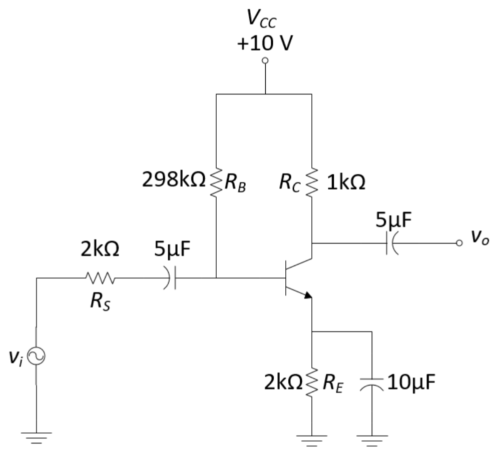
26 Nov 2021 20:00 – 21:30

Instructor: Dr A. K. Topallı

EEE 331 ANALOG ELECTRONICS

MIDTERM EXAMINATION 1 SOLUTIONS

Question 1 (35 points). Consider the BJT amplifier given below.



Transistor values are:

β = 100 $\quad$ V_T = 26 mV

V_{BE}(on) = 0.7 V $\quad$ V_A = 200 V

$$r_\pi = \frac{V_T}{I_{BQ}} \quad g_m = \frac{I_{CQ}}{V_T} \quad r_O = \frac{V_A}{I_{CQ}}$$

a. (**5 points)** Draw the DC equivalent of the given circuit by replacing the AC source and capacitances with their corresponding equivalents (i.e., open or short circuits).

b. (10 points) By DC analysis, find I_{BQ}, I_{CQ}, I_{EQ}, V_{CEQ}.

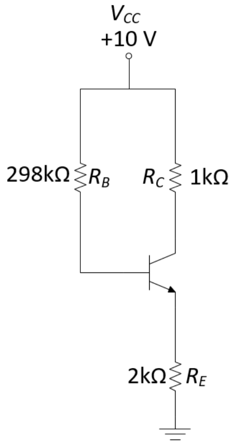
c. (5 points) Draw the AC equivalent of the given circuit with the small-signal model of the transistor. Replace the DC source and capacitances with their corresponding equivalents (i.e., open or short circuits). Assume ideal transistor (i.e., do not take any internal capacitors into consideration).

d. (10 points) Derive the equation for voltage gain A_v by using Kirchhoff's & Ohm's laws and calculate its value.

e. (5 points) What is the polarity of A_v? What does it mean?

Answer 1.

a. AC voltage source → short circuit, Capacitors → open circuits



b. KVL at B-E side: $V_{CC} = R_B I_{BQ} + V_{BE}(\text{on}) + R_E I_{EQ}$ → $10 = (298\text{ k})I_{BQ} + V_{BE}(\text{on}) + (2\text{ k})I_{EQ}$

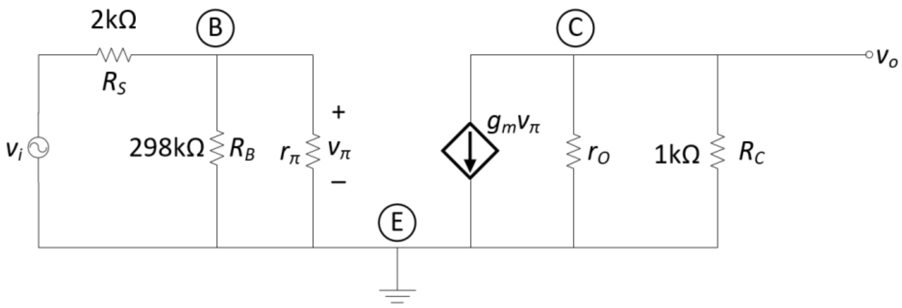
$I_{EQ} = (\beta + 1)I_{BQ}$ → $10 - 0.7 = (298\text{ k} + (100 + 1)\cdot 2\text{ k})I_{BQ}$ → $I_{BQ} = \frac{9.3}{500\text{ k}} = 18.6\ \mu\text{A}$

$I_{CQ} = \beta . I_{BQ} = 1.86\text{ mA}$ $\quad I_{EQ} = (\beta + 1). I_{BQ} = 1.88\text{ mA}$

KVL at C-E side: $V_{CC} = R_C I_{CQ} + V_{CEQ} + R_E I_{EQ}$ → $10 = (1\text{ k})I_{CQ} + V_{CEQ} + (2\text{ k})I_{EQ}$

$V_{CEQ} = 10 - 1.86 - 3.76 = 4.38\text{ V}$

c. DC voltage source → short circuit, Capacitors → short circuits



d. $r_\pi = \frac{26\text{ m}}{18.6\ \mu} = 1.4\text{ k}\Omega$ $\quad g_m = \frac{1.86\text{ m}}{26\text{ m}} = 71.5\text{ mA/V}$ $\quad r_O = \frac{200}{1.86\text{ m}} = 107.5\text{ k}\Omega$

