MATH 240
Probability for Engineers

Exercises

Example 1

In the process of producing engine valves, the valves are subjected to a first grind. Valves whose thicknesses are within the specification are ready for installation. Those valves whose thicknesses are above the specification are reground, while those whose thicknesses are below the specification are scrapped. Assume that after the first grind, 70% of the valves meet the specification, 20% are reground, and 10% are scrapped. Furthermore, assume that of those valves that are reground, 90% meet the specification, and 10% are scrapped.

a. Find the probability that a valve is ground only once.

b. Given that a valve is not reground, what is the probability that it is scrapped?

c. Find the probability that a valve is scrapped.

d. Given that a valve is scrapped, what is the probability that it was ground twice?

e. Find the probability that the valve meets the specification (after either the first or second grind).

f. Given that a valve meets the specification (after either the first or second grind), what is the probability that it was ground twice?

g. Given that a valve meets the specification, what is the probability that it was ground only once?

Example 2

The following table presents probabilities for the number of times that a certain computer system will crash in the course of a week. Let *A* be the event that there are more than two crashes during the week, and let *B* be the event that the system crashes at least once. Find a sample space. Then find the subsets of the sample space that correspond to the events *A* and *B*. Then find *P(A)* and *P(B)*.

Number of Crashes	Probability
0	0.60
1	0.30
2	0.05
3	0.04
4	0.01

Example 3

In how many ways can 7 graduate students be assigned to 1 triple and 2 double hotel rooms during a conference?

Example 4

A California study concluded that following 7 simple health rules can extend a man's life by 11 years on the average and a woman's life by 7 years. These 7 rules are as follows: no smoking, get regular exercise, use alcohol only in moderation, get 7 to 8 hours of sleep, maintain proper weight, eat breakfast, and do not eat between meals. In how many ways can a person adopt 5 of these rules to follow

(a) if the person presently violates all 7 rules?

(b) if the person never drinks and always eats breakfast?

Example 5

A particular automatic sprinkler system has two different

types of activation devices for each sprinkler head. One type has a reliability of 0.9; that is, the probability that it will activate the sprinkler when it should is 0.9. The other type, which operates independently of the first type, has a reliability of 0.8. If either device is triggered, the sprinkler will activate. Suppose a fire starts near a sprinkler head.

Example 5 (Contin.)

a. What is the probability that the sprinkler head will be activated?

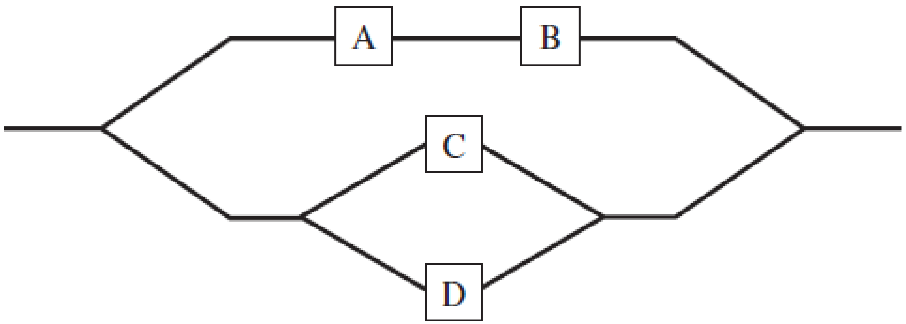
b. What is the probability that the sprinkler head will not be activated?

c. What is the probability that both activation devices will work properly?

d. What is the probability that only the device with reliability 0.9 will work properly?

Example 6

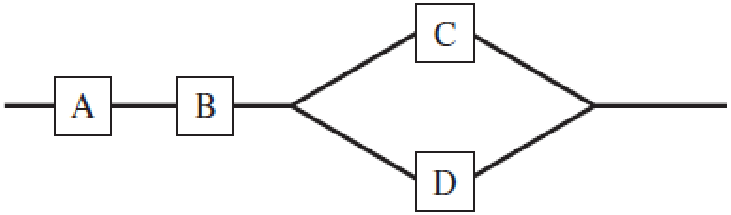
A system consists of four components connected as shown in the following diagram:



Assume A, B, C, and D function independently. If the probabilities that A, B, C, and D fail are 0.10, 0.05, 0.10, and 0.20, respectively, what is the probability that the system functions?

