May 10, 2022

EEE 208 ELECTRIC CIRCUIT ANALYSIS II
EEE 202 ELECTRIC CIRCUITS II

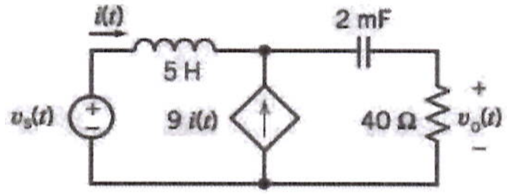
SPRING'22 - MIDTERM EXAM

SOLUTIONS

Name: ________________________________, **Signature:** ________________________

Closed books and closed notes. Use of mobile phones is considered cheating. Show your work clearly in the spaces provided. Otherwise, no credit will be given.

(22 pts) **Q.1** The input to the circuit shown in the following figure is the source voltage $v_s(t)$. The output is the resistor voltage $v_o(t)$. Determine the output voltage when the circuit is at steady state and the input is $v_s(t) = 25\cos(100t - 15°)$ V.

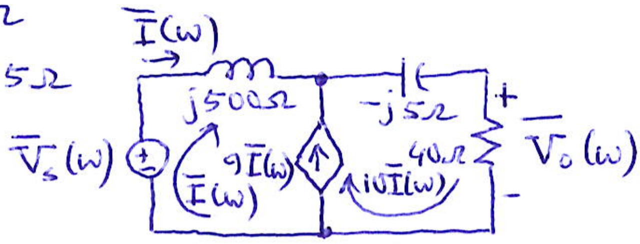


Using phasor method, we get

$\bar{V}_s(\omega) = 25\angle{-15°}$ V.

$\bar{Z}_L = j\omega L = j100 \times 5 = j500\,\Omega$

$\bar{Z}_C = \dfrac{-j}{\omega C} = \dfrac{-j}{100 \times 2 \times 10^{-3}} = -j5\,\Omega$



The circuit in the frequency domain

Applying KVL to the supermesh, we obtain

$j500\bar{I}(\omega) - j5 \times 10\bar{I}(\omega) + 40 \times 10\bar{I}(\omega) = 25\angle{-15}$

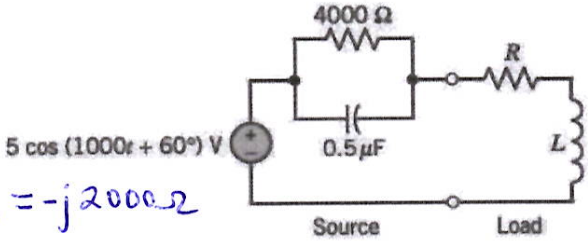
Thus, $\bar{I}(\omega) = \dfrac{25\angle{-15°}}{400 + j450} = \dfrac{25\angle{-15°}}{602\angle{48.37°}}$

$= 0.004152\angle{-63.37°}$ A

$\bar{V}_o(\omega) = 40 \times 10\bar{I}(\omega) = 16.61\angle{-63.37°}$ V

Therefore, $v_o(t) = 16.61\cos(100t - 63.37°)$ V

(22 pts) **Q.2** Determine values of R and L for the circuit shown in the following figure that cause maximum power transfer to the load.



$$\bar{Z}_c = \frac{-j}{\omega C} = \frac{-j}{10^3 \times 0.5 \times 10^{-6}} = -j2000\,\Omega$$

$$\bar{Z}_t = (4000 \,//\, -j2000)\,\Omega$$
$$= \frac{4000(-j2000)}{4000 - j2000} = 800 - j1600\,\Omega$$

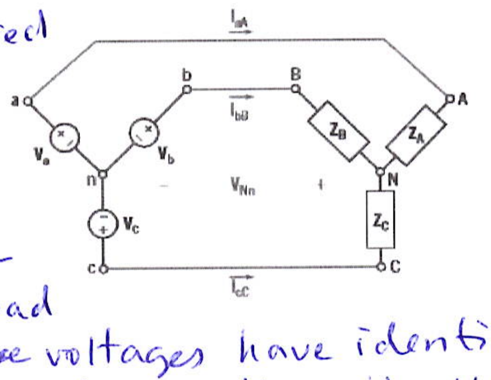
$$\bar{Z}_L = \bar{Z}_t^{*} = 800 + j1600\,\Omega$$

Therefore,

$$\bar{Z}_L = R + j1000L = 800 + j1600$$

$$R = 800\,\Omega \quad \text{and} \quad L = \frac{1600}{1000} = 1.6\text{ H}$$

(20 pts) **Q.3** Determine the complex power delivered to the three-phase load of a three-wire Y-to-Y circuit such as the one shown in the following figure. The phase voltages of the Y-connected source are $\mathbf{V}_a = 110 \angle 0°$ V rms, $\mathbf{V}_b = 110 \angle -120°$ V rms, and $\mathbf{V}_c = 110 \angle 120°$ V rms. The load impedances are $\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 50 + j80\ \Omega$.



The complex power delivered to the three-phase load is three times the complex power delivered to the single-phase load if the circuit is a balanced three-phase circuit. As given, load impedances are equal; phase voltages have identical amplitude and are out of phase with each other by 120°. Thus, the given circuit is a balanced three-phase one. Then,

$$\bar{I}_{aA} = \frac{\bar{V}_a}{\bar{Z}_A} = \frac{110\angle 0°}{50 + j80} = \frac{110\angle 0°}{94.34\angle 58°} = 1.16\angle -58°\text{ A rms}$$

The complex power delivered to the three-phase load is

$$3\bar{S}_A = 3\bar{V}_a\bar{I}_{aA}^{*} = 3(110\angle 0°)(1.16\angle -58°)^{*}$$
$$= 3 \times 110 \times 1.16\angle 58° = 382.80\angle 58° = 203 + j325\text{ VA}$$

(20 pts) **Q.4** Two electrical loads are connected in parallel to a 400-V rms, 50-Hz supply. The first load is 12 kVA at 0.7 lagging power factor; the second load is 10 kVA at 0.8 lagging power factor. Find the average power, the apparent power, and the power factor of the two combined loads.

