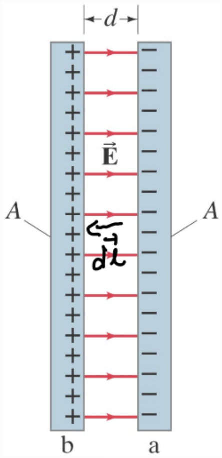
Chapter 24: Capacitance and Energy Storage

Capacitance:

$Q = C V$ → capacitance

$C = \dfrac{Q}{V}$, measured in SI in C/V (farad).



* A single plate has $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{Q}{2A\varepsilon_0}$.

* A parallel plate capacitor has $E = Q/A\varepsilon_0$.

$$V = -\int_b^a \vec{E}\cdot\vec{dl} = V_b - V_a = \int_b^a E\,dl = E\int_b^a dl = Ed = \frac{Qd}{A\varepsilon_0}$$

$$C = Q/V = A\varepsilon_0/d$$

Example 24.1

$A = (20\times10^{-2}\,m)(3\times10^{-2}\,m)$

$d = 10^{-3}\,m$

$V = 12\,V$

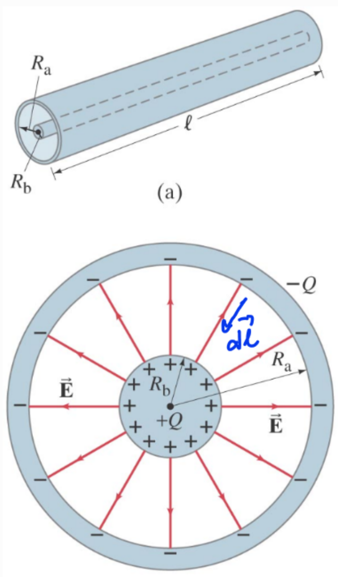
$\varepsilon_0 = 8.85\times10^{-12}\,C^2/Nm^2$

a) $C = A\varepsilon_0/d = 53\times10^{-12}\,F$

b) $Q = CV = 6.4\times10^{-10}\,C$

c) $E = V/d = 1.2\times10^{4}\,V/m$

Example 24.2



$\vec{E}$-field in the region $R_b < r < R_a$: use Gauss's Law.

$$\oint \vec{E}\cdot\vec{dA} = Q_{enc}/\varepsilon_0 \Rightarrow E\,2\pi R\cancel{\ell} = \lambda\cancel{\ell}/\varepsilon_0 \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R}$$

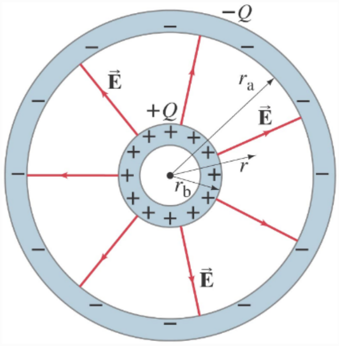
$$E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{\ell R}$$

$$V_b - V_a = -\int_a^b \vec{E}\cdot\vec{dl} = -\frac{Q}{2\pi\varepsilon_0\ell}\int_{R_a}^{R_b}\frac{dR}{R} = -\frac{Q}{2\pi\varepsilon_0\ell}\ln\frac{R_b}{R_a}$$

$$V = \frac{Q}{2\pi\varepsilon_0\ell}\ln\frac{R_a}{R_b} \quad , \quad C = Q/V$$

$$C = \frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)}$$

Example 24.3

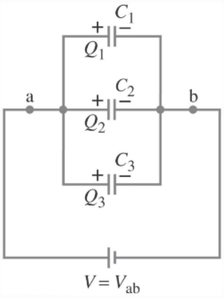


$\vec{E}$-field in $r_b < r < r_a$: $E(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_a r_b}\right), \quad C = \frac{Q}{V} = 4\pi\varepsilon_0\left(\frac{r_a r_b}{r_a - r_b}\right)$$

Copocitors is Series and Parallel:

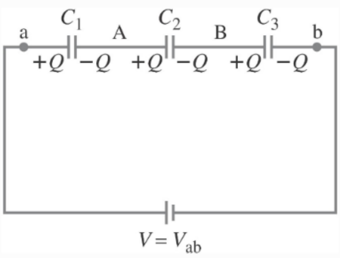


Parallel Configuration

$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V = (C_1 + C_2 + C_3)V$$

$$Q = C_{eq}V = (C_1 + C_2 + C_3)V \Rightarrow C_{eq} = C_1 + C_2 + C_3$$

Serial Configuration



$$V = V_1 + V_2 + V_3, \quad Q = C_1V_1 = C_2V_2 = C_3V_3$$

$$Q = C_{eq}V \Rightarrow V = Q/C_{eq} = Q/C_1 + Q/C_2 + Q/C_3$$

$$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3$$

Electric Enegy Storage:

$$\int dW = \int_0^Q V\,dq \Rightarrow W = U = \frac{1}{C}\int_0^Q q\,dq \Rightarrow U = \frac{1}{2}\frac{Q^2}{C}$$

$$V = q/C \Rightarrow U = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example 24.8

a) $U = \frac{1}{2}CV^2 \Rightarrow U = \frac{1}{2}(150\times10^{-6}F)(200V)^2 = 3.0J$

b) $P = U/\Delta t \Rightarrow P = 3.0J/10^{-3}s = 3000W$

Example 24.9

$Q \rightarrow$ constant

$$\underset{2\downarrow}{C} = \frac{A\varepsilon_0}{d \uparrow 2}$$

Increases by a factor of 2 ~ $U = \frac{1}{2}\frac{Q^2}{C}$

$Q^2 \rightarrow$ constant

$C \rightarrow$ drops by a factor of 2

Energy Density:

* consider the parallel plate capacitor. $C = A\varepsilon_0/d$ and $V = Ed$

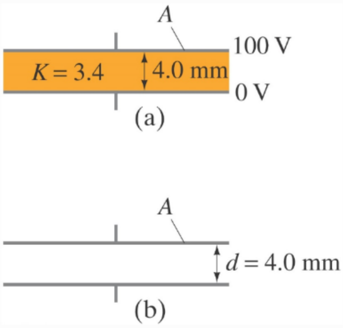
$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)(E^2d^2) = \frac{1}{2}\varepsilon_0 E^2 \underbrace{Ad}_{\text{Volume}} \Rightarrow u = \frac{\text{energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

Dielectrics:

* In a parallel plate capacitor, $C = \underbrace{K}_{} \varepsilon_0 \frac{A}{d}$

$\hookrightarrow$ permittivity

Example 24.11



a) $C = K\varepsilon_0 \frac{A}{d} = 3.0 \times 10^{-8}\,F$ $\quad Q = CV = 3.0 \times 10^{-6}\,C$

$E = V/d = 25 \times 10^{3}\,V/m$ $\quad U = \frac{1}{2}CV^2 = 1.5 \times 10^{-4}\,J$

b) $C_0 = C/K$ - - - - -