Example 7

A system consists of four components, connected as shown in the diagram. Suppose that the components function independently, and that the probabilities of failure are 0.05 for A, 0.03 for B, 0.07 for C, and 0.14 for D. Find the probability that the system functions.



Example 8

An extrusion die is used to produce aluminum rods. Specifications are given for the length and the diameter of the rods. For each rod, the length is classified as too short, too long, or OK, and the diameter is classified as too thin, too thick, or OK. In a population of 1000 rods, the number of rods in each class is as follows:

	Diameter		
Length	**Too Thin**	**OK**	**Too Thick**
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

a- A rod is sampled at random from this population. What is the probability that it is too short?

b- If a rod is sampled at random, what is the probability that it is either too short or too thick?

Example 9

Six hundred paving stones were examined for cracks, and 15 were found to be cracked. The same 600 stones were then examined for discoloration, and 27 were found to be discolored. A total of 562 stones were neither cracked nor discolored. One of the 600 stones is selected at random.

a. Find the probability that it is cracked, discolored, or both.

b. Find the probability that it is both cracked and discolored.

c. Find the probability that it is cracked but not discolored.

Example 10

Human blood may contain either or both of two antigens, A and B. Blood that contains only the A antigen is called type A, blood that contains only the B antigen is called type B, blood that contains both antigens is called type AB, and blood that contains neither antigen is called type O. At a certain blood bank, 35% of the blood donors have type A blood, 10% have type B, and 5% have type AB.

a. What is the probability that a randomly chosen blood donor is type O?

b. A recipient with type A blood may safely receive blood from a donor whose blood does not contain

Example 9

The number of flaws in a 1-inch length of copper wire manufactured by a certain process varies from wire to wire. Overall, 48% of the wires produced have no flaws, 39% have one flaw, 12% have two flaws, and 1% have three flaws.

a. Plot the cumulative distribution function of the random variable X that represents the number of flaws in a randomly chosen wire.

b. Evaluate mean and standard deviation of X.

c. Evaluate median of X.

Example 10

A survey of cars on a certain stretch of highway during morning commute hours showed that 70% had only one occupant, 15% had 2, 10% had 3, 3% had 4, and 2% had 5. Let X represent the number of occupants in a randomly chosen car.

a. Find the probability mass function of X.

b. Find $P(X \leq 2)$.

c. Find $P(X > 3)$.

d. Find μ_X.

e. Find σ_X.

Example 11

The element titanium has five stable occurring isotopes, differing from each other in the number of neutrons an atom contains. If X is the number of neutrons in a randomly chosen titanium atom, the probability mass function of X is given as follows:

x	24	25	26	27	28
$p(x)$	0.0825	0.0744	0.7372	0.0541	0.0518

a. Find μ_X.

b. Find σ_X.

Example 12

After manufacture, computer disks are tested for errors. Let X be the number of errors detected on a randomly chosen disk. The following table presents values of the cumulative distribution function $F(x)$ of X.

x	$F(x)$
0	0.41
1	0.72
2	0.83
3	0.95
4	1.00

a. What is the probability that two or fewer errors are detected?

b. What is the probability that more than three errors are detected?

c. What is the probability that exactly one error is detected?

d. What is the probability that no errors are detected?

e. What is the most probable number of errors to be detected?

Example 13

The main bearing clearance (in mm) in a certain type of engine is a random variable with probability density function

$$f(x) = \begin{cases} 625x & 0 < x \leq 0.04 \\ 50 - 625x & 0.04 < x \leq 0.08 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability that the clearance is less than 0.02 mm?

b. Find the mean clearance.

c. Find the standard deviation of the clearances.

