EEE 302 Principles of Communications
MIDTERM EXAM ANSWERS

Q1.(15 pts) The bandpass signal s(t) is given as

$$s(t) = 200\mathrm{sinc}(200t)\cos(2\pi f_c t + \theta)$$

where $f_c = 10^6\,Hz, and\ \theta = \pi/4 rad.$

(a) Compute and sketch the spectrum of *s*(*t*).

(b) Determine its complex envelope signal, $\tilde{s}(t)$.

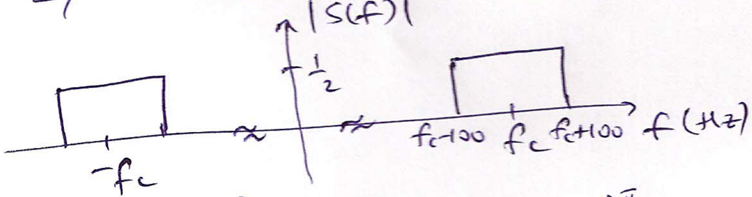
(c) If *s*(*t*) is input to an *ideal* bandpass filter with the frequency response *H*(*f*) given by

$$H(f) = \Pi[(f - f_c)/B]e^{-j2\pi f t_d}, f > 0$$

determine the output signal *y*(*t*) using the complex envelope techniques.

A1. a) $S(f) = \frac{1}{2}\left\{\mathrm{rect}\left(\frac{f-f_c}{200}\right)e^{j\frac{\pi}{4}} + \mathrm{rect}\left(\frac{f+f_c}{200}\right)e^{-j\frac{\pi}{4}}\right\}$, $f_c = 10^6\,Hz$

(5)



b) $s(t) = \mathrm{Re}\left\{s_{lp}(t)e^{j2\pi f_c t}\right\} \Rightarrow s_{lp}(t) = e^{j\frac{\pi}{4}}\,200\,\mathrm{sinc}(200t)$ (5)

c) $H_{lp}(f) = \Pi\left(\frac{f}{B}\right)e^{-j2\pi(f+f_c)t_d} \xrightarrow{\mathcal{F}^{-1}} h_{lp}(t) = B e^{-j2\pi f_c t_d}\mathrm{sinc}[B(t-t_d)]$.

$$y_{lp}(t) = \frac{1}{2}h_{lp}(t) * s_{lp}(t)$$
$$= \frac{1}{2}B e^{-j2\pi f_c t_d}\mathrm{sinc}[B(t-t_d)] * e^{j\frac{\pi}{4}}200\,\mathrm{sinc}(200t)$$

$$y(t) = \mathrm{Re}\left\{y_{lp}(t)e^{j2\pi f_c t}\right\} = \frac{200 B}{2}\left\{\mathrm{sinc}[B(t-t_d)] * \mathrm{sinc}(200t)\right\}\cdot\cos\left[2\pi f_c(t-t_d)+\frac{\pi}{4}\right]$$

(5)

Q2. (25 pts) The message signal $m(t) = 2\cos(2\pi 40t) + \cos(2\pi 90t)$ is to be transmitted using DSB-SC modulation technique. The carrier wave is given as $c(t) = 10\cos(2\pi 1000t)$.

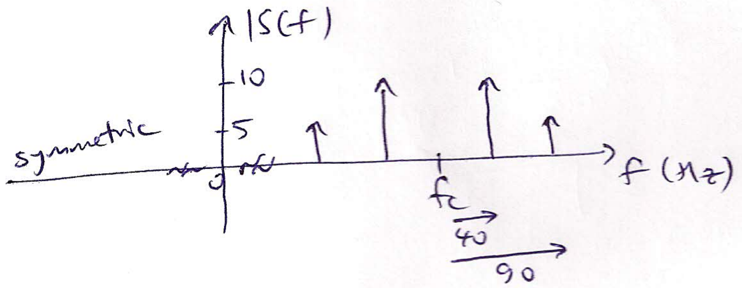
(a) Determine the analytic expression for the modulated signal. Sketch the amplitude spectrum of the modulated signal and the phasor diagram (using the carrier signal as reference).

(b) Determine the transmission bandwidth and power of the DSB-SC modulated signal.

(c) Suppose now the same message signal $m(t)$ is input to a SSB-LS modulation system with the same carrier wave $c(t)$. Determine the analytic expression for the modulated signal. Sketch its amplitude spectrum.

