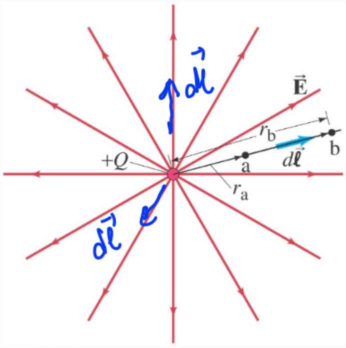
Chapter 23: Electric Potential

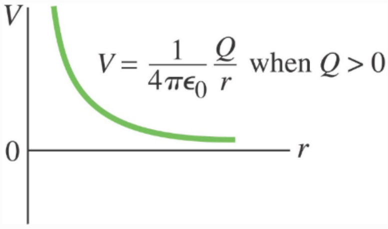
Electric Potential due to a point charge:

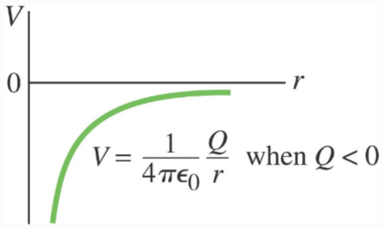


$$\Delta V = V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\cdot\vec{dl} = -\int_{r_a}^{r_b} E\,dr = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2}$$

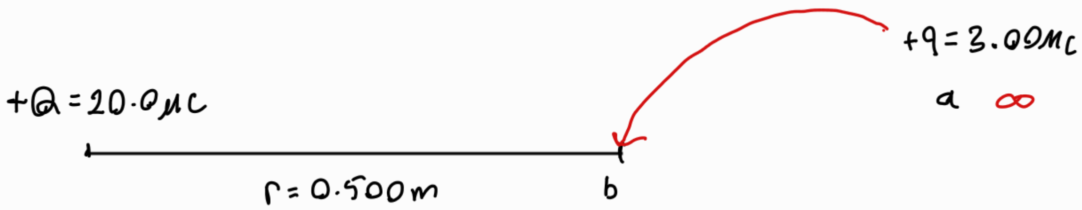
$$V_b - V_a = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$ let $V_b = 0$ at $r_b = \infty$

$$V = (1/4\pi\varepsilon_0)(Q/r)$$





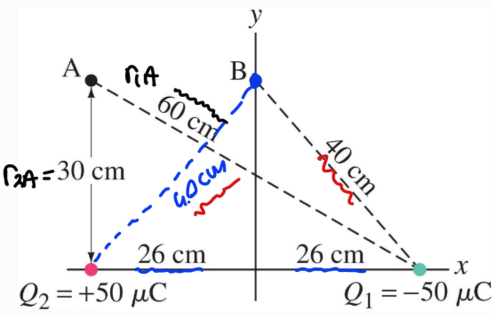
Example 23.6



$\Delta V = V_b - V_a$, $W = \Delta U = q\Delta V = q(V_b - V_a) = q\left(\frac{kQ}{r_b} - \cancel{\frac{kQ}{r_a}}^{0}\right)$

$$W = (3.00\times10^{-6}C)\,\frac{(8.99\times10^{9}\,Nm^2/C^2)(2.00\times10^{-5}C)}{0.500\,m} = 1.08\,J$$ set $r_a = \infty \Rightarrow 1/r_a = 0$

Example 23.7



a) $V_A = V_{A2} + V_{A1} = k\frac{Q_2}{r_{2A}} + k\frac{Q_1}{r_{1A}}$

$Q_1 = -5.0\times10^{5}C$, $Q_2 = +5.0\times10^{5}C$

$r_{1A} = 0.6m$, $r_{2A} = 0.3$ m, $k = 9.0\times10^{9}\,Nm^2/C^2$

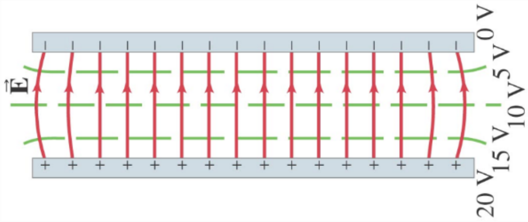
$$V = 7.5\times10^{5}\,V$$

b) $V_B = V_{B2} + V_{B1} = k\frac{Q_2}{r_{2B}} + k\frac{Q_1}{r_{1B}}$ } $V_B = 0$

$Q_1 = -Q_2$, $r_{2B} = r_{1B}$

* At all points on the y-axis $V = 0$ due to symmetry!

Equipotential Surfaces



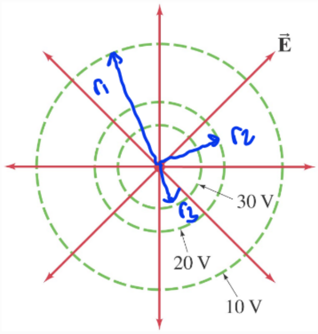
* An equipotential is a line or surface over which the electric potential is constant.

* Electric field lines are perpendicular to equipotential lines or surfaces.

$$\Delta V = \underbrace{V_b - V_a}_{0} = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l}$$

If $\vec{E}$-field $\neq 0$, then $\vec{E}\cdot d\vec{l} = 0 \Rightarrow \cos\theta = 0$, $\theta = 90^\circ$.

Example 23.10



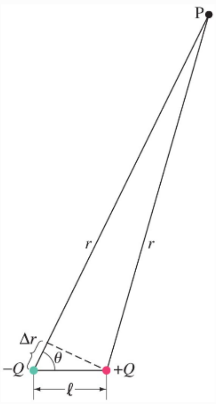
$Q = 4.0 \times 10^{-9}\,C$

$V = k\dfrac{Q}{r} \Rightarrow r = \dfrac{kQ}{V}$, If $V_1 = 10V$, $r_1 = \dfrac{(9.0\times10^9)(4.0\times10^{-9})}{10}\,m$

$r_1 = 3.6\,m$

$r_2 = \dfrac{(9.0\times10^9)(4.0\times10^{-9})}{20}\,m = 1.8\,m$

Electric Potential due to a Electric Dipole:



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{(r+\Delta r)} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{\Delta r}{r(r+\Delta r)}$$

Assuming that $r \gg l$, $\Delta r \approx l\cos\theta$ and $r \gg \Delta r = l\cos\theta$.

$$V \approx \frac{1}{4\pi\varepsilon_0}\frac{Ql\cos\theta}{r^2} \Rightarrow V \approx \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}$$

electric dipole

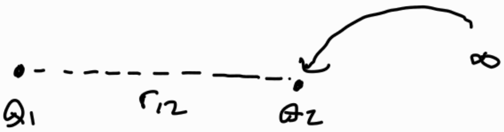
$\vec{E}$ determined from V:

$$\Delta V = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} \Rightarrow dV = -\vec{E}\cdot d\vec{l} = E_l\,dl \Rightarrow E_l = -\frac{dV}{dl}$$

$$\vec{E} = \underbrace{-\frac{\partial V}{\partial x}}_{E_x}\hat{i} - \underbrace{\frac{\partial V}{\partial y}}_{E_y}\hat{j} - \underbrace{\frac{\partial V}{\partial z}}_{E_z}\hat{k}$$

* If you are given $V(x,y,z)$, then you can calculate the electric field vector.

Electrostatic Potential Energy:



$$U = qV \Rightarrow U = Q_2 V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$$