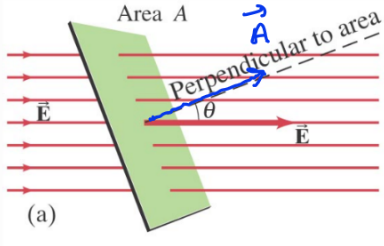
Chapter 22 : Gauss's Law

Electric Flux :

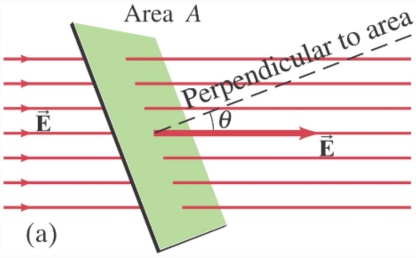
$$\Phi_E = E_\perp A = E A_\perp = EA\cos\theta$$

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

(E-field is uniform)



Example 22-1

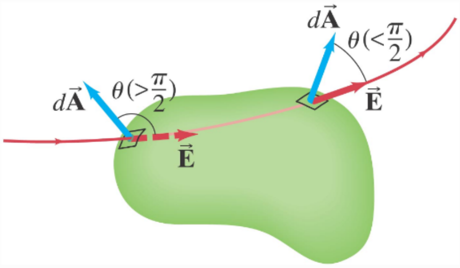


$$A = (0.1\,m)\times(0.2\,m) = 0.02\,m^2$$

$$\cos\theta = \cos 30^\circ = 0.866\,, \quad E = 200\,N/C$$

$$\Phi_E = EA\cos\theta = (0.02\,m^2)(200\,N/C)(0.866) \approx 3.5\,Nm^2/C$$

Electric Flux through a closed surface:



$$\Phi_E = \sum_{i=1}^{n} \vec{E}_i\cdot\Delta\vec{A}_i = \sum_{i=1}^{n} E_i\,\Delta A_i\cos\theta_i$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$

→ infinitely small area element

→ more general than $\Phi_E = \vec{E}\cdot\vec{A}$

Gauss's Law

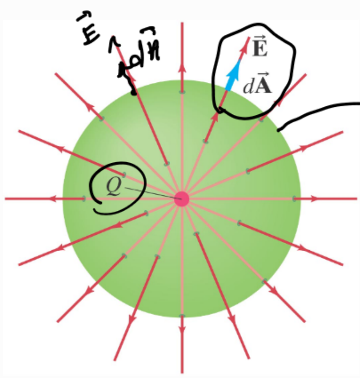
The net number of E-field lines passing through a closed surface is proportional to the net charge enclosed in this closed surface.

$$\underbrace{\oint \vec{E}\cdot d\vec{A}}_{\Phi_E} = \frac{Q_{enc}}{\varepsilon_0}$$

Q_{enc} → enclosed charge

ε_0 → permitivity of free space

E-field due to a point charge:



→ Gaussian surface (closed)

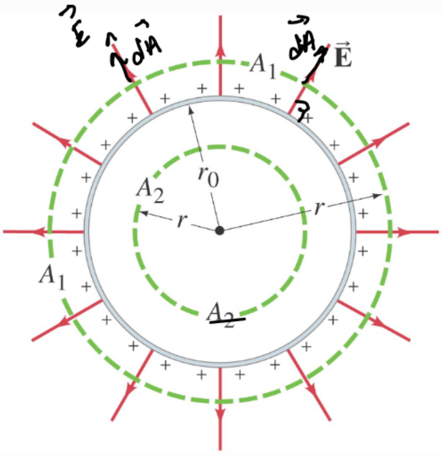
$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA\,\overset{1}{\cos 0} = E\oint dA = E\,4\pi r^2$$

$$Q_{enc} = Q \Rightarrow E\,4\pi r^2 = Q/\varepsilon_0 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

* If a gaussian surface encloses several point charges, then we have

$$\oint \vec{E}\cdot d\vec{A} = \oint \left(\sum_i \vec{E}_i\right)\cdot d\vec{A} = \sum \frac{Q_i}{\varepsilon_0} = \frac{Q_{enc}}{\varepsilon_0}$$