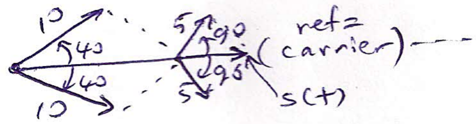
(d) Determine the transmission bandwidth and power of the SSB-LS modulated signal.

A2. a) $s(t) = m(t).c(t) = 10\cos(2\pi 960t) + 10\cos(2\pi 1040t) + 5\cos(2\pi 910t) + 5\cos(2\pi 1090t)$



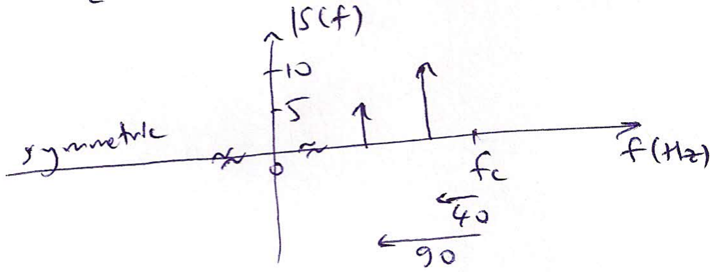
$f_c = 1000 Hz$

7.5



b) $B_T = 2 \times 90 = 180$ Hz, $P_T = 250$ W. 5

c) $s(t) = [5\cos(2\pi 960t) + 2.5\cos(2\pi 910t)] \cdot 2$



7.5

d) $B_T = 90$ Hz, $P_T = 125$ W. 5

Q3. (30 pts) A message signal $m(t)$ with power spectral density given by

$$S_M(f) = \begin{cases} a\dfrac{|f|}{W}, & |f| \le W \\ 0, & otherwise \end{cases}$$

where a=2 and W=10 kHz, is to be modulated using DSB-SC technique and transmitted over a noisy channel with power spectral density of noise given as

$$S_N(f) = \begin{cases} N_o(1-\dfrac{|f|}{B}), & |f| < B \\ 0, & otherwise \end{cases}$$

with B=400 kHz and $N_o = 10^{-6} W/Hz$. Assume A_c=1V, f_c=200kHz, and the channel attenuates signal power by a factor of 1000 (=30 dB). Assume that a suitable band-pass filter is used at the receiver to limit out-of-band noise.

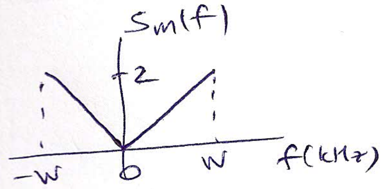
(a) Determine the average signal power at the receiver input?

(b) Determine the channel signal-to-noise ratio of the receiver, $(SNR)_C$?

(c) Determine the output signal-to-noise ratio, $(SNR)_O$, and figure-of-merit $(FOM = \dfrac{SNR_O}{SNR_C})$ of the receiver?

A3. a)

$$P_m = \int_{-W}^{W} S_m(f)\,df = 20\,kW$$



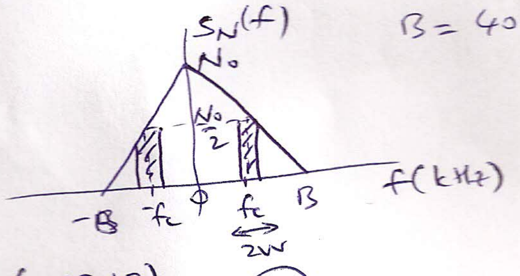
W = 10 kHz

$$P_{s,ave} = \frac{1}{2}A_c^2 P_m \cdot \frac{1}{L_{ch}} = \frac{1}{2} 20.10^3 . \frac{1}{10^3} = 10\,W.$$ (10)

↑ 30dB channel attenuation

b)

$$P_{N,ave} \simeq \frac{N_o}{2}.2W.2 = 20\,mW$$



B = 400 kHz

$$(SNR)_c = \frac{P_{s,ave}}{P_{N,ave}} = \frac{10}{20.10^{-3}} = 500 \ (\sim 27dB)$$ (10)

c) For DSB-SC receiver, $(SNR)_o = (SNR)_c = 500$

$$(FOM)_{DSB-SC} = \frac{(SNR)_o}{(SNR)_c} = 1.$$ (10)

Q4. (30 pts) A carrier wave of unit amplitude and frequency 100 kHz is *frequency-modulated* by a sinusoidal wave of amplitude A_m and frequency f_m. The frequency sensitivity of the modulator is 100 kHz per volt.

