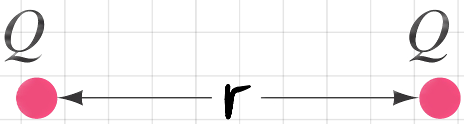
Chapter 21: Electric Charge and Field

* There are two types of charges in nature "Positive" and "negative".
* Like charges repel each other, opposite charges attract each other.
* The electric charge is conserved in any interaction.
* In conductors charges can freely flow, in insulators they cannot.

Coloumbs Law



$F \propto Q_1 Q_2$ and $F \propto 1/r^2 \Rightarrow F \propto \frac{Q_1 Q_2}{r^2}$

Experimentally determining the proportionality constant k, we have

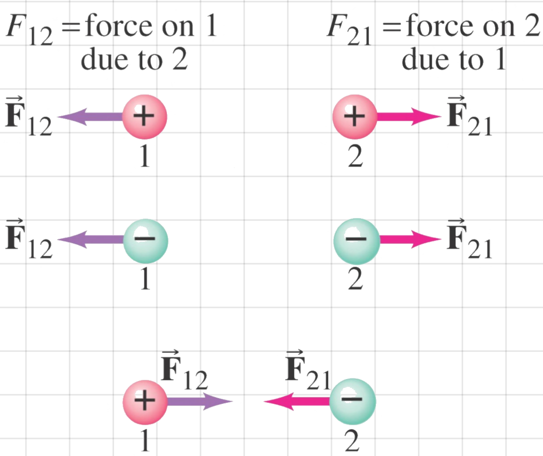
$F = k\frac{Q_1 Q_2}{r^2}$ where $k = 8.99 \times 10^9 \ Nm^2/C^2$

$\hookrightarrow (1/4\pi\varepsilon_0)$

Magnitude of $\vec{F}$

$1e = 1.602 \times 10^{-19} C$

* The force between the charges is along the line connecting the two charges.



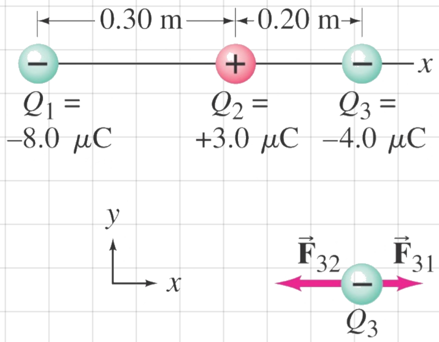
Conceptual Example 21-1: Which charge exerts the greater force?

Two positive point charges, $Q1 = 50\ \mu C$ and $Q2 = 1\ \mu C$, are separated by a distance ℓ. Which is larger in magnitude, the force that $Q1$ exerts on $Q2$ or the force that $Q2$ exerts on $Q1$?

$F_{12} = k\frac{Q_1 Q_2}{\ell^2}$, $F_{21} = k\frac{Q_1 Q_2}{\ell^2} \Rightarrow F_{12} = F_{21}$

Example 21-2: Three charges in a line.

Three charged particles are arranged in a line, as shown. Calculate the net electrostatic force on particle 3 (the -4.0 μC on the right) due to the other two charges.



$$F_{31} = k\frac{Q_1 Q_2}{r_{31}^2} = \frac{(9.0\times10^9\,\mathrm{Nm^2/c^2})(4.0\times10^{-6}c)(8.0\times10^{-6}c)}{(0.50\,m)^2}$$

