EEE211: Computational Programming for Engineers
Midterm Examination

Date: 11.12.2020 Time: 18:00 – 20:00

Q1 (30 points)	
Q2 (30 points)	
Q3 (40 points)	
Total (100)	

General Rules:

1. **<u>Only</u>** scientific calculators are allowed

2. You have to **switch on** your camera

3. You have to upload your answers as **one .pdf** file the

4. We **do not accept** any files sent by email

5. Your handwriting should be clear

6. First page of your answers sheets should include a **photo of your student ID**, your name and student ID (written clearly)

Bisection iteration num formula

$$n \geq \frac{\log\left(\frac{b-a}{\varepsilon}\right)}{\log 2}$$

7. **This exam is not open book**

8. **You have to show all your work in details**

9. **Exam duration is 110 min. and last 10 min. for uploading your file to the blackboard system**

Bisection
Error calculation

- The iteration can be stopped using the following error measure:

$$\varepsilon_a = \left| \frac{x_r^{\text{new}} - x_r^{\text{old}}}{x_r^{\text{new}}} \right| 100\%$$

Where: x_r^{new} and x_r^{old} are the roots from the present iteration and from the previous iteration, respectively.

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$\varepsilon = \left| \frac{d}{dR} + \Delta \right|$

$$\varepsilon = \left| \underbrace{(2xy^2+1)}_{0.9} 0.1 \right| + \left| \underbrace{(2yx^2+1)}_{0.125} 0.025 \right|$$

Question # 1 (30 points):

a) **(15 points)** Compute the error for the following function:

$\left| \frac{d}{dx} \cdot \Delta x \right| + \frac{d}{dy} \times \Delta y$

$$F(x,y) = x^2y^2 + x + y \qquad \text{at point (-1,2)}$$

$2xy^2 + 1$

Assume the errors in x and y are: $\Delta x = 0.1$ and $\Delta y = 0.025$ 0.825

b) **(15 points)** The area of a triangle with sides a,b and c and an angle θ is given by:

$$S = ½\ bc\ sin(\theta)$$

$\frac{30 \times \pi}{180}$ 0.5236 $\frac{0.01 \times \pi}{180}$ 1.7453×10^{-4}

$n \geq \frac{\log\left(\frac{b-a}{\varepsilon}\right)}{\log 2}$

Assuming: b = 4.0 ± 0.005 m , c = 3.0 ± 0.005 and θ = $30^0 \pm 0.01^0$
What is the maximum error in S?.
(Hint: the value of the angle θ should be in radians, $1^0 = \pi/180$)

a	b	f(a)	f(b)	c	f(c)
2.5	4	-0.375	6	3.25	0.703125
2.5	3.25	-0.375	0.732125	2.875	-0.205078125
2.875	3.25				

Question # 2 (30 points):

a) (**10 points**) Determine the minimum number of iteration required by the bisection method to obtain the root of the following function:

$a = 2.5$, $b = 4$ $\quad 2^n = 1500$, $n = \log_2 1500$ $\quad \frac{b-a}{2^n} < \text{tol}$ $\quad \frac{4-2.5}{0.001} < 2^n$

$$F(x) = x^3 - 6x^2 + 11x - 6 = 0$$

In the interval [2.5,4] with error tolerance $\varepsilon_r = 0.001$

b) (**20 points**) Using the Newton-Raphson method, determine a positive real root of the following function:

$X_1 = X_0 - \frac{f(x_0)}{f'(x_0)}$ $\quad x_0 = 0.5$

$$F(x) = cos\ x - x^3 = 0$$

$f'(x) = -\sin x - 3x^2 = 0$

$x_1 = 0.5 - \frac{0.87496}{-0.7%%}$, $x_1 = -0.57660$

Start the iterations with the initial value of $x_0 = 0.5$
Stop the iterations if the absolute relative error is $|\varepsilon_a| = 0.001$

$x_2 = -0.5766 - \frac{1.1816}{-0.32240}$, $x_2 = 2.1311$

$x_1 = 0.57642$
$x_2 = 0.5786$
$x_3 = 0.57863$

Question # 3 (40 points):

a) (**15 points**) The distance of a moving object as a function of time is given by the following table:

fdd at x = 0.5
$\frac{6.80 - 3.65}{0.5} = 6.3$ 6.8
bdd at x = 0.5 $\frac{3.65 - 0}{0.5} = 7.3$