Example 14

A computer sends a packet of information along a channel and waits for a return signal acknowledging that the packet has been received. If no acknowledgment is received within a certain time, the packet is re-sent. Let *X* represent the number of times the packet is sent. Assume that the probability mass function of *X* is given by

$$p(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4, \text{ or } 5 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

a. Find the value of the constant c so that p(x) is a probability mass function.

b. Find P(X = 2).

c. Find the mean number of times the packet is sent.

d. Find the variance of the number of times the packet is sent.

e. Find the standard deviation of the number of times the packet is sent.

Example 15

The probability density function for the random variable *X* is determined as follows.

$$f(x) = \begin{cases} \dfrac{x}{10}; & 0 \le x < 2 \\ \dfrac{(10-x)}{40}; & 2 \le x \le 10 \end{cases}$$

Calculate the mean and the median of *X*.

Example 16

In a lot of 10 components, 2 are sampled at random for inspection. Assume that in fact exactly 2 of the 10 components in the lot are defective. Let X be the number of sampled components that are defective.

a. Find $P(X = 0)$.

b. Find $P(X = 1)$.

c. Find $P(X = 2)$.

d. Find the probability mass function of X.

e. Find the mean of X.

f. Find the standard deviation of X.

Example 17

Let A and B be events with $P(A) = 0.3$ and $P(A \cup B) = 0.7$.

a. For what value of $P(B)$ will A and B be mutually exclusive?

b. For what value of $P(B)$ will A and B be independent?

Example 18

Silicon wafers are used in the manufacture of integrated circuits. Of the wafers manufactured by a certain process, 10% have resistances below specification and 5% have resistances above specification.

a. What is the probability that the resistance of a randomly chosen wafer does not meet the specification?

b. If a randomly chosen wafer has a resistance that does not meet the specification, what is the probability that it is too low?

Example 19

A certain plant runs three shifts per day. Of all the items produced by the plant, 50% of them are produced on the first shift, 30% on the second shift, and 20% on the third shift. Of all the items produced on the first shift, 1% are defective, while 2% of the items produced on the second shift and 3% of the items produced on the third shift are defective.

a. An item is sampled at random from the day's production, and it turns out to be defective. What is the probability that it was manufactured during the first shift?

b. An item is sampled at random from the day's production, and it turns out not to be defective. What is the probability that it was manufactured during the third shift?

Example 20

A production facility contains two machines that are used to rework items that are initially defective. Let X be the number of hours that the first machine is in use, and let Y be the number of hours that the second machine is in use, on a randomly chosen day. Assume that X and Y have joint probability density function given by

$$f(x) = \begin{cases} \frac{3}{2}(x^2 + y^2) & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability that both machines are in operation for more than half an hour?

b. Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$.

c. Are X and Y independent? Explain.

Example 20 (contin.)

Find $\mathrm{Cov}(X, Y)$.

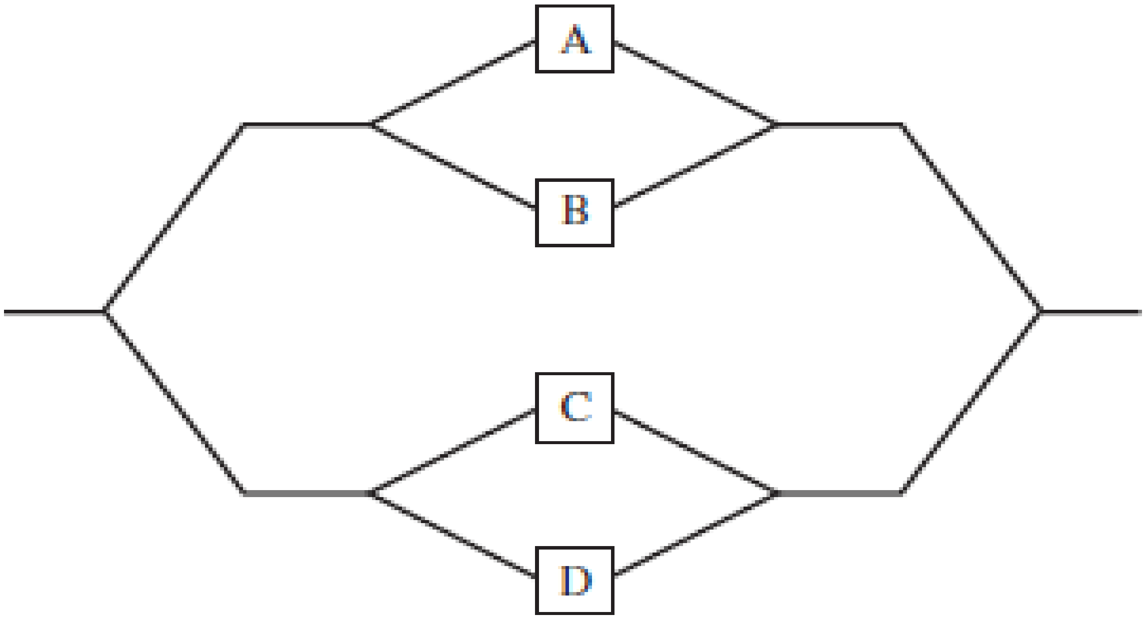
Find $\rho_{X,Y}$.