$$= 1.2\,N$$

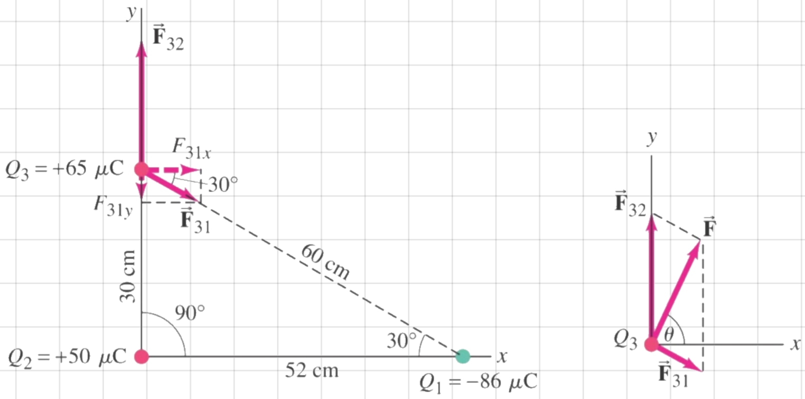
$$F_{32} = k\frac{Q_1 Q_2}{r_{32}^2} = \frac{(9.0\times10^9\,\mathrm{Nm^2/c^2})(4.0\times10^{-6}c)(3.0\times10^{-6}c)}{(0.20\,m)^2}$$

$$= 2.7\,N$$

$$F = F_{31} - F_{32} = 1.2\,N - 2.7\,N = -1.5\,N$$

Example 21-3: Electric force using vector components.

Calculate the net electrostatic force on charge $Q3$ shown in the figure due to the charges $Q1$ and $Q2$.



$$F_{31} = k\frac{Q_1 Q_2}{r_{31}^2}, \quad r_{31} = 0.60\,m$$

$$F_{32} = k\frac{Q_1 Q_2}{r_{32}^2}, \quad r_{32} = 0.30\,m$$

$$F_{31} = (9.0\times10^9\,\mathrm{Nm^2/c^2})\frac{(6.5\times10^{-5}c)(8.5\times10^{-5}c)}{(0.60\,m)^2} = 140\,N$$

$$F_{32} = (9.0\times10^9\,\mathrm{Nm^2/c^2})\frac{(6.5\times10^{-5}c)(8.5\times10^{-5}c)}{(0.30\,m)^2} = 330\,N$$

Determine the components of $\vec{F}_{31}$:

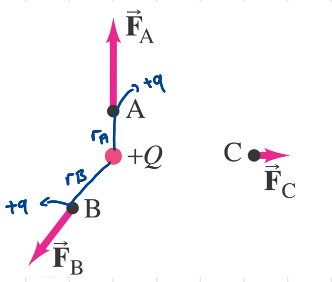
$F_{31x} = F_{31}\cos 30^\circ = (140\,N)\cos 30^\circ = 120\,N$, $\quad F_{31y} = -F_{31}\sin 30^\circ = (-330\,N)\sin 30^\circ = -70\,N$

Summing the two vectors component by component,

$F_x = F_{31x} = 120\,N$ and $F_y = F_{32} + F_{31y} = 330\,N - 70\,N = 260\,N$

$F = (F_x^2 + F_y^2)^{1/2} = 290\,N$ and $\theta = \tan^{-1}(F_y/F_x) = 65^\circ$

Electric Field:

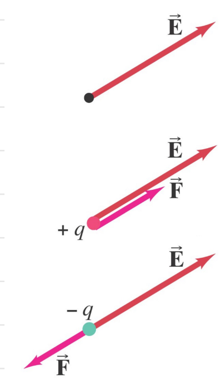


$$E_A = \frac{\vec{F}_A}{q}, \quad \vec{F}_A = k\frac{qQ}{r_A^2} \Rightarrow E_A = k\frac{Q}{r_A^2}$$

$$E_B = \frac{\vec{F}_B}{q}, \quad \vec{F}_B = k\frac{qQ}{r_B^2} \Rightarrow E_B = k\frac{Q}{r_B^2}$$

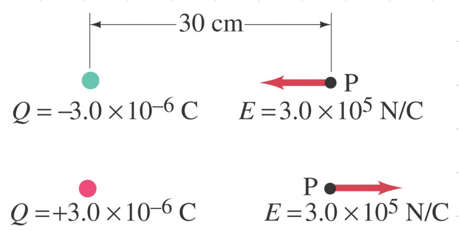
For a single point charge : $E = \underbrace{(1/4\pi\varepsilon_0)}_{k} \frac{Q}{r^2}$

Force on a charge Particle in E field : $\vec{F} = q\vec{E}$



* For a positively charged particle, $\vec{E}$ and $\vec{F}$ are in the same direction
* For a negatively charged particle $\vec{E}$ and $\vec{F}$ are in opposite directions.

