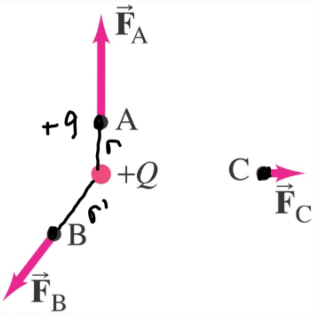
Chapter 21: Electric Charge and Electric Field

Electric Field:



$$\vec{E}_A = \frac{\vec{F}_A}{q}, \quad F_A = k\frac{qQ}{r^2} \Rightarrow E_A = k\frac{Q}{r^2}$$

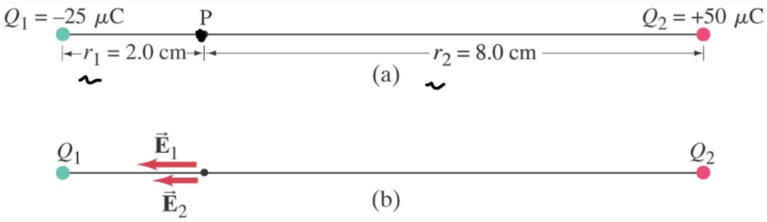
$$E_B = k\frac{Q}{r'^2}, \quad \text{E-field} \propto Q/r^2$$

* For a single point charge, $E = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{k}\frac{Q}{r^2}$

Force on a charge particle in E-field: $\vec{F} = q\vec{E}$

* for a positively charged particle $\vec{E}$ and $\vec{F}$ are in the same direction.
* for a negatively charged particle $\vec{E}$ and $\vec{F}$ are in opposite directions.

Example 21-7



$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

(superposition principle)

$Q_1 = 25 \times 10^{-6}\,C \quad r_1 = 2 \times 10^{-2}\,m$

$Q_2 = 50 \times 10^{-6}\,C \quad r_2 = 8 \times 10^{-2}\,m$

a) $$E_p = k\frac{Q_1}{r_1^2} + k\frac{Q_2}{r_2^2} = k\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right)$$

$$E_p = \underbrace{(9.0 \times 10^9\,N\,m^2/C^2)}_{k}\left(\frac{25 \times 10^{-6}C}{(2 \times 10^{-2}m)^2} + \frac{50 \times 10^{-6}C}{(8 \times 10^{-2}m)^2}\right) \Rightarrow E = 6.3 \times 10^8\,N/C$$

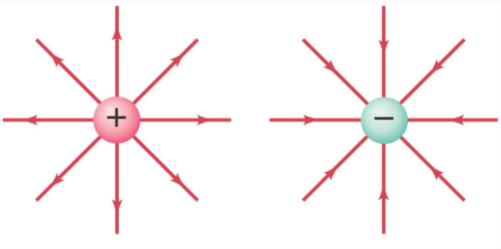
b) $$a = \frac{F}{m} = \frac{qE_p}{m} \Rightarrow a = \frac{\overbrace{(1.6 \times 10^{-19}C)}^{q}(6.3 \times 10^8 N/C)}{\underbrace{9.11 \times 10^{-31}\,kg}_{m}} = 1.1 \times 10^{20}\,m/s^2$$

As the electron is negatively charged $\vec{F}$ and $\vec{E}_p$ will be in opposite directions!

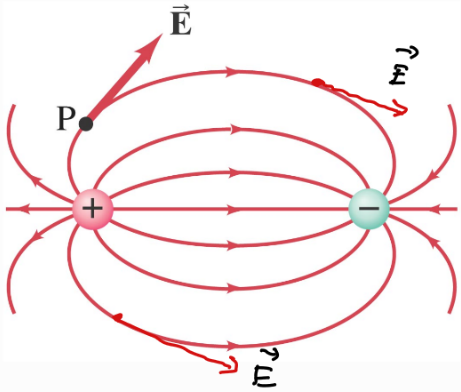
$\vec{E}$-field due to continuous charge distributions:

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2} \Rightarrow \vec{E} = \int d\vec{E}$$

Electric Field Lines:



For a dipole:



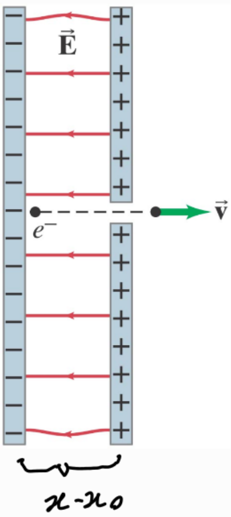
* For a conductor, the static E-field inside is zero.

