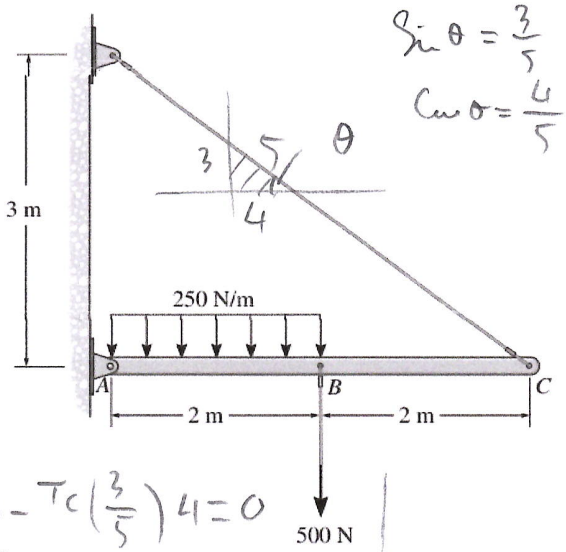
(5)

4. **Draw** the shear and moment diagrams for the beam. **Draw the free body diagrams of the relevant sections to guide you for the solution process. It is compulsory to clearly write the $V(x)$ and $M(x)$ expressions for the relevant sections (25 POINTS).**

FBD of the System:




$f_e = 500\ N$




$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$+\circlearrowleft \sum M_A = 0: \quad 500(1) + 500(2) - T_C\left(\frac{3}{5}\right)4 = 0$

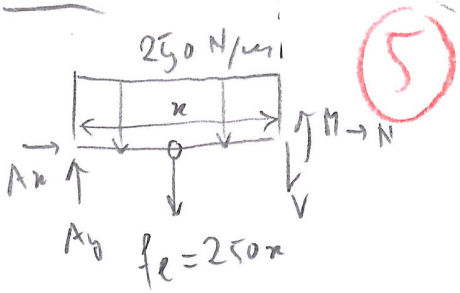
$T_C = 625\ N$ (2)

$+\rightarrow \sum F_x = 0: \quad A_x - T_C(4/5) = 0 \quad \boxed{A_x = 500\ N}$ (1)

$+\uparrow \sum F_y = 0: \quad A_y + 625(3/5) - 500 - 500 = 0$

$A_y = 625\ N$ (1)

CUT 1 : $C_1$




(5)

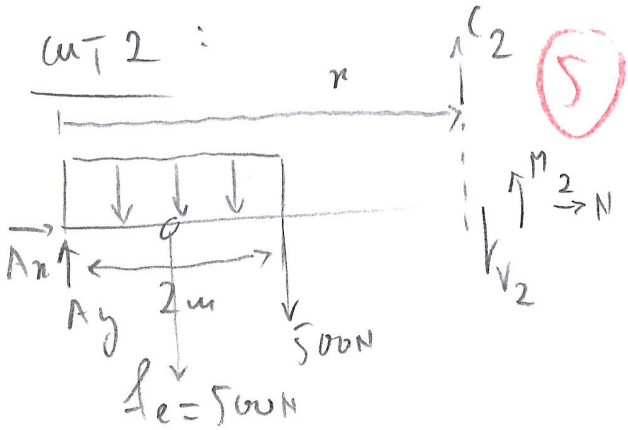
$+\uparrow \sum F_y = 0: \quad A_y - 250x - V = 0$

$V_1(x) = 625 - 250x$ (2)

$+\circlearrowleft \sum M_{C1} = 0: \quad A_y x - 250x\frac{x}{2} - M_1(x) = 0$

$M_1(x) = 625x - 125x^2$ (2)

CUT 2 :




(5)

