İZMİR EKONOMİ ÜNİVERSİTESİ

29 Dec 2021 11:00 – 12:30

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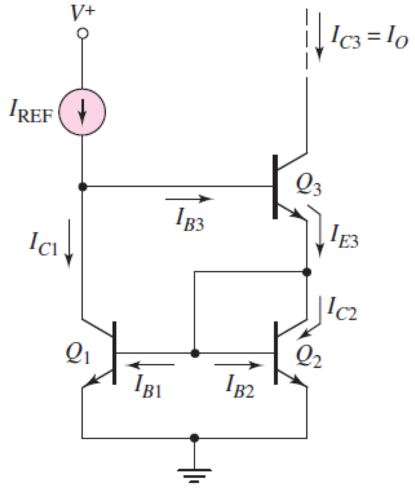
EEE 331 ANALOG ELECTRONICS

MIDTERM EXAMINATION 2 SOLUTIONS

Question 1 (30 points). Consider below Wilson current source. Assume identical transistors.



a. (20 points) Express the output current in terms of the reference current and transistor current gain β.

b. (5 points) Repeat part (a) for $\beta = 100$.

c. (5 points) Why is this circuit also called a "current mirror"?

Answer 1.

a. Transistors are identical.

$I_{B1} = I_{B2}$ $\quad$ $I_{C1} = I_{C2}$ $\quad$ $\beta_1 = \beta_2 = \beta_3 \equiv \beta$ $\quad$ $I_{E3} = I_{C2} + 2I_{B2} = I_{C2}\left(1 + \frac{2}{\beta}\right)$

$$I_{C2} = \frac{I_{E3}}{\left(1 + \frac{2}{\beta}\right)} = \frac{1}{\left(1 + \frac{2}{\beta}\right)} \times \left(\frac{1+\beta}{\beta}\right) I_{C3} = \left(\frac{1+\beta}{2+\beta}\right) I_{C3}$$

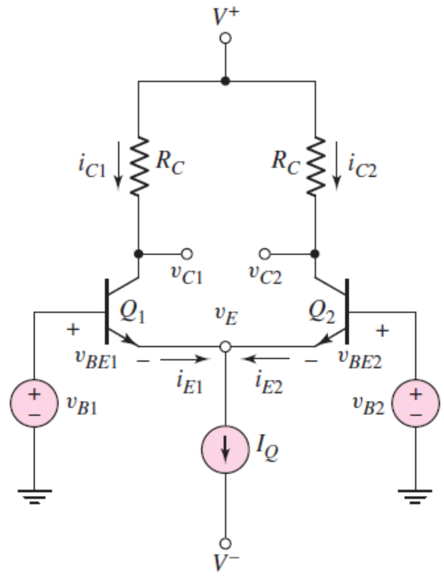
$$I_{REF} = I_{C1} + I_{B3} = I_{C2} + I_{B3} = \left(\frac{1+\beta}{2+\beta}\right) I_{C3} + \frac{I_{C3}}{\beta}$$

$$I_{C3} = I_O = I_{REF} \times \frac{1}{1 + \frac{2}{\beta(2+\beta)}} = I_{REF} \times \frac{\beta(2+\beta)}{2 + \beta(2+\beta)}$$

b. $\beta = 100$ ➔ $I_O = I_{REF} \times \frac{1}{1+\frac{2}{100(2+100)}} = 0.9998 \cdot I_{REF}$

c. Because I_{REF} can almost be mirrored at the output side.

Question 2 (40 points). Consider below basic BJT differential amplifier. Assume identical transistors.



a. (10 points) Draw the AC equivalent of the given circuit with the small-signal model of the transistors. Represent constant-current source by the resistor R_O.

b. (10 points) Using AC equivalent circuit, express the output voltage V_{c2} (one-sided output) in the form of

$$V_{c2} = K_1 V_{b1} - K_2 V_{b2}$$

Write K_1 and K_2 in terms of circuit and transistor parameters (do not use any numeric values).

c. (5 points) Calculate K_1 and K_2 of part (b) for $\beta = 100$, $I_Q = 1.04\ \text{mA}$, $R_B = 5\ \text{k}\Omega$, $R_C = 20\ \text{k}\Omega$, $R_O = 40\ \text{k}\Omega$, $V_T = 26\ \text{mV}$, $V_A = \infty$. Ignore base currents.

d. (10 points) Express V_{c2} in the form of

$$V_{c2} = A_d V_d + A_{cm} V_{cm}$$

Write differential gain A_d and common-mode gain A_{cm} in terms of K_1 and K_2 and calculate their values (**Hint:** Use the relation between the input voltages V_{b1}, V_{b2} and V_d, V_{cm}).