$v_o = -g_m v_\pi (r_O \parallel R_C)$ $\quad v_\pi = \frac{(R_B \| r_\pi)}{R_S + (R_B \| r_\pi)} v_i$ $\quad v_i = \frac{R_S + (R_B \| r_\pi)}{(R_B \| r_\pi)} v_\pi$

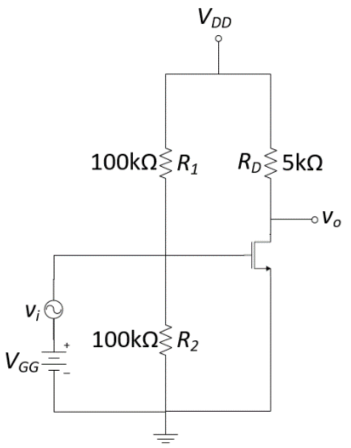
$$r_O \parallel R_C = \frac{r_O \cdot R_C}{r_O + R_C} = \frac{(107.5\text{ k})(1\text{ k})}{108.5\text{ k}} = 0.99\text{ k}\Omega$$

$$r_\pi \parallel R_B = \frac{r_\pi \cdot R_B}{r_\pi + R_B} = \frac{(1.4\text{ k})(298\text{ k})}{299.4\text{ k}} = 1.39\text{ k}\Omega$$

$$A_V = \frac{v_o}{v_i} = \frac{-g_m (r_O \parallel R_C)(R_B \parallel r_\pi)}{R_S + (R_B \parallel r_\pi)} = -\frac{(71.5\text{ m})(0.99\text{ k})(1.39\text{ k})}{(2\text{ k}) + (1.39)\text{k}} = -\frac{98.39\text{ k}}{3.39\text{ k}} = -29$$

e. Polarity of A_v is negative. It means there is a 180° phase shift between output and input voltages.

Question 2 (35 points). Consider the NMOS amplifier given below.



Transistor values are:

$$V_{TN} = 1 \text{ V}$$

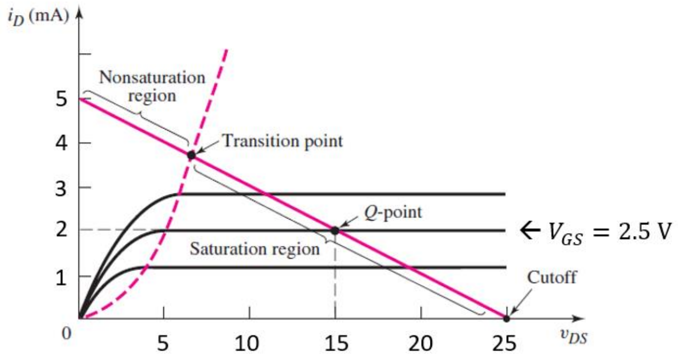
$$K_n = 1 \text{ mA/V}^2$$

$$V_A = \infty$$

$$g_m = 2K_n(V_{GSQ} - V_{TN})$$

$$r_O = V_A/I_{DQ}$$

The operating point Q is shown in the graph below.



a. (5 points) Draw the AC equivalent of the given circuit with the small-signal model of the transistor. Replace the DC sources with their corresponding equivalents (i.e., open or short circuits). Assume ideal transistor (i.e., do not take any internal capacitors into consideration).

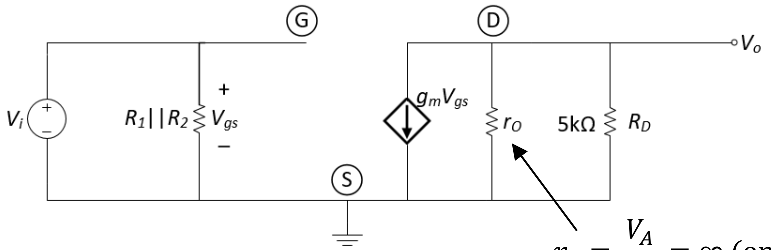
b. (10 points) Calculate the value of the input resistance R_i to the amplifier?

c. (10 points) Calculate the value of the output resistance R_o looking back into the output terminals?

d. (10 points) Derive the equation for voltage gain A_V by using Kirchhoff's and Ohm's Laws and calculate its value.