Ex 21.6: Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge $Q = -3.0 \times 10^{-6}$ C.

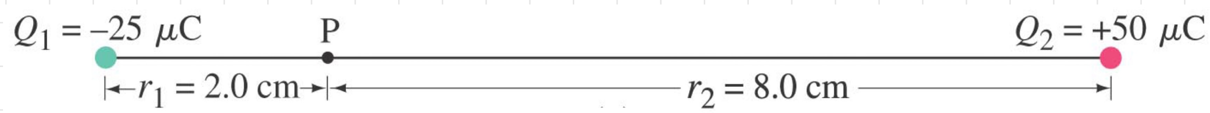


$$E = k\frac{Q}{r^2} \quad (9.0 \times 10^9\ Nm^2/C^2)\frac{(3.0 \times 10^{-6} C)}{(0.30 m)^2}$$

$E = 3.0 \times 10^5$ N/C ↗ magnitude

* The field $\vec{E}$ is toward the charged Particle Q.

Ex Two point charges are separated by a distance of 10.0 cm. One has a charge of -25 μC and the other +50 μC. (a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge. (b) If an electron (mass = 9.11 x 10-31 kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?



$\vec{E}_P = \vec{E}_1 + \vec{E}_2$
(superposition principle)

a) $E_P = k\dfrac{Q_1}{r_1^2} + k\dfrac{Q_2}{r_2^2} = k\left(\dfrac{Q_1}{r_1^2} + \dfrac{Q_2}{r_2^2}\right)$

$= (9.0\times10^9\ Nm^2/C^2)\left[\dfrac{25\times10^{-6}C}{(2\times10^{-2}m)^2} + \dfrac{50\times10^{-6}C}{(18\times10^{-2}m)^2}\right]$

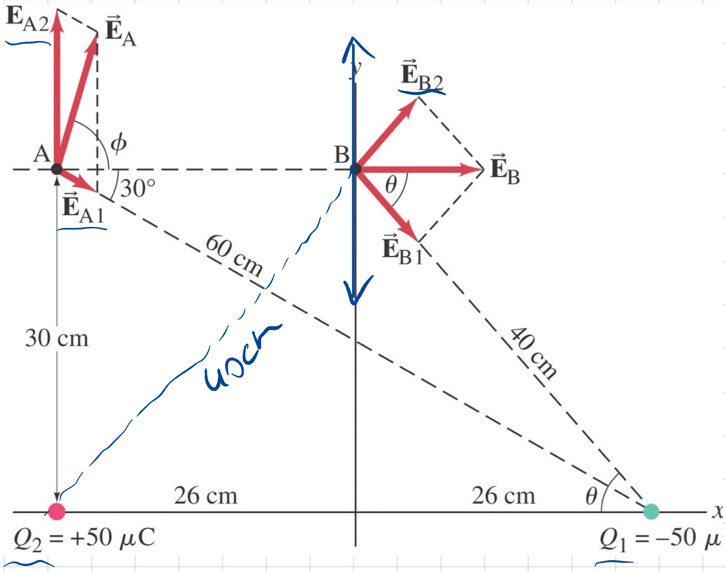
$E_P = 6.3\times10^8\ N/C$ * E_P is along the $-x$ direction!

b) $a = \dfrac{F}{m} = \dfrac{qE_P}{m} \Rightarrow a = \dfrac{\overbrace{(1.6\times10^{-19}C)}^{q\text{ (charge of an electron)}}(6.3\times10^8\ N/C)}{\underbrace{9.11\times10^{-31}kg}_{m}} \Rightarrow a = 1.1\times10^{20}\ m/s^2$

* $\vec{a}$ vector is along the $+x$ direction

Ex

Example 21-8: above two point charges. Calculate the total electric field
(a) at point A and
(b) at point B in the figure due to both charges, Q1 and Q2.