Load 1: $P_1 = |\bar{S}|\cos\theta = (12\text{ kVA})(0.7) = 8.4\text{ kW}$

$Q_1 = |\bar{S}|\sin(\cos^{-1}0.7) = (12\text{ kVA})\sin 45.6^\circ$

$= 8.57\text{ kVAR}$

Load 2: $P_2 = (10\text{ kVA})(0.8) = 8\text{ kW}$

$Q_2 = (10\text{ kVA})\sin(\cos^{-1}0.8) = 10\sin 36.9^\circ$

$= 6\text{ kVAR}$

Total: $\bar{S} = P + jQ = (8.4+8) + j(8.57+6)$

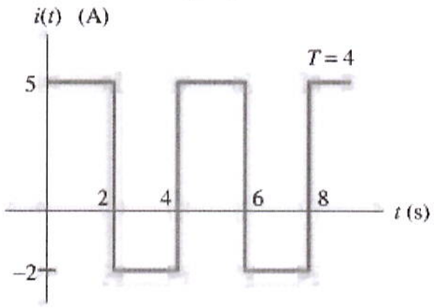
$= 16.4 + j14.57 = 21.9\angle 41.6^\circ\text{ kVA}$

The average power is $P = 16.4\text{ kW}$

The apparent power is $S = |\bar{S}| = \sqrt{P^2+Q^2} = 21.9\text{ kVA}$

The power factor is $pf = \cos 41.6^\circ = 0.75$ lagging.

(16 pts) **Q.5** Find the rms value of the current waveform shown in the following figure.



$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}, \quad T = 4s$

$= \sqrt{\frac{1}{4}\left(5^2\times 2 + (-2)^2\times 2\right)}$

$= \sqrt{\frac{1}{4}(50+8)}$

$= \sqrt{\frac{58}{4}}$

$= 3.808\text{ A}.$

Useful Info:

Phasors:

$\mathbf{V}(\omega) = V_m e^{j\theta_v} = V_m \angle \theta_v$ $\quad$ $\mathbf{I}(\omega) = I_m e^{j\theta_i} = I_m \angle \theta_i$ $\quad$ $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$\mathbf{V}_L(\omega) = \mathrm{Z}_L \mathbf{I}_L(\omega)$ $\quad$ $\mathbf{V}_C(\omega) = \mathrm{Z}_C \mathbf{I}_C(\omega)$ $\quad$ $\mathbf{V}_R(\omega) = \mathrm{Z}_R \mathbf{I}_R(\omega)$

$\mathrm{Z}_L = j\omega L$ $\quad$ $\mathrm{Z}_C = 1/j\omega C$ $\quad$ $\mathrm{Z}_R = R$

Note that ω is the angular frequency, which is $2\pi f$.

The **rms** value of a periodic waveform $w(t)$ is defined as: $\quad W_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} w^2(t)dt}$

For a **sinusoidal waveform** $v(t) = V_m \cos(\omega t + \theta)$: $\quad V_{rms} = \frac{V_m}{\sqrt{2}}$

Equivalent impedance when the impedances are in series $Z_{eq} = \sum_{i=1}^{k} Z_i = Z_1 + Z_2 + \cdots + Z_k$.

Equivalent impedance when the impedances are in parallel $\frac{1}{Z_{eq}} = \sum_{i=1}^{k} \frac{1}{Z_i} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_k}$.

The **average power** $P = V_{rms} I_{rms} \cos(\theta_V - \theta_I)$. The **reactive power** $Q = V_{rms} I_{rms} \sin(\theta_V - \theta_I)$.

The **complex power** $\mathbf{S} = \frac{\mathbf{VI}^*}{2} = \frac{V_m I_m}{2} \angle(\theta_V - \theta_I) = V_{rms} I_{rms} \angle(\theta_V - \theta_I) = P + jQ$.

The **apparent power** is $|\mathbf{S}| = \frac{V_m I_m}{2}$. $\quad$ The **power factor** $pf = \cos(\theta_V - \theta_I)$.

For the **maximum power transfer**, $\mathrm{Z}_L = \mathrm{Z}_t^*$, where Z_L is the load impedance and Z_t^* is the complex conjugate of the Thévenin equivalent impedance Z_t of the circuit.

For **coupled inductors** in the time domain $v_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$ and $v_2 = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$

For an **ideal transformer**, $\mathbf{V}_2 = \frac{N_2}{N_1} \mathbf{V}_1$ and $\mathbf{I}_1 = -\frac{N_2}{N_1} \mathbf{I}_2$.

Total complex power in a **balanced three-phase** circuits: $\boldsymbol{S}_\mathrm{T} = 3\boldsymbol{S}_\mathrm{p} = 3\frac{\mathbf{VI}^*}{2}$.

In **balanced three-phase** circuits, $V_{LL} = \sqrt{3} V_p$ and $I_L = I_p$ for Y connection; $V_{LL} = V_p$ and $I_L = \sqrt{3} I_p$ for Δ connection.

In an **ideal op-amp**: $\quad i_+ = i_- = 0 \quad\quad v_+ = v_-$

Nano (**n**) → 10^{-9} $\quad$ Micro (**μ**) → 10^{-6} $\quad$ Milli (**m**) → 10^{-3} $\quad$ Kilo (**k**) → 10^{3} $\quad$ Mega (**M**) → 10^{6}