$f'(x) = \frac{(x_{i+1}) - (x_{i-1})}{2h}$
$f''(x) = \frac{(x_{i+1}) - 2(x_i) + (x_{i-1})}{h^2}$

Step = 0.25

t (sec)	x(m)
0	0
0.5	3.65
1.0	6.80
1.5	9.90
2.0	12.15

Use the central divided difference to compute the approximate velocity of the object at t = 0.5 s and t = 1.25 s.

cdd
$\frac{9.90 - 6.80}{0.5} = 6.2$

$h = \frac{\Delta x}{3}$

Secant
$x_{n+1} = x_n - f(x_n) \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$

$\frac{b-a}{3n}$

$x_2 = x_1 - y_1 \frac{x_1 - x_0}{y_1 - y_0}$

$x_2 = x_1 - y_1 \dfrac{x_1 - x_0}{y_1 - y_0}$

- Use secant method to estimate the root of

f(x) = ln x.

- Start the computation with value of
  - $x_i = x_{i-1} = 0.5$
  - $x_u = x_i = 5.0$
- Solution

$x_2 = 5.0 - 1.609 \dfrac{5.0 - 0.5}{1.609 + (0.693)}$

The secant method

Newton

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

b) (**15 points**) Use the multi-segment trapezoidal rule to determine the value of the following integration:

$\Delta x = \dfrac{b-a}{n}$, $n = 6$, $\Delta x = \dfrac{1.8-0}{6} = 0.3$

$$\int_0^{1.8} f(x)dx$$

0.15

$\dfrac{0.3}{2}\left[f(0) + 2f(0.3) + 2f(0.6) + 2f(0.9) + 2f(1.2) + 2f(1.5) + f(1.8)\right] = 6.53$

For the following data set:

x	$f(x)$
0	0.5
0.3	0.6
0.6	0.8
0.9	1.3
1.2	2
1.5	3.2
1.8	4.8

$\Delta x = \dfrac{b-a}{n} = 0.3$

$6x^3$

$36x$ $\quad 0.2 < x < a$

$0.15\left[f(n_0) + 2f(n_1) - - - 2f(n_{n-1}) + f(n_n)\right]$

$E \le \dfrac{(b-a)^3}{12n^2}\left[\max f''(x)\right]$

$f''(x)$

c) (**10 points**)Use the one-interval Simpson's 1/3 rule to compute the integration of the following function:

$$f(x) = 0.2 + 25x + 3x^2 + 2x^4$$

$n = 1$ $\quad \Delta x = 2$

between $a = 0$ to $b = 2$

Determine the true relative error (with respect to the ture value)

$x_0 = 0$ $\quad x_2 = 2$

$x_1 = 1$

$h = \dfrac{b-a}{2n}$

$\dfrac{2}{2} = 1$

$\dfrac{1}{3}\left[f(0) + 4f(1) + f(2)\right]$

$\dfrac{h}{3}\left[\quad\right]$

$h = \dfrac{b-a}{2n}$

$\left(f(a) + 2f(a)\right)$

$x_1 = x_0 - y_0 \dfrac{x_0 - x_{-1}}{y_0 - y_{-1}}$

fdd

$f(x)' = \dfrac{f(x_{+1}) - f(x)}{x_{+1} - x}$

b) (**15 points**) Use the multi-segment trapezoidal rule to determine the value of the following integration:

$$\int_0^{1.8} f(x)dx$$

For the following data set:

x	*f(x)*
0	0.5
0.3	0.6
0.6	0.8
0.9	1.3
1.2	2
1.5	3.2
1.8	4.8

c) (**10 points**)Use the one-interval Simpson's 1/3 rule to compute the integration of the following function:

$$f(x) = 0.2 + 25x + 3x^2 + 2x^4$$

between $a = 0$ to $b = 2$
Determine the true relative error (with respect to the ture value)

$$f''(x) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$

$$\frac{b-a}{3n}\left[f(0) + 4f(1) + 2f(2) \cdots f(n)\right]$$