Assume A_m=0.05, f_m=10 kHz for the parts (a) and (b):
(a) Write down the analytic expression for a transmit FM signal, $s_{FM}(t)$. What is the peak frequency deviation from the carrier, f_c. Find the frequency modulation index, β?

(b) Determine and sketch the amplitude spectrum of $s_{FM}(t)$ and evaluate its bandwidth using the narrow-band FM approximation.

(c) For A_m=0.5, determine and sketch the spectrum of $s_{FM}(t)$ and evaluate its approximate bandwidth by using Carson's rule.

(d) Show block diagram of the receiver to recover the message signal.

Note: You can use some of the following values of the Bessel function of the first kind, $J_k(\beta)$ as given below:

n	β	$J_k(\beta)$
0	0.5	0.9385
1	0.5	0.2423
2	0.5	0.0306

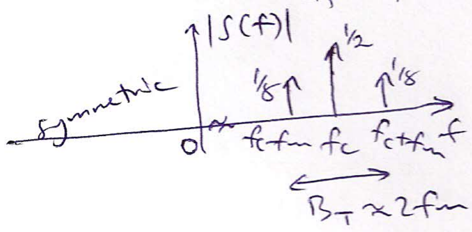
n	β	$J_k(\beta)$
0	5	-0.1776
1	5	-0.3276
2	5	0.0466
3	5	0.3648
4	5	0.3912
5	5	0.2611
6	5	0.1310
7	5	0.0534
8	5	0.0184
9	5	0.0055

A4

a) $s(t) = \cos(2\pi \cdot 100 \cdot 10^3 t + \underbrace{0{,}5}_{\beta} \sin(2\pi \cdot 10 \cdot 10^3 t))$ (10)

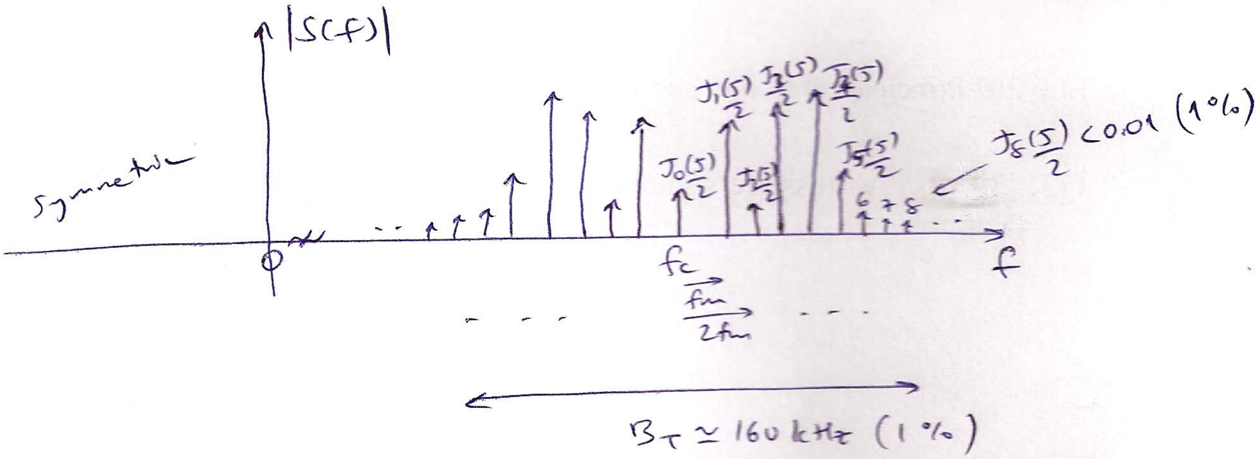
$\Delta f = \beta \cdot f_m = 0{,}5 \cdot 10 \cdot 10^3 = 5$ kHz.

b)



$B_T \approx 20$ kHz (7.5)

c) $A_m = 0{,}5$: $\beta = \frac{k_f A_m}{f_m} = 5 \gg 1$ (WBFM)



(7.5)

Carson's rule: $B_T \simeq 2(\Delta f + f_m) \simeq 120$ kHz

d) FM receiver using Balanced Frequency Discriminator circuit (using slope network and envelope detector), see class notes.

(5)