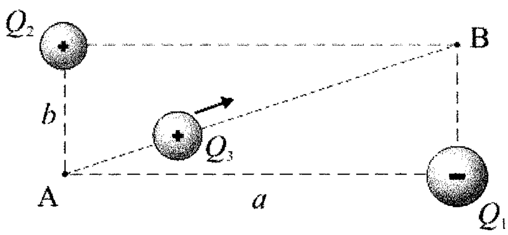
PHYS102 Midterm Exam **05/04/2019**

Izmir University of Economics

Name ____________________ **Student No** ____________________

Each question is worth 20 points. You have 90 minutes.

Problem 1. Two charges Q_1 and Q_2 are placed in vertices of a rectangle as shown in the figure. (a) Determine the electric potential at points A and B due to the charges Q_1 and Q_2. (b) Find the electric field vectors at points A and B due to the charges Q_1 and Q_2. (c) Find the work that needs to be done to move a third charge Q_3 diagonally from the point A to B. (Express your answers in terms of the charges Q_1, Q_2 and Q_3, and the distances a and b.)



8

a) $V = k\frac{Q}{r}$

$$V_A = \frac{k(-Q_1)}{a} + k\frac{Q_2}{b} = k\left(\frac{Q_2}{b} - \frac{Q_1}{a}\right)$$

$$V_B = k\frac{Q_2}{a} + k\frac{(-Q_1)}{b} = k\left(\frac{Q_2}{a} - \frac{Q_1}{b}\right)$$

8

b) $E = k\frac{Q}{r^2}$

$$E_A = k\frac{Q_2}{b^2}(-\hat{j}) + k\frac{Q_1}{a^2}\hat{\imath} = k\left(\frac{-Q_2}{b^2}\hat{j} + \frac{Q_1\hat{\imath}}{a^2}\right)$$

$$E_B = k\frac{Q_2}{a^2}\hat{\imath} + k\frac{Q_1}{b^2}(-\hat{j}) = k\left(\frac{Q_2\hat{\imath}}{a^2} - \frac{Q_1\hat{j}}{b^2}\right)$$

4

c) $W = q(V_b - V_a)$

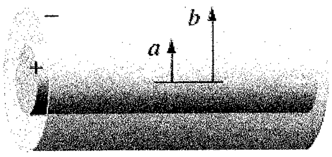
$$W = Q_3\left(k\frac{Q_2}{a} - k\frac{Q_1}{b} - k\frac{Q_2}{b} + k\frac{Q_1}{a}\right)$$

$$W = Q_3 k\left(\frac{1}{a}(Q_2 + Q_1) - \frac{1}{b}(Q_2 + Q_1)\right)$$

$$W = Q_3 k (Q_2 + Q_1)\left(\frac{1}{a} - \frac{1}{b}\right)$$

Problem 2. A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform surface charge density on the outer cylindrical shell (radius b). This surface charge density is negative and of the just right magnitude so that the cable as a whole is electrical neutral. Find the electric field in each of the three regions.

(a) Inside the inner cylinder ($s < a$).
(b) Between the cylinders ($a < s < b$).
(c) Outside the cable ($s > b$)



(8) a) $\oint \vec{E}\cdot d\vec{A} = E\,2\pi s.l = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho\pi s^2 l}{\varepsilon_0}$

$E = \frac{\rho s}{2\varepsilon_0}$

(8) b) $\oint \vec{E}\cdot d\vec{A} = E\,2\pi s.\,l = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho\pi a^2 l}{\varepsilon_0}$

$E = \frac{\rho a^2}{2\varepsilon_0 s}$

(4) c) $\oint \vec{E}\cdot d\vec{A} = E.\,2\pi s.l = \frac{Q_{encl}}{\varepsilon_0} = 0$

$E = 0$

Problem 3. A parallel plate air capacitor of area 25 cm^2 and with plates 1 mm apart is charged to a potential difference of 100 V. (a) Calculate the energy stored in the capacitor. (b) The plates of the capacitor are now moved a further 1 mm apart with the power supply connected. Find the change in the stored energy. (c) If the power supply had been disconnected before the plates had been moved apart, what would have been the energy change in this case?

