Sample Questions

Questions 1-2 are related with the given integral

$$\int \frac{x}{(1-4x^2)^{2/3}}\,dx$$

1) Which of the following(s) is/are correct for the given integral

$$\int \frac{x}{(1-4x^2)^{2/3}}\,dx$$

I. The integral can be correctly solved by method of substitution with $u = 1 - 4x^2$
II. The integral can be correctly solved by inverse substitution with $2x = sin\theta$
III. The integral can be correctly solved by inverse substitution with $1 - 4x^2 = u^3$
IV. The integral can be correctly solved by integration by parts with $u = (1 - 4x^2)^{2/3}$ and $dv = xdx$
V. The integral can be correctly solved by integration by parts with $u = x$ and $dv = (1 - 4x^2)^{2/3}dx$

A) I – II – III B) I – II C) Only I D) Only IV E) Only V

Answer: A

2) Evaluate the integral

$$\int \frac{x}{(1-4x^2)^{2/3}}\,dx$$

Find the correct answer

A) $-\frac{1}{4}\,(1-4x^2)^{2/3} + C$
B) $-\frac{3}{8}\,(1-4x^2)^{1/3} + C$
C) $-\frac{3}{8}\frac{1}{(1-4x^2)^{1/3}} + C$
D) $-\frac{1}{4}\,(1-4x^2)^{4/3} + C$
E) $-\frac{3}{8}\frac{x}{(1-4x^2)^{1/3}} + C$

Answer: B

Questions 3-4 are related with the given integral

$$\int x\cos(x^2+1)\ dx$$

3) Which of the following techniques can be used to correctly solve the given integral

$$\int x\cos(x^2+1)\ dx$$

A) Inverse substitution $x = tan\theta$

B) Method of substitution with $x = cos\theta$

C) Integration by parts with $u = \cos(x^2+1)$ and $dv = xdx$

D) Integration by parts with $u = x$ and $dv = \cos(x^2+1)\ dx$

E) Method of substitution with $u = x^2+1$

Answer: E

4) Evaluate the given integral

$$\int x\cos(x^2+1)\ dx$$

Find the correct answer

A) $-\frac{1}{2}\cos(x^2+1)+C$

B) $-\frac{1}{2}\cos^2(x^2+1)+C$

C) $\frac{1}{2}\sin(x^2+1)+C$

D) $\frac{1}{2}\sin^2(x)+C$

E) $\frac{1}{2}(x^2+1)\sin x+C$

Answer: C

5) Evaluate the integral

$$I = \int \frac{x}{1-\sqrt{x}}\ dx$$

Select the correct answer below.

A) $I = -2\left(\frac{x}{2}-\sqrt{x}+\ln\left|\sqrt{x}-1\right|\right)+C$

B) $I = \left(\frac{x}{2}+\sqrt{x}-\frac{1}{1-\sqrt{x}}\right)+C$

C) $I = \left(\sqrt{x}+\ln\left|\sqrt{x}-1\right|\right)+C$

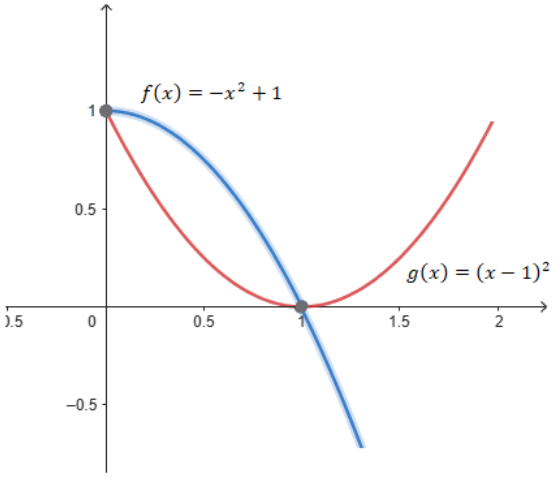
D) $I = -2\left(\frac{x^{\frac{3}{2}}}{3}+\frac{x}{2}+\sqrt{x}+\ln\left|\sqrt{x}-1\right|\right)+C$

E) $I = \left(x^{\frac{3}{2}}-\frac{x}{2}+\ln\left|\sqrt{x}-1\right|\right)+C$

Answer: D

Questions 6-7 are related with the plane region described in Question 5.

6) Let a plane region R be bounded above by $f(x) = -x^2 + 1$ and bounded below by $g(x) = (x-1)^2$ on $[0,1]$. If the graph of $y = f(x)$ and $y = g(x)$ are given below, which of the integrals correctly computes the volume of the solid obtained by rotating the region R about y-axis?



A. $V = 2\pi \int_0^1 [(-x^2 + 1) - (x-1)^2]\ dx$

B. $V = 2\pi \int_0^1 x[(-x^2 + 1) - (x-1)^2]\ dx$

C. $V = \pi \int_0^1 [(-x^2 + 1)^2 - (x-1)]^2\ dx$

D. $V = \pi \int_0^1 x[(-x^2 + 1)^2 - (x-1)^4]\ dx$

E. $V = 2\pi \int_0^1 [(x-1)^2 - (-x^2 + 1)]\ dx$

Answer: B

7) Find the volume **of the solid given in Question 6.** Select the correct answer below.

A) $\frac{\pi}{5}$

B) $\frac{3\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{2\pi}{3}$

E) $\frac{\pi}{6}$

Answer: C

8) Find Taylor series of $f(x) = \sin(2x + \pi)$ about $x = \frac{\pi}{2}$. Select the correct answer below.

A) $2x + 4x^2 - \frac{8}{6}x^3 + \cdots$

B) $2\left(x - \frac{\pi}{2}\right) - \frac{8}{6}\left(x - \frac{\pi}{2}\right)^3 + \cdots$

C) $1 + 2\left(x - \frac{\pi}{2}\right) - 8\left(x - \frac{\pi}{2}\right)^3 + \cdots$

D) $2\left(x - \frac{\pi}{2}\right) - 2\left(x - \frac{\pi}{2}\right)^2 - 8\left(x - \frac{\pi}{2}\right)^3 + \cdots$

E) $1 + 2\left(x - \frac{\pi}{2}\right) + 4\left(x - \frac{\pi}{2}\right)^2 - \frac{8}{6}\left(x - \frac{\pi}{2}\right)^3 + \cdots$

Answer: B

9) Use second order Taylor polynomial of $f(x) = \sqrt{x}$ about $x = 4$ to approximate $\sqrt{5}$. Find the correct answer below.

A) $\sqrt{5} \approx 2 + \frac{1}{8} - \frac{1}{64}$

B) $\sqrt{5} \approx 2 - \frac{1}{4} + \frac{1}{32}$

C) $\sqrt{5} \approx 2 + \frac{1}{4} - \frac{1}{32}$

D) $\sqrt{5} \approx 2 + \frac{1}{4} - \frac{1}{64}$

E) $\sqrt{5} \approx 2 - \frac{1}{8} - \frac{1}{64}$

Answer: D

10) Which of the followings is/are correct for the integral

$$\int_{\frac{1}{2}}^{\frac{e}{2}} \frac{1}{x(\ln(2x))^2}\, dx$$

I. This is an improper integral.
II. This is not an improper integral.
III. It is a convergent improper integral.
IV. This improper integral diverges to ∞.
V. This improper integral diverges to $-\infty$.

A) I – III B) Only II C) Only I D) I – V E) I – IV

Answer: E