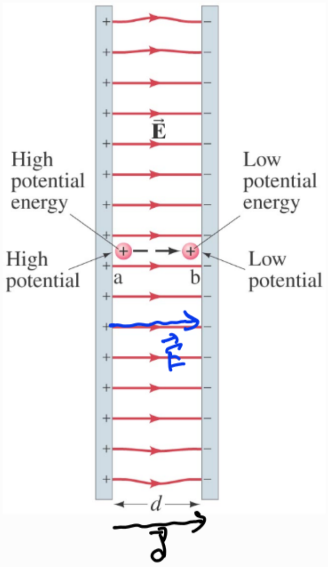
Chapter 23: Electric Potential



* The electrostatic force is conservative!

$U_b - U_a = -W = \Delta U$ and $W_{ba} = \vec{F}\cdot\vec{d} = q\vec{E}\cdot\vec{d}$

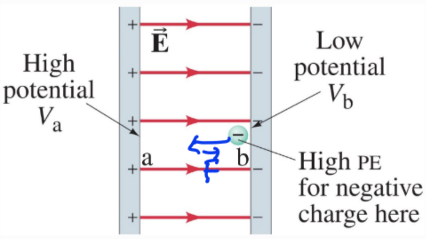
$\Delta U = U_b - U_a = -qEd$ $\qquad = qEd \quad (\vec{E} \parallel \vec{d})$

* Electric potential: potential energy / unit charge

$V = U/q$ (measured in Joules / coulomb ≡ Volts)

$$V_b - V_a = \Delta V = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}$$

Example 23.1

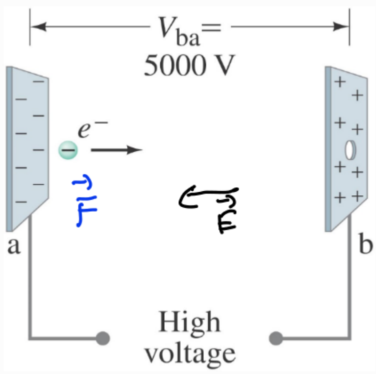


a) kinetic increases as electron moves left.

$U_b > U_a \Rightarrow \Delta U = U_a - U_b < 0$

b) $V_a - V_b = \Delta V = \dfrac{\Delta U}{-q} \Rightarrow \Delta V > 0$

Example 23.2



$\vec{F} = q\vec{E} \Rightarrow \vec{F} = -e\vec{E}$

a) $V_{ba} = V_b - V_a = 5000\,V$, $\Delta U = U_b - U_a = qV_{ba}$

$\Delta U = (-1.6\times10^{-19}C)(5000V) = -8.0\times10^{-16}\,J$

b) since electric force is conservative, we have

$\Delta U + \Delta K = 0 \Rightarrow \Delta K = -\Delta U \Rightarrow K_b - \cancel{K_a}^{\,0} = -(U_b - U_a)$

$\frac{1}{2}mv^2 = -(-8.0\times10^{-16}J) \Rightarrow v = \left(\dfrac{2\times(8.0\times10^{-16}J)}{9.1\times10^{-31}kg}\right)^{\frac{1}{2}}$

$v = 4.2\times10^{7}\,m/s$

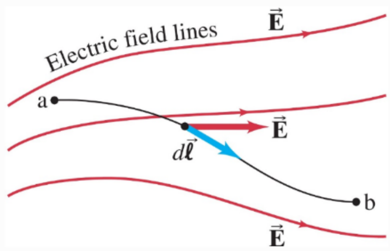
Relation between Electric potential and E-field:

* for a conservative force, we have $\underbrace{U_b - U_a}_{q(V_b - V_a)} = -\int_a^b \vec{F}\cdot d\vec{l}$, $\vec{F} \to q\vec{E}$

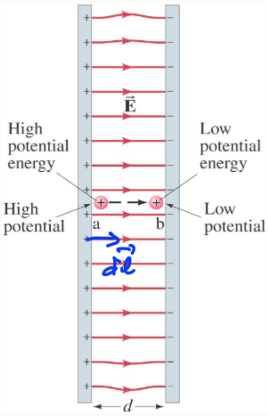
$V_{ba} = V_b - V_a = \dfrac{U_b - U_a}{q}$

$\vec{F} = q\vec{E}$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$



A charged particle in a uniform field:

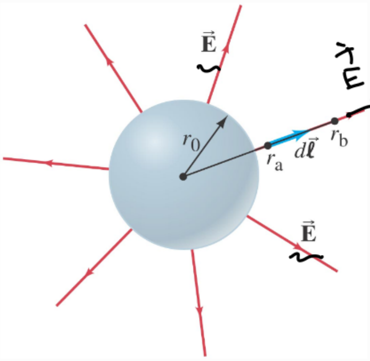


$$V_{ba} = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = -E\int_a^b \underbrace{dl}_{\text{line element}} = -E\overbrace{(b-a)}^{d}$$

$$V_{ba} = -Ed$$

check the relation between ΔU and ΔV: $qV_{ba} = -qEd = \Delta U$

Example 23.4



* First, recall that $E=0$ when $r<r_0$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ when $r>r_0$.

a) $$V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right)$$

$$\vec{E}\cdot d\vec{l} = E\,dr\cos 0 = E\,dr$$ (set $V_b=0$ at $r_b=\infty$)

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

b) $V(r_0) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0}$ (constant)

c) $$V_b - V_a = -\overbrace{\int_{r_a}^{r_0}\vec{E}\cdot d\vec{l}}^{0} - \int_{r_0}^{r_b}\vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{r_0}^{r_b}\frac{dr}{r^2}$$

$$\cancel{V_b}^{\,0} - V_a = \frac{Q}{4\pi\epsilon_0}\left(\cancel{\frac{1}{r_b}}^{\,0} - \frac{1}{r_0}\right)$$ (set $V_b=0$ at $r_b=\infty$)

$$V = \frac{Q}{4\pi\epsilon_0}\frac{1}{r_0}$$ (constant)