a) $\vec{E}_A = \vec{E}_{A_1} + \vec{E}_{A_2}$

$E_{A_1} = k\dfrac{Q_1}{r_{A_1}^2} = (9.0\times10^9\ Nm^2/C^2)\ \dfrac{50\times10^{-6}C}{(0.60m)^2} = 1.25\times10^6\ N/C$

$E_{A_2} = k\dfrac{Q_2}{r_{A_2}^2} = (9.0\times10^9\ Nm^2/C^2)\ \dfrac{50\times10^{-6}C}{(0.30m)^2} = 5.0\times10^6\ N/C$

$E_{Ax} = E_{A_1}\cdot\cos30 = (1.25\times10^6\ N/C)\cos30° = 1.1\times10^6\ N/C$

$E_{Ay} = E_{A2} - E_{A1}\sin30° = (5.0\times10^6\ N/C) - (1.25\times10^6\ N/C)\sin30° = 4.4\times10^6\ N/C$

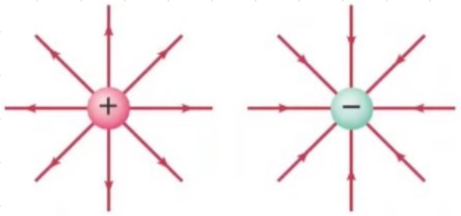
$E_A = \left(E_{Ax}^2 + E_{Ay}^2\right)^{1/2} = 4.5\times10^6\ N/C$, $\phi = \arctan(E_{Ay}/E_{Ax}) = 76°$

b) $E_{B1} = E_{B2} = (9.0\times10^9\ Nm^2/C^2)\ \dfrac{50\times10^{-6}C}{(0.40m)^2} = 2.8\times10^6\ N/C$, $E_B = 2E_{B1}\cos\theta$

Electric field of a continuous charge distribution:

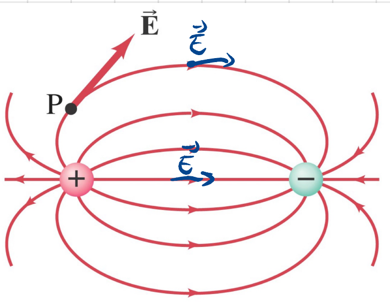
$dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{r^2} \Rightarrow \vec{E} = \int d\vec{E}$ # Need a separate integral for each component

Electric field lines:



Electric field lines starts on Positive charges and end on negative charges.

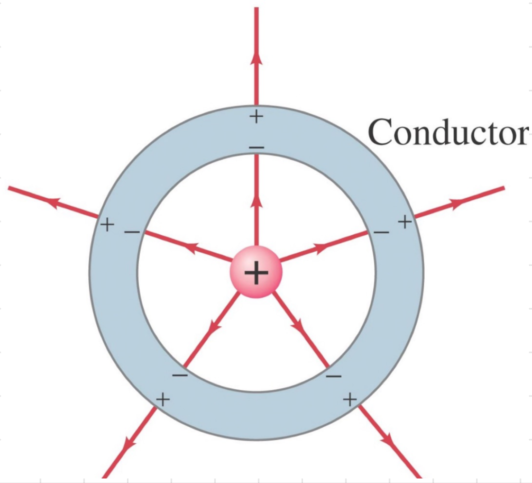
If you have a dipole, two opposite charges in sign



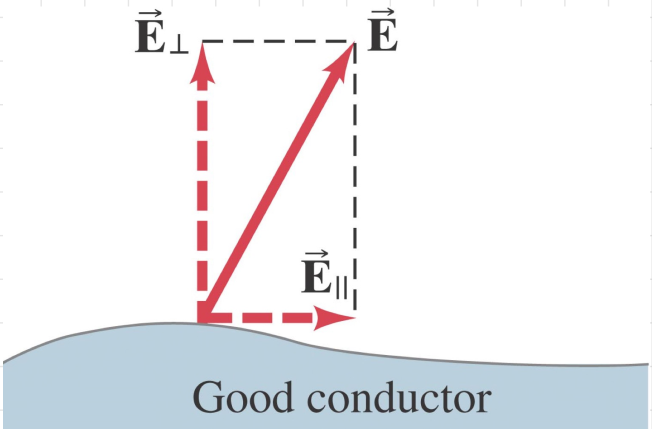
★ $\vec{E}$. field field vector will be always tangent to the electric field lines!