Find the conditional probability density function $f_{Y|X}(y \mid 0.5)$.

Find the conditional expectation $E(Y \mid X = 0.5)$.

Example 21

A system consists of four components connected as shown.



Assume A, B, C, and D function independently. If the probabilities that A, B, C, and D fail are 0.1, 0.2, 0.05, and 0.3, respectively, what is the probability that the system functions?

Example 22

A commuter must pass through three traffic lights on her way to work. For each light, the probability that it is green when she arrives is 0.6. The lights are independent.

- What is the probability that all three lights are green?
- The commuter goes to work five days per week. Let X be the number of times out of the five days in a given week that all three lights are green. Assume the days are independent of one another. What is the distribution of X?
- Find $P(X = 3)$.

Example 23

Porcelain figurines are sold for \$10 if flawless, and for \$3 if there are minor cosmetic flaws. Of the figurines made by a certain company, 90% are flawless and 10% have minor cosmetic flaws. In a sample of 100 figurines that are sold, let Y be the revenue earned by selling them and let X be the number of them that are flawless.

- Express Y as a function of X.
- Find P(X=4), E(X) and Var(X)
- Find μ_Y.
- Find σ_Y.

Example 24

The number of hits on a certain website follows a Poisson distribution with a mean rate of 4 per minute.

- What is the probability that 5 messages are received in a given minute?
- What is the probability that 9 messages are received in 1.5 minutes?
- What is the probability that fewer than 3 messages are received in a period of 30 seconds?

Example 25

Twenty air-conditioning units have been brought in for service. Twelve of them have broken compressors, and eight have broken fans. Seven units are chosen at random to be worked on. What is the probability that three of them have broken fans?

Example 26

A traffic light at a certain intersection is green 50% of the time, yellow 10% of the time, and red 40% of the time. A car approaches this intersection once each day. Let X represent the number of days that pass up to and including the first time the car encounters a red light. Assume that each day represents an independent trial.

- Find $P(X = 3)$.
- Find $P(X \leq 3)$.
- Find μ_X
- Find σ_X^2

Example 27

A process that fills packages is stopped whenever a package is detected whose weight falls outside the specification. Assume that each package has probability 0.01 of falling outside the specification and that the weights of the packages are independent.

- Find the mean number of packages that will be filled before the process is stopped.
- Find the variance of the number of packages that will be filled before the process is stopped.
- Assume that the process will not be stopped until four packages whose weight falls outside the specification are detected. Find the mean and variance of the number of packages that will be filled before the process is stopped.

Example 28

Of customers ordering a certain type of personal computer, 20% order an upgraded graphics card, 30% order extra memory, 15% order both the upgraded graphics card and extra memory, and 35% order neither. Fifteen orders are selected at random. Let X_1, X_2, X_3, X_4 denote the respective numbers of orders in the four given categories.

- Find $P(X_1 = 3, X_2 = 4, X_3 = 2, \text{ and } X_4 = 6)$.
- Find $P(X_1 = 3)$.

Example 29

Scores on a standardized test are approximately normally distributed with a mean of 480 and a standard deviation of 90.