Example 22-3



a) $E = ?$ when $r > r_0$. Draw the gaussian surface outside the shell.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = E\,4\pi r^2$$

$(\vec{E} \parallel d\vec{A})$

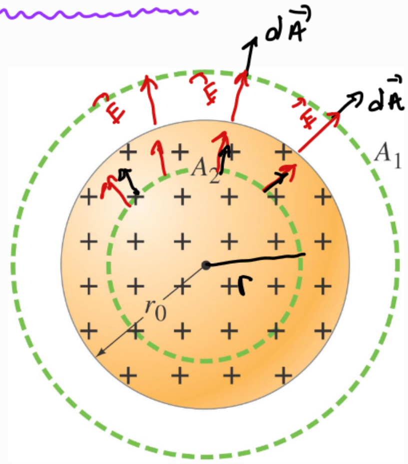
$$Q_{enc} = Q \Rightarrow E\,4\pi r^2 = Q/\varepsilon_0 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

b) $E = ?$ when $r < r_0$. Draw the surface inside the shell.

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{Q_{enc}}_{\to 0}/\varepsilon_0 \Rightarrow E = 0$$

c) Nothing would change!

Example 22-4



a) $E = ?$ when $r > r_0$. Consider the surface A_1.

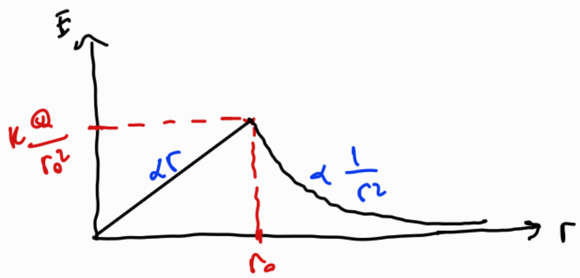
$$\oint \vec{E}\cdot d\vec{A} = E\oint dA = E\,4\pi r^2 = Q/\varepsilon_0 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

b) $E = ?$ when $r < r_0$. consider the surface A_2.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = E\,4\pi r^2 \text{ since } \vec{E} \parallel d\vec{A}.$$

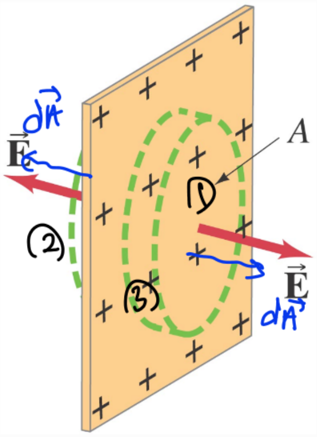
Charge density $\longrightarrow$ $\rho = \dfrac{Q}{\frac{4}{3}\pi r_0^3}$ $\Rightarrow$ $\dfrac{Q = \cancel{\rho}\left(\cancel{\frac{4}{3}}\cancel{\pi} r_0^3\right)}{Q_{enc} = \cancel{\rho}\left(\cancel{\frac{4}{3}}\cancel{\pi} r^3\right)}$ $\Rightarrow Q_{enc} = Q\dfrac{r^3}{r_0^3}$

$$\Rightarrow E\,4\pi \cancel{r^2} = \frac{Q}{\varepsilon_0}\frac{r^{\cancel{3}}}{r_0^3} \Rightarrow E = \frac{Q\,r}{4\pi\varepsilon_0 r_0^3}$$



Example 22-7

uniformly charge sheet of charge with σ = charge / Area



* we choose a closed cylindrical gaussian surface A.

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_1 \vec{E}\cdot d\vec{A}}_{\vec{E} \parallel d\vec{A}} + \underbrace{\int_2 \vec{E}\cdot d\vec{A}}_{\vec{E} \parallel d\vec{A}} + \overbrace{\underbrace{\int_3 \vec{E}\cdot d\vec{A}}_{\vec{E} \perp d\vec{A}}}^{0}$$

$$\oint \vec{E}\cdot d\vec{A} = E\int_1 dA + E\int_2 dA = 2E\cancel{A} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$E = \sigma / 2\varepsilon_0$$