IZMIR UNIVERSITY OF ECONOMICS

Faculty of Engineering

Term	**: 2024-2025 Fall**
Course ID	**: EEE 211/213: "Computational Methods for Engineers"**
Exam	**: Midterm Exam**
Date	**: 11.11.2024**
Duration	**: 120 min.**
Instructor	**: Prof. Dr. Diaa Gadelmavla**

Full Name : ....Sample Answers..

Student ID : ....

Classroom : .... Section : ....

Information on exam rules

Electronic devices such as laptops, mobile phones, and smartwatches are generally prohibited in the examination room. However, exceptions can be made for individuals with special needs, provided they have valid medical documentation. Requests for exceptions must be submitted with prior written approval from the academic advisor, and they should include details on the necessary measures to maintain the integrity and security of the examination.

Please refrain from engaging in cheating or any other prohibited activities during the examination. Suspected cheating may result in a score of zero on your exam, and any students found cheating may face disciplinary actions in accordance with law #2547. This includes actions such as using unauthorized electronic devices, communicating with classmates, exchanging exam or formula sheets, or using unauthorized written materials during the exam, all of which qualify as attempted cheating.

Directions:

1. You are only allowed to use a standard scientific calculator.

2. Use four-decimal digits accuracy in your computations.

Declaration

I affirm that the activities and assessments completed as part of this examination are entirely my own work and comply with all relevant rules regarding copyright, plagiarism, and cheating. I acknowledge that if there is any question regarding the authenticity of any portion of my assessment, I may be subject to oral examination. The signatory of evidence records may also be contacted, or a disciplinary process may be initiated as per law #2547.

Signature of Student:

Question	1	2	3	4				
Score	/25	/25	/25	/25				
Total	/100							

Question # 1:

Assume a function $f = \frac{x^2 y}{u^2}$ with the errors in *x*, *y*, and *u* being 1%, -1% and 2%, respectively. Find the approximate percentage relative error in the function *f*.

Answer:

First find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial u}$:

$$\frac{\partial f}{\partial x} = \frac{3x^2 y}{u^2}, \quad \frac{\partial f}{\partial y} = \frac{x^3}{u^2}, \quad \frac{\partial f}{\partial u} = -\frac{2x^3 y}{u^3}.$$

Now write down an expression for δf

$$\delta f \simeq \frac{3x^2 y}{u^2}\delta x + \frac{x^3}{u^2}\delta y - \frac{2x^3 y}{u^3}\delta u$$

Hence write down an expression for the percentage relative error in f:

$$\frac{\delta f}{f} \times 100 \simeq \frac{3x^2 y}{u^2} \times \frac{u^2}{x^3 y}\delta x \times 100 + \frac{x^3}{u^2} \times \frac{u^2}{x^3 y}\delta y \times 100 - \frac{2x^3 y}{u^3} \times \frac{u^2}{x^3 y}\delta u \times 100$$

Finally, calculate the value of the percentage relative error:

$$\begin{aligned} \frac{\delta f}{f} \times 100 &\simeq 3\frac{\delta x}{x} \times 100 + \frac{\delta y}{y} \times 100 - 2\frac{\delta u}{u} \times 100 \\ &= 3(1) - 1 - 2(2) = -2\% \end{aligned}$$

Question # 2:

Find one positive root for the following function using the Newton's method:

$$F(x) = \cos(x) - x^3 = 0$$

Start the iterations with an initial value of $x_0 = 0.5$. Stop the iterations after 4 significant digits are equal between two successive iterations.

Answer:

Newton's Method:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Compute $f'(x) = -\sin x - 3x^2$.

Iteration 1. k=0: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{\cos(0.5)-(0.5)^3}{-\sin(0.5)-3(0.5)^2} = 1.1121.$

Iteration 2. k=1: $x_2 = x_1 - \frac{f(x_1}{f'(x_1)} = 0.9097.$

Iteration 3. k=2: $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.8672.$

Iteration 4. k=3: $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.8654.$

Iteration 5. k=4: $x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.8654.$

Question # 3:

Determine one initial interval that contains a root of *tan(x) – 2x +1* = 0. Based on the interval you obtain, what is the minimum bisection iterations ***n*** required to get this root in this interval with accuracy $\epsilon = 10^{-6}$. Show your work in details.

Answer:

Let $f(x) = \tan x - 2x + 1 = 0$. There are many intervals containing a solution of f. We show here only one of them. Since

$f(1) = \tan 1 - 2(1) + 1 = 0.55740 > 0$ and $f(2) = \tan 2 - 2(2) + 1 = -5.1850 < 0$

there exists a root of f between 1 and 2. So requested interval is $\underline{[1, 2]}$.

Since $|p - p_n| \le \frac{b-a}{2^n}$, then for $a = 1$, $b = 2$ we have

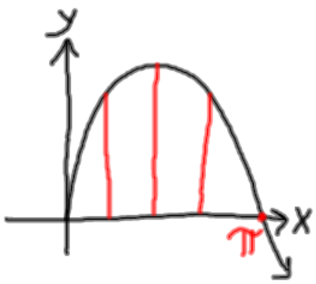
$$\frac{2-1}{2^n} \le \varepsilon = 10^{-6} \Rightarrow 10^6 \le 2^n \Rightarrow 6 \log 10 \le n \log 2 \Rightarrow \frac{6}{\log 2} \le n \Rightarrow \frac{6}{0.30103} \le n$$

$$\underline{n \ge 20}.$$

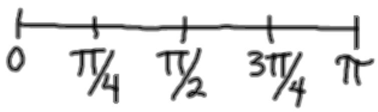
<u>Question # 4:</u>

Compute the area under the curve given by *f(x) = sin (x)* on the interval [*0, π*] using the Trapezoidal rule with *n* = 4 traperzoids.

Answer:



$$\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$$



$$T = \frac{b-a}{2n}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$$T = \frac{\pi}{4(2)}\left[0 + 2\left(\frac{\sqrt{2}}{2}\right) + 2(1) + 2\left(\frac{\sqrt{2}}{2}\right) + 0\right]$$

$$T = \frac{\pi}{8}\left[0 + \sqrt{2} + 2 + \sqrt{2} + 0\right]$$

$$T = \frac{\pi}{8}\left[2 + 2\sqrt{2}\right]$$

$$T \approx 1.896$$