- What proportion of the scores are above 700?
- What is the 25th percentile of the scores?
- If someone's score is 600, what percentile is she on?
- What proportion of the scores are between 420 and 520?

Example 30

The molarity of a solute in solution is defined to be the number of moles of solute per liter of solution (1 mole = 6.02×10^{23} molecules). If X is the molarity of a solution of sodium chloride (NaCl), and Y is the molarity of a solution of sodium carbonate (Na_2CO_3), the molarity of sodium ion (Na^+) inasolution made of equal parts NaCl and Na_2CO_3 is given by $M = 0.5X + Y$. Assume X and Y are independent and normally distributed, and that X has mean 0.450 and standard deviation 0.050, and Y has mean 0.250 and standard deviation 0.025.

- What is the distribution of M?
- Find $P(M > 0.5)$.

Example 31

A catalyst researcher states that the diameters, in microns, of the pores in a new product she has made have the exponential distribution with parameter $\lambda = 0.25$.

- What is the mean pore diameter?
- What is the standard deviation of the pore diameters?
- What proportion of the pores are less than 3 microns in diameter?
- What proportion of the pores are greater than 11 microns in diameter?

Example 32

The number of traffic accidents at a certain intersection is thought to be well modeled by a Poisson process with a mean of 3 accidents per year.

- Find the mean waiting time between accidents.
- Find the standard deviation of the waiting times between accidents.
- Find the probability that more than one year elapses between accidents.
- Find the probability that less than one month elapses between accidents.
- If no accidents have occurred within the last six months, what is the probability that an accident will occur within the next year?

Example 33

Resistors are labeled 100 Ω. In fact, the actual resistances are uniformly distributed on the interval (95, 103).

- Find the mean resistance.
- Find the standard deviation of the resistances.
- Find the probability that the resistance is between 98 and 102 Ω.

Example 34

The probability that a certain kind of component will survive a shock test is $3/4$. Find the probability that exactly 2 of the next 4 components tested survive.

Example 35

The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?

Example 36

The complexity of arrivals and departures of planes at an airport is such that computer simulation is often used to model the "ideal" conditions. For a certain airport with three runways, it is known that in the ideal setting the following are the probabilities that the individual runways are accessed by a randomly arriving commercial jet:

Runway 1: $p_1 = 2/9$,
Runway 2: $p_2 = 1/6$,
Runway 3: $p_3 = 11/18$.

What is the probability that 6 randomly arriving airplanes are distributed in the following fashion?

Runway 1: 2 airplanes,
Runway 2: 1 airplane,
Runway 3: 3 airplanes

5.23 The probabilities are 0.4, 0.1, 0.3, and 0.2, respectively, that a delegate to a certain convention arrived by air, bus, automobile, or train. What is the probability that among 10 delegates randomly selected at this convention, 3 arrived by air, 3 arrived by bus, 2 arrived by automobile, and 2 arrived by train?

5.11 The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 4 of the next 6 patients having this operation survive?

5.15 It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that

(a) none contracts the disease;

(b) fewer than 2 contract the disease;

(c) more than 3 contract the disease.

Example 37

An automated egg carton loader has a 1% probability of cracking an egg, and a customer will complain if more than one egg per dozen is cracked. Assume that each egg load is an independent event.

a) What is the probability that a carton of a dozen eggs results in a complaint?
b) **b)** What is the mean of the number of cracked eggs in a carton of a dozen eggs?

5.43 A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that

(a) all nationalities are represented;

(b) all nationalities except Italian are represented.

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5.31 A random committee of size 4 is selected from 4 doctors and 2 nurses. Write a formula for the probability distribution of the random variable X representing the number of doctors on the committee. Find $P(2 \leq X \leq 3)$.

Example 38

For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

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Using the geometric distribution with $x = 5$ and $p = 0.01$, we have

$$g(5; 0.01) = (0.01)(0.99)^4 = 0.0096.$$

Example 39

At a "busy time," a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to make a connection. Suppose that we let $p = 0.05$ be the probability of a connection during a busy time. We are interested in knowing the probability that 5 attempts are necessary for a successful call.

