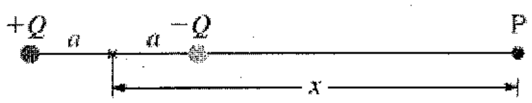
PHYS102 Midterm Make-up 09/05/2019

Izmir University of Economics

Name ________________ Student No ____________ Section _________

Each question is worth 20 points. You have 90 minutes.

Problem 1. Determine the electric potential and the magnitude and direction of the electric field vector at point P. The two charges are separated by a distance *2a*, and the point P is a distance *x* from the midpoint between the two charges. Express your answer in terms of Q, x, a and the constant k.



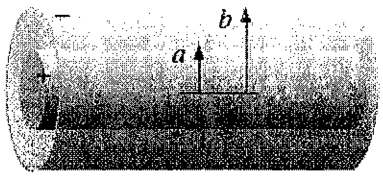
$$E = k\frac{Q}{(x+a)^2} - k\frac{Q}{(x-a)^2} = kQ\left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right)$$

$$= \frac{-4kQxa}{(x^2-a^2)^2} \quad \text{to the left}$$

$$V = k\frac{Q}{x+a} - k\frac{Q}{x-a} = kQ\left(\frac{1}{x+a} - \frac{1}{x-a}\right)$$

$$= \frac{-2kQa}{x^2-a^2}$$

Problem 2. A cylindrical capacitor consists of a cylinder of radius a surrounded by a coaxial shell of inner radius b. Both cylinders have length l which we assume is much greater than the separation of the cylinders, b-a, so we can neglect end effects. The capacitor is charged so that one cylinder has a charge +Q and the other one a charge -Q. (a) Find the electric field between the two cylinders. (b) Calculate the electric potential difference between the two cylinders. (c) Obtain an expression for the capacitance.



a) $\oint \vec{E} \cdot d\vec{A} = EA = E\, 2\pi r l = \dfrac{\lambda l}{\varepsilon_0}$

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$

b) $V_b - V_a = -\displaystyle\int_a^b E_r\, dr = \frac{-\lambda}{2\pi \varepsilon_0} \int_a^b \frac{dr}{r} = \frac{-\lambda}{2\pi \varepsilon_0} \ln\left(\frac{b}{a}\right)$

c) $C = \dfrac{Q}{|\Delta V|} = \dfrac{\lambda L}{\dfrac{\lambda \ln\left(\frac{b}{a}\right)}{2\pi \varepsilon_0}} = \dfrac{2\pi \varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$

Problem 3. How much energy must a 28-V battery expand to charge a 0.45-mF and a 0.20mF capacitor fully when they are placed in (a) parallel and (b) in series? How much charge flowed from the battery in each case?

a) $U = \frac{1}{2} C V^2 = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2}(0.45 \times 10^{-3} + 0.20 \times 10^{-3})(28)^2$

$= 294.8 \times 10^{-3}$ J

b) $U = \frac{1}{2} C V^2 = \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) V^2 = \frac{28^2}{2} \left(\frac{0.45 \times 10^{-3} \times 0.20 \times 10^{-3}}{0.65 \times 10^{-3}} \right)$

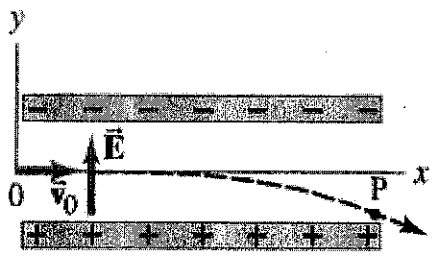
$= 54.27 \times 10^{-3}$ J

$U = \frac{1}{2} Q V \qquad Q = \frac{2U}{V}$

$Q_{parallel} = \frac{2 \cdot (294.8 \times 10^{-3} J)}{28} = 18.2 \times 10^{-3}$ C

$Q_{series} = \frac{2 \cdot 54.27 \times 10^{-3}}{28} = 3.87 \times 10^{-3}$ C

Problem 4. Suppose an electron travelling with speed v_0 enters a uniform electric field **E**, which is at right angles to $\mathbf{v_0}$ as shown in the figure. Describe its motion by giving the equation of its path while in the electric field in terms of the given parameters in the problem. Ignore gravity.



$y = \frac{1}{2} a_y t^2 = -\frac{eE}{2m} t^2$

$x = v_0 t$

$y = \frac{-eE}{2m} \frac{x^2}{v_0^2}$

Problem 5. An electric charge Q is distributed uniformly throughout a non-conducting solid sphere of radius r_0. Determine the electric field vector using Gauss's law (a) outside the sphere ($r > r_0$) and (b) inside the sphere ($r < r_0$).

a) $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$ $\quad E\, 4\pi r^2 = \frac{Q}{\varepsilon_0}$ $\quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$

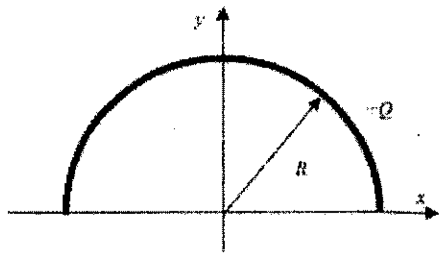
b) $\oint \vec{E} \cdot d\vec{A} = E\,(4\pi r^2)$

$$Q_{encl} = \frac{\frac{4}{3}\pi r^3 \rho_E}{\frac{4}{3}\pi r_0^3 \rho_E} \cdot Q = \frac{r^3}{r_0^3} Q$$

$$E\, 4\pi r^2 = \frac{Q_{encl}}{\varepsilon_0} = \frac{r^3}{r_0^3}\frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{r_0^3}$$

Problem 6 (Bonus). A curved plastic rod of charge +Q forms a semi-circle of radius R as shown in the figure. The charge is uniformly distributed across the rod. Determine the electric field vector E at the origin.



$$dq = \lambda R\, d\theta$$

$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{R^2}\sin\theta\, d\theta$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{180} \sin\theta\, d\theta$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 R}\left(-\cos\theta\right)\Big|_0^{180} = \frac{\lambda}{4\pi\varepsilon_0 R}\left(-\cos 180 + \cos 0\right)$$

$$E_y = \frac{2\lambda}{4\pi\varepsilon_0 R}$$