★ The number of lines starting on a positive charge is proportional to the amount of charge.

★ The electric field is stronger where the lines are closer together.



★ The net charge on a conducter resides on its outer surface.
Static $\vec{E}$ field in a conductor is always zero.



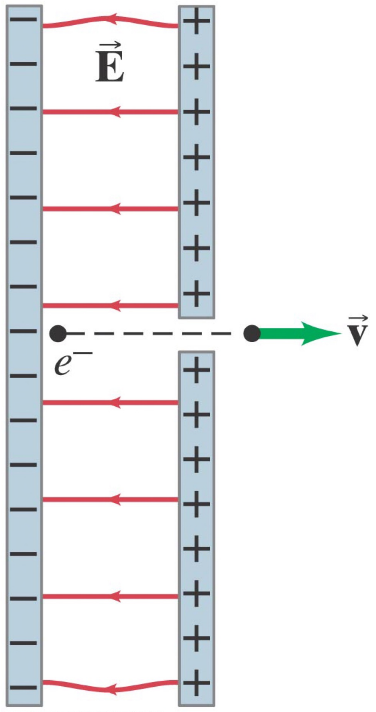
★ The electric field is always perpendicular to the surface of a good conductor.

Motion of a charged particle in electric field:

The force on a charged particle in $\vec{E}$. field: $\vec{F} = q\vec{E}$, $\vec{a} = \frac{\vec{F}}{m}$

Ex

21-14: An electron (mass m = 9.11 x 10-31 kg) is accelerated in the uniform field (E = 2.0 x 104 N/C) between two parallel charged plates. The separation of the plates is 1.5 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.



a) $a = \frac{qE}{m} = \frac{(1.6\times10^{-19}C)(2.0\times10^{4}N/C)}{(9.11\times10^{-31}kg)} \Rightarrow a = 3.5\times10^{15}m/s^2$

$V_0 = 0, \quad V = ?$
$d = 1.5\times10^{-2}m$

$V^2 = \cancel{V_0^2}^{\,0} + 2ad$

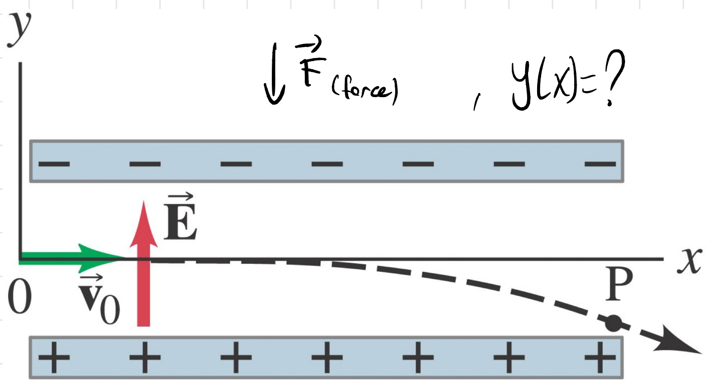
$V = \left[2(3.5\times10^{15}m/s^2)(1.5\times10^{-2}m)\right]^{1/2} \simeq 10^7 m/s$

b) $F_E = qE = (1.6\times10^{-19}C)(2.0\times10^{4}N/C) = 3.2\times10^{-15}N$

$F_G = mg = (9.11\times10^{-31}kg)(9.8m/s^2) = 8.9\times10^{-30}N$ Too small

Ex

21-16: Suppose an electron traveling with speed v0 = 1.0 x 107 m/s enters a uniform electric field, which is at right angles to v0 as shown. Describe its motion by giving the equation of its path while in the electric field. Ignore gravity.