* For a conductor, E-field is always perpendicular to the surface.

Motion of a charged particle in E-field:

$$\vec{F} = q\vec{E} \quad \text{and} \quad \vec{F} = m\vec{a} \quad \Rightarrow \quad a = \frac{qE}{m}$$

Example 21-15



a) $a = \dfrac{(1.6 \times 10^{-19}\,C)(2.0 \times 10^{4}\,N/C)}{(9.11 \times 10^{-31}\,kg)} \Rightarrow a = 3.5 \times 10^{15}\,m/s^2$

$V_0 = 0$, $V = ?$

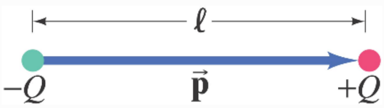
$x - x_0 = 1.5 \times 10^{-2}\,m$

$$V^2 = \cancel{V_0^2}^{\,0} + 2a(x - x_0)$$

$$V = \left(2 \times (3.5 \times 10^{15}\,m/s^2)(1.5 \times 10^{-2}\,m)\right)^{1/2} \approx 10^{7}\,m/s$$

b) $F_E = qE = 3.2 \times 10^{-15}\,N$, $F_G = mg = 8.9 \times 10^{-30}\,N$

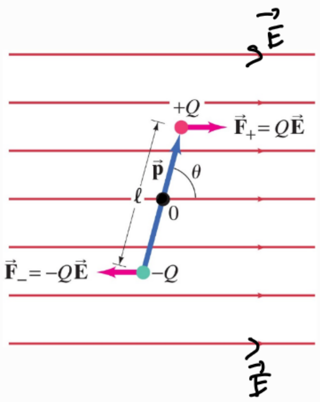
Electric Dipole



$|\vec{p}| = Q\ell$

$\Downarrow$

Electric Dipole in $\vec{E}$-field



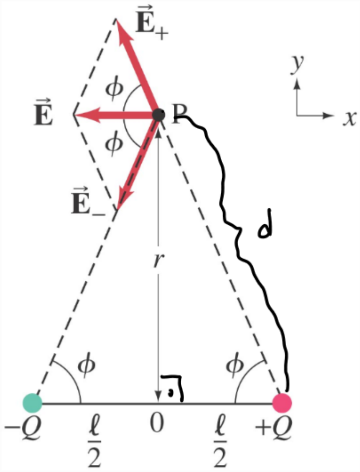
* $Q_{NET} = Q - Q = 0$

$\Rightarrow$ No net force on dipole

$$\tau = \frac{\ell}{2}\sin\theta\, QE + \frac{\ell}{2}\sin\theta\, QE$$

$$\tau = pE\sin\theta \Rightarrow \vec{\tau} = \vec{p} \times \vec{E}$$

Electric Field due to a dipole:



$\vec{E} = \vec{E}_+ + \vec{E}_-$ and $E_+ = E_- = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q}{r^2 + \ell^2/4}$

$d^2 = r^2 + \dfrac{\ell^2}{4}$, Due to the symmetry at P, $E = 2E_+ \cos\phi$
$= 2E_- \cos\phi$

$$E = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q}{r^2+\ell^2/4}\right)\left(\frac{\ell}{2(r^2+\ell^2/4)^{1/2}}\right)$$

Recalling that $p = Q\ell$, $E = \dfrac{1}{4\pi\varepsilon_0} \dfrac{p}{(r^2+\ell^2/4)^{3/2}}$

In the limit $r \gg \ell$, $E \approx \dfrac{1}{4\pi\varepsilon_0} \dfrac{p}{r^3}$