Example 40

The probability that an eagle kills a rabbit in a day of hunting is 10%. Assume that results are independent for each day.

a) What is the probability that the first succesful hunt occurs on the fourth day?
b) What is the probability that the first succesful hunt occurs after three days?

5.56 On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection

(a) exactly 5 accidents will occur?

(b) fewer than 3 accidents will occur?

(c) at least 2 accidents will occur?

Example 41

Suppose that the log-ons to a computer network follow a Poisson process with an average of five counts per minute.

a) What is the mean time between log-ons?
b) Determine the probability density and cumulative distribution functions for the time between log-ons?
c) Determine x such that the probability that at least one log-on occurs before time x minutes is 0.95.
d) What is the probability that more than 20 log-ons occur in 5 minutes?

Example 42

The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

a) What is the probability that a laser fails before 6000 hours?
b) What is the life in hours that 95% of the lasers exceed?

6.13 A group of individuals with standard health conditions are put on an experimental diet for one month. The gain in weights of these individuals after a month is normally distributed. They average 1450 grams, with a standard deviation of 250 grams. Find the probability that the gain in weight for a given individual will be

(a) more than 1700 grams;

(b) less than 1250 grams;

(c) between 1100 and 1600 grams.

6.14 The inside diameter of the washers produced by a certain company is normally distributed with a mean of 0.60 centimeters and standard deviation of 0.004 centimeters.

(a) What percentage of the washers have an inside diameter of more than 0.61 centimeters?

(b) Obtain the probability that the inside diameter of the washers will be between 0.595 and 0.605 centimeters.

(c) Below what value of the inside diameter will 20% of the washers fall?

Example 44

In a company, the time a postal clerk spends with his or her customer has an exponential distribution with a mean of four minutes. Suppose a customer has spent four minutes with a postal clerk. What is the probability that he or she will spend at least an additional three minutes with the postal clerk?

Example 45

At a police station in a large city, calls come in at an average rate of four calls per minute. Assume that the time that elapses from one call to the next has the exponential distribution. Take note that we are concerned only with the rate at which calls come in, and we are ignoring the time spent on the phone. We must also assume that the times spent between calls are independent. This means that a particularly long delay between two calls does not mean that there will be a shorter waiting period for the next call. We may then deduce that the total number of calls received during a time period has the Poisson distribution.

1. Find the average time between two successive calls.
2. Find the probability that after a call is received, the next call occurs in less than ten seconds.
3. Find the probability that exactly five calls occur within a minute.
4. Find the probability that less than five calls occur within a minute.
5. Find the probability that more than 40 calls occur in an eight-minute period.

1. On average there are four calls occur per minute, so 15 seconds, or $\frac{15}{60}$= 0.25 minutes occur between successive calls on average.

2. Let T = time elapsed between calls. From part a, $\mu = 0.25$, so m = $\frac{1}{0.25}$ = 4. Thus, T ~ Exp(4). The cumulative distribution function is $P(T < t) = 1 - e^{-4t}$. The probability that the next call occurs in less than ten seconds (ten seconds = 1/6 minute) is $P(T < \frac{1}{6}) = 1 - e^{-4\frac{1}{6}} \approx 0.4866$

3. Let X = the number of calls per minute. As previously stated, the number of calls per minute has a Poisson distribution, with a mean of four calls per minute. Therefore, $X \sim Poisson(4)$, and so $P(X = 5) = \frac{4^5 e^{-4}}{5!} \approx 0.1563$. (5! = (5)(4)(3)(2)(1))

4. Keep in mind that X must be a whole number, so $P(X < 5) = P(X \leq 4)$.
   To compute this, we could take $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$.
   Using technology, we see that $P(X \leq 4) = 0.6288$.

5. Let Y = the number of calls that occur during an eight minute period.
   Since there is an average of four calls per minute, there is an average of (8)(4) = 32 calls during each eight minute period.
   Hence, $Y \sim Poisson(32)$. Therefore, $P(Y > 40) = 1 - P(Y \leq 40) = 1 - 0.9294 = 0.0707$.