(10) a) $U = \frac{1}{2}CV^2 \qquad C = \frac{Q}{\Delta V} = \frac{Q\varepsilon_0 A}{Q.d} = \varepsilon_0\frac{A}{d}$

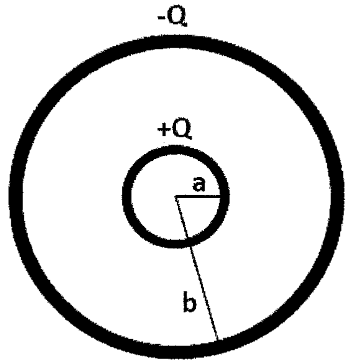
$U = \frac{1}{2}\varepsilon_0\frac{A}{d}V^2 = \frac{1}{2}\,\frac{8.85\times10^{-12}\times 25\times10^{-4}\times(100)^2}{0.001} = 1.106\times10^{-7}\,J$

(5) b) $U = \frac{1}{2}\times 1.106\times10^{-7} = 0.553\times10^{-7}\,J$

(5) c) $U = \frac{1}{2}QV = \frac{1}{2}Q\,\frac{Qd}{\varepsilon_0 A} = \frac{1}{2}\frac{Q^2 d}{\varepsilon_0 A} \qquad d = 0.002\,m$

$V = E.d = \frac{Qd}{\varepsilon_0 A} \qquad U = 2\times 1.106\times10^{-7} = 2.212\times10^{-7}\,J$

Problem 4. A spherical capacitor consists of two thin concentric spherical conducting shells of radius a and b as shown. The inner shall carries a uniformly distributed charge $+Q$ on its surface, and the outer shell an equal but opposite charge $-Q$. (a) Determine the electric field in three regions, i.e., $r < a$, $a < r < b$, and $r > b$. (b) Determine the electric potential difference between the two thin shells, V_{ab}. (c) Determine the capacitance of the capacitor.



(8) a) $r < a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad E\, 4\pi r^2 = \frac{Q_{encl}}{\varepsilon_0} \qquad E = 0$$

$a < r < b$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad E\, 4\pi r^2 = \frac{Q}{\varepsilon_0} \qquad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$r > b$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad E\, 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} \qquad E = 0$$

(8) b) $$V_{ba} = -\int_b^a \vec{E} \cdot d\vec{\ell} = -\frac{Q}{4\pi\varepsilon_0} \int_{r_b}^{r_a} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

$$= \frac{Q}{4\pi\varepsilon_0} \left(\frac{r_b - r_a}{r_a \cdot r_b} \right)$$

(4) c) $$C = \frac{Q}{V_{ba}} = 4\pi\varepsilon_0 \frac{r_a r_b}{r_b - r_a}$$

Problem 5. An electron moving to the right at 7.5 x 10^5 m/s enters a uniform electric field parallel to its direction. If the electron is to be brought to rest in the space of 4.0 cm, (a) what is the direction of the electric field? (b) What is the strength of the field? (m_e = 9.1 x 10^{-31} kg and e = 1.6 x 10^{-19} C)

5

a) The electric field must be in the same direction as the initial velocity of the electron. So, it is to the right.

15

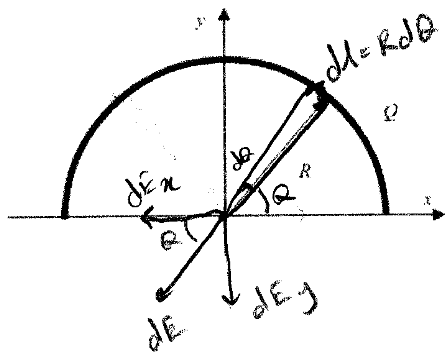
b) $F = qE = ma \qquad a = \frac{qE}{m}$

$v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x$

$E = \frac{m(v^2 - v_0^2)}{2q\Delta x} = \frac{-m v_0^2}{2q\Delta x}$

$E = -\frac{9.1\times10^{-31} \cdot (7.5\times10^5)^2}{2\times1.6\times10^{-19} \times 0.04} = 40 N/C$

Problem 6 (Bonus). A curved plastic rod of charge +*Q* forms a semi-circle of radius *R* as shown in the figure. The charge is uniformly distributed across the rod. Determine the electric field vector *E* at the origin.



$dq = \lambda R d\theta$

$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}\cos\theta$

$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{180}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_0^{180}$

$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}(\sin 180 - \sin 0) = 0$ 5

$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{R^2}\sin\theta\, d\theta$

$E_y = \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{180}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(-\cos\theta)\Big|_0^{180} = \frac{\lambda}{4\pi\varepsilon_0 R}(-\cos 180 + \cos 0)$

$E_y = \frac{2\lambda}{4\pi\varepsilon_0 R}$ 15