$a_y = \frac{F}{m} = qE/m \Rightarrow a_y = -\frac{eE}{m}$

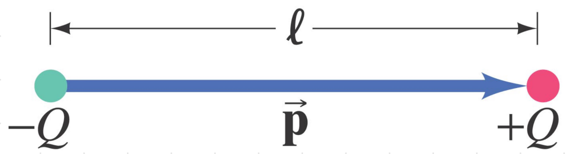
Since F_x is zero, $a_x = 0$

$y = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$ and $x = \cancel{x_0}^{\,0} + V_{0x}t$

$\hookrightarrow t = x/V_0$

$y(x) = \frac{eE}{2mV_0^2}x^2$

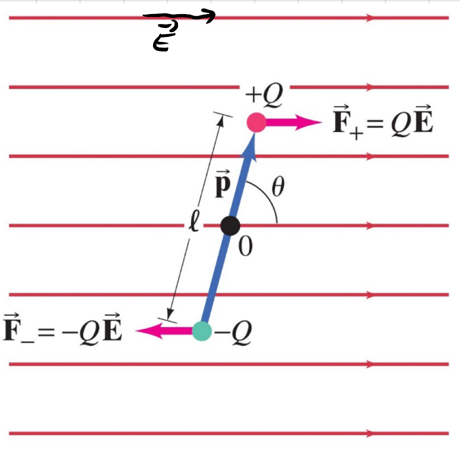
Electric Dipole:



$\vec{P} = Q\,\vec{\ell}$

↳ Dipole moment vector

Electric Dipole in E. field:



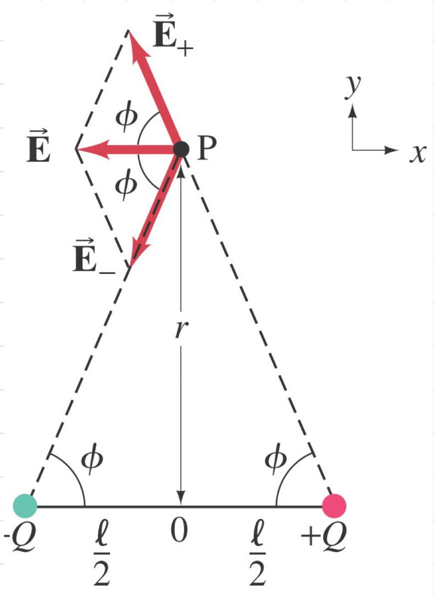
$Q_{net} = Q - Q = 0 \Rightarrow \vec{F}_{Net} = \vec{F}_+ + \vec{F}_- = 0$

What about the torque?

$\tau = \frac{1}{2}\sin\theta\, QE + \frac{1}{2}\sin\theta\, QE = \underbrace{Q\ell}_{|\vec{P}|} E\sin\theta = PE\sin\theta$

$\vec{\tau} = \vec{P} \times \vec{E}$

Electric Field due to a Dipole:



$\vec{E} = \vec{E}_+ + \vec{E}_-$ and $E_+ = E_- = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{K}\frac{Q}{r^2+\ell^2/4}$

$d^2 = r^2 + \ell^2/4$ Due to the symetry at point P,

$E = 2E_+\cos\phi = 2E_-\cos\phi$

$$E = 2\underbrace{\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q}{r^2+\ell^2/4}\right)}_{E_+}\underbrace{\left(\frac{\ell}{2\left(r^2+\ell^2/4\right)^{1/2}}\right)}_{\cos\phi}$$

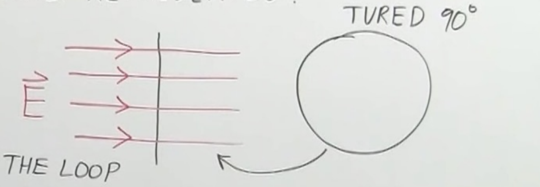
Recalling that $P = Q\ell$, $E = \frac{1}{4\pi\varepsilon_0}\frac{P}{\left(r^2+\ell^2/4\right)^{3/2}}$

★ Assuming $r \gg \ell$:

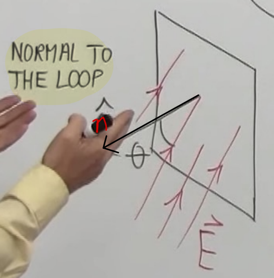
$E \approx \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}$