Answer 2.

a. DC voltage source → short circuit



$$r_O = \frac{V_A}{I_{DQ}} = \infty \text{ (open circuit)}$$

b. $R_i = R_1 \parallel R_2 = (100\text{k}) \parallel (100\text{k}) = 50\text{k}\Omega$

c. $R_o = R_D \parallel r_O = (5\text{k}) \parallel (\infty) = 5\text{k}\Omega$

d. From the graph: V_{GSQ} = 2.5 V

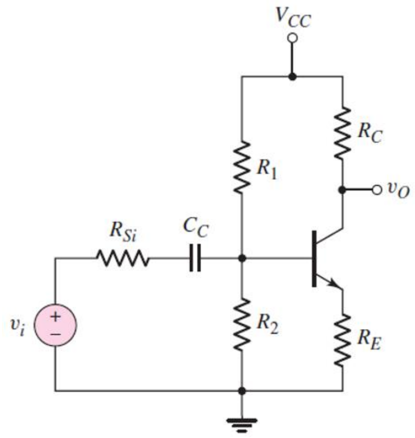
$g_m = 2K_n(V_{GSQ} - V_{TN}) = 2 \cdot 1 \cdot (2.5 - 1) = 3 \text{ mA/V}$

$V_o = -R_o g_m V_{gs}$

$V_{gs} = V_i$

$$A_V = \frac{V_o}{V_i} = -g_m R_o = -(3\text{ m})(5\text{ k}) = -15$$

Question 3 (30 points). Consider the bipolar common-emitter circuit with a coupling capacitor given below.



R_{Si} = 0.5 kΩ

R_1 = 50 kΩ

R_2 = 10 kΩ

R_C = 1 kΩ

R_E = 0.5 kΩ

C_C = 2 μF

V_{CC} = 12 V

V_{BE}(on) = 0.7 V

β = 100

a. (10 points) Draw the small-signal equivalent circuit with the coupling capacitor C_C. Assume ideal transistor (i.e., do not take any internal capacitors into consideration).

b. (20 points) Calculate the corner frequency in Hz and maximum gain in dB.

Hint 1: You can take: g_m = 85 mA/V r_π = 1.18 kΩ $r_o = \infty$

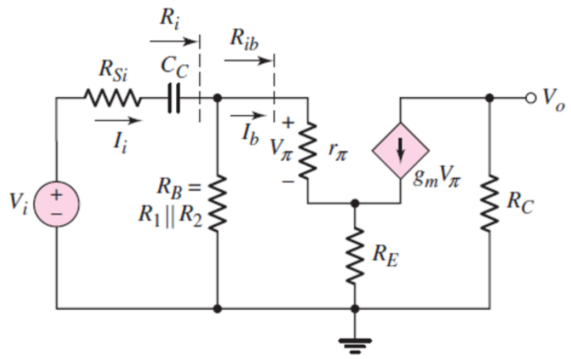
Hint 2: You can use $$A_v = \frac{V_o(s)}{V_i(s)} = -\frac{g_m r_\pi R_C R_B \tau_s s}{(R_{Si}+R_i)(R_B+R_{ib})(1+\tau_s s)}$$

where R_i is the small-signal input resistance to the amplifier, and R_{ib} is the small-signal input resistance of the transistor seen from the base terminal.

Time constant: $\tau_s = C_C(R_{Si} + R_i)$

Answer 3.

a.



b.

$V_b = r_\pi I_b + R_E(\beta + 1)I_b = [r_\pi + (\beta + 1)R_E]I_b$

$R_{ib} = \dfrac{V_b}{I_b} = r_\pi + (\beta + 1)R_E = 1.18\ \text{k} + (100 + 1)0.5\ \text{k} = 51.68\ \text{k}\Omega$

$R_B = R_1 \parallel R_2 = \frac{(50\ \text{k})(10\ \text{k})}{50\ \text{k}+10\ \text{k}} = 8.33\ \text{k}\Omega$ $\qquad$ $R_i = R_B \parallel R_{ib} = \frac{(8.33\ \text{k})(51.68\ \text{k})}{8.33\ \text{k}+51.68\ \text{k}} = 7.17\ \text{k}\Omega$

$\tau_s = C_C(R_{Si} + R_i) = (2\ \mu)(0.5\ \text{k} + 7.17\ \text{k}) = 15.3\ \text{ms}$

Corner frequency: $\quad f_L = \dfrac{1}{2\pi\tau_s} = \dfrac{1}{2(3.14)(15.3\ \text{m})} = 10.4\ \text{Hz}$

$$|A_v|_{max} = \frac{g_m r_\pi R_C R_B}{(R_{Si} + R_i)(R_B + R_{ib})} = \frac{(85\ \text{m})(1.18\ k)(1\ \text{k})(8.33\ \text{k})}{(0.5\ \text{k} + 7.17\ \text{k})(8.33\ \text{k} + 51.68\ \text{k})} = 1.82$$

Maximum gain in dB = $20\log_{10}(1.82) = 5.2$