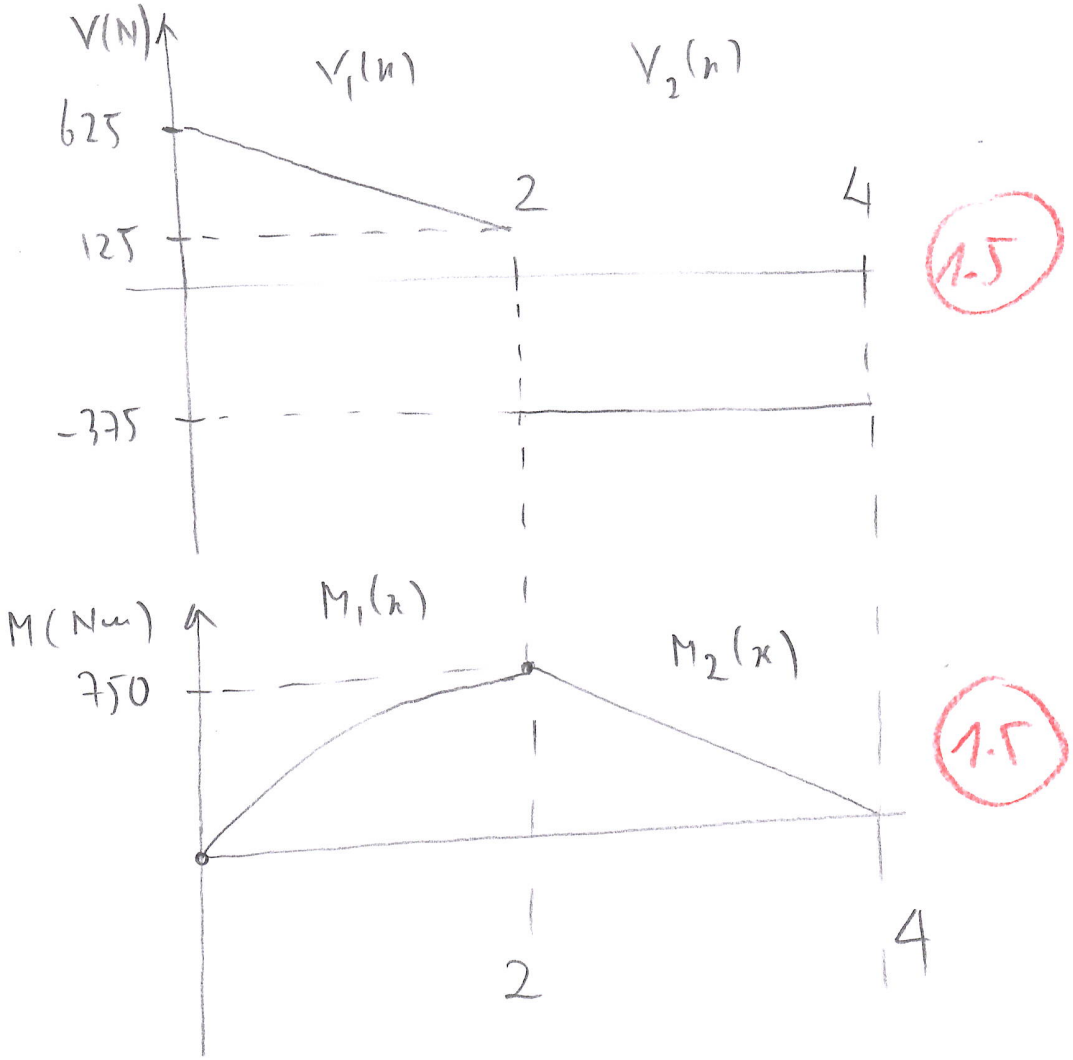
$+\uparrow \sum F_y = 0: \quad A_y - 500 - 500 - V_2(x) = 0$

$V_2(x) = -375\ N$ (2)

$+\circlearrowleft \sum M_{C2} = 0: \quad A_y x - 500(x-1) - 500(x-2) - M_2(x) = 0$

$M_2(x) = 625x - 500x + 500 - 500x + 1000$

$M_2(x) = 1500 - 375x$ (2)


V(N)
$V_1(x)$
$V_2(x)$
625
125
-375
2
4
1.5
M(Nm)
$M_1(x)$
$M_2(x)$
750
1.5
2
4