IF A LOOP OF AREA = A IS PLACED WITHIN AN ELECTRIC FIELD WITH THE PLANE OF THE LOOP PERPENDICULAR TO THE ELECTRIC FIELD THEN THE ELECTRIC FLUX THROUGH THE LOOP IS DEFINED AS: $\boxed{\Phi_E = EA}$ AND CAN BE GRAPHICALLY REPRESENTED:

PROPORTIONAL TO # OF ELECTRIC FIELD LINES PASSING THROUGH THE LOOP

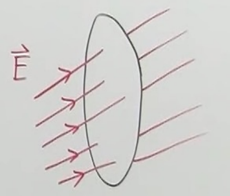


IF THE ELECTRIC FIELD IS NOT PERPENDICULAR TO THE PLANE OF THE LOOP:



THEN $\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot\hat{n}A = EA\cos\theta$

$$\boxed{\Phi_E = EA\cos\theta}$$



EX: IF $A = 5m^2$ $E = 200\frac{N}{C}$ $\theta = 60°$

$$\Phi_E = \left(200\tfrac{N}{C}\right)(5m^2)(\cos 60°) = 500\,\frac{Nm^2}{C}$$

IF A LOOP OF AREA = A IS PLACED WITHIN AN ELECTRIC FIELD WITH THE PLANE OF THE LOOP PERPENDICULAR TO THE ELECTRIC FIELD THEN THE ELECTRIC FLUX THROUGH THE LOOP IS DEFINED AS: $\boxed{\Phi_E = EA}$ (OR IF NOT PERPENDICULAR) $\boxed{\Phi_E = EA\cos\theta}$

$\vec{E} = 2000\frac{N}{C}\hat{x}$ $A = 5m^2$ $\Phi_E = 10000\frac{N}{C}m^2$

$\vec{E} = 50\frac{N}{C}\hat{x}$ $A = 80m^2$ $\Phi_E = 4000\frac{N}{C}m^2$

$\vec{E} = 2000\frac{N}{C}\hat{x}$ $A = 0.1m^2$ $\Phi_E = 200\frac{N}{C}m^2$

$\vec{E} = 50\frac{N}{C}\hat{x}$ $A = 0.2m^2$ $\Phi_E = 10\frac{N}{C}m^2$

IF AN IMAGINARY SPHERE IS PLACED AROUND A POINT CHARGE, WITH THE POINT CHARGE AT THE CENTER OF THE SPHERE THEN GAUSS'S LAW CAN BE DEFINED BY THE EQUATION

$$\boxed{EA = \frac{Q_{INSIDE}}{\epsilon_0}}$$

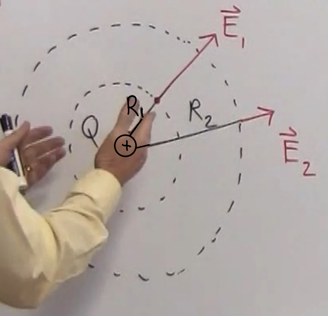
WHERE E IS THE MAGNITUDE OF THE ELECTRIC FIELD AT THE SURFACE OF THE SPHERE (KNOWN AS THE GAUSSIAN SURFACE) AND DIRECTED PERPENDICULAR TO THE SURFACE

$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{Q_{IN}}{\epsilon_0}}$$

A = SURFACE AREA OF THE GAUSSIAN SURFACE

Q_{INSIDE} = CHARGE ENCLOSED BY THE GAUSSIAN SURFACE

ϵ_0 = PERMITIVITY OF FREE SPACE $= 8.85\times10^{-12}\frac{C^2}{Nm^2}$



THE NUMBER OF ELECTRIC FIELD LINES PASSING THROUGH ANY SIZE SURFACE REMAINS CONSTANT $\Rightarrow \Phi_E$ = CONSTANT

$\therefore \Phi_E = EA \propto Q_{IN}$ $EA = \frac{Q_{IN}}{\epsilon_0}$