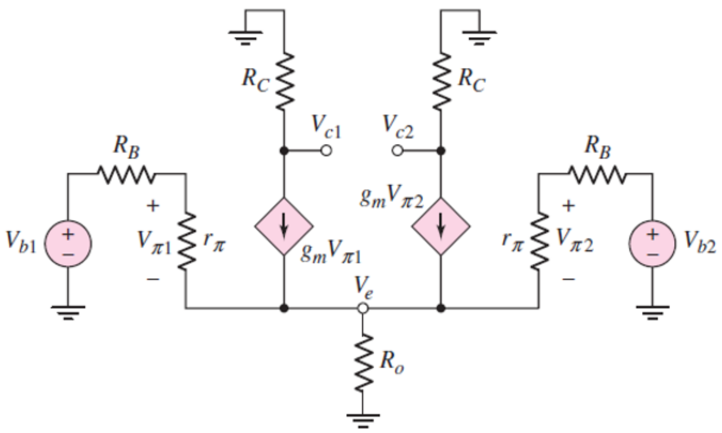
e. (5 points) Calculate the common-mode rejection ratio CMRR in dB.

Hints: $r_\pi = \dfrac{\beta V_T}{I_{CQ}}$ $\qquad g_m r_\pi = \beta$ $\qquad r_O = \dfrac{V_A}{I_{CQ}}$

Answer 2.

Note: Below answer assumes a nonzero R_B. If you take it zero, it is also accepted.

a.



b.

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_\pi} = \frac{V_e}{R_o}$$

$$g_m r_\pi = \beta$$

$$V_{\pi 1}\left(\frac{1+\beta}{r_\pi}\right) + V_{\pi 2}\left(\frac{1+\beta}{r_\pi}\right) = \frac{V_e}{R_o}$$

$$\frac{V_{\pi 1}}{r_\pi} = \frac{V_{b1} - V_e}{r_\pi + R_B} \quad \text{and} \quad \frac{V_{\pi 2}}{r_\pi} = \frac{V_{b2} - V_e}{r_\pi + R_B}$$

$$V_e = \frac{V_{b1} + V_{b2}}{2 + \dfrac{r_\pi + R_B}{(1+\beta)R_o}}$$

One-Sided Output

$$V_o = V_{c2} = -(g_m V_{\pi 2})R_C = -\frac{\beta R_C (V_{b2} - V_e)}{r_\pi + R_B} = -\frac{\beta R_C}{r_\pi + R_B}\left\{\frac{V_{b2}\left[1 + \dfrac{r_\pi + R_B}{(1+\beta)R_o}\right] - V_{b1}}{2 + \dfrac{r_\pi + R_B}{(1+\beta)R_o}}\right\}$$

$$V_{c2} = \underbrace{\frac{\beta R_C}{r_\pi + R_B} \cdot \frac{(1+\beta)R_o}{2(1+\beta)R_o + r_\pi + R_B}}_{K_1} \cdot V_{b1} - \underbrace{\frac{\beta R_C}{r_\pi + R_B} \cdot \frac{(1+\beta)R_o + r_\pi + R_B}{2(1+\beta)R_o + r_\pi + R_B}}_{K_2} \cdot V_{b2}$$

c.

$$I_{E1} = I_{E2} = \frac{I_Q}{2} = \frac{(1.04\text{ m})}{2} = 0.52\text{ mA}$$

$$I_{C1} \cong I_{E1} = 0.52\text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(26\text{ m})}{(0.52\text{ m})} = 5\text{ k}\Omega$$

$$K_1 = \frac{(100)(20\text{ k})}{(5\text{ k}) + (5\text{ k})} \cdot \frac{(101)(40\text{ k})}{2(101)(40\text{ k}) + (5\text{ k}) + (5\text{ k})} = 99.88$$

$$K_2 = \frac{(100)(20\text{ k})}{(5\text{ k}) + (5\text{ k})} \cdot \frac{(101)(40\text{ k}) + (5\text{ k}) + (5\text{ k})}{2(101)(40\text{ k}) + (5\text{ k}) + (5\text{ k})} = 100.12$$

$$V_{c2} = 99.88 \cdot V_{b1} - 100.12 \cdot V_{b2}$$

d.

$$V_{b1} = V_{cm} + \frac{V_d}{2}$$

$$V_{b2} = V_{cm} - \frac{V_d}{2}$$

$$V_{c2} = K_1\left(V_{cm} + \frac{V_d}{2}\right) - K_2\left(V_{cm} - \frac{V_d}{2}\right) = \frac{K_1 + K_2}{2} V_d + (K_1 - K_2)V_{cm}$$

$$V_{c2} = A_d V_d + A_{cm} V_{cm}$$

$$A_d = \frac{K_1 + K_2}{2} = \frac{99.88 + 100.12}{2} = 100$$

$$A_{cm} = K_1 - K_2 = 99.88 - 100.12 = -0.24$$

$$V_{c2} = 100V_d - 0.24V_{cm}$$

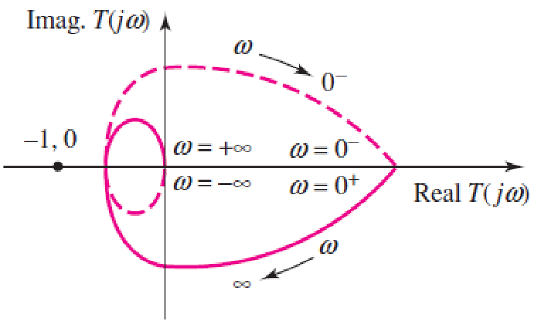
e.

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{100}{0.24} = 416.7$$

$$CMRR_{dB} = 20\log_{10}(416.7) = 52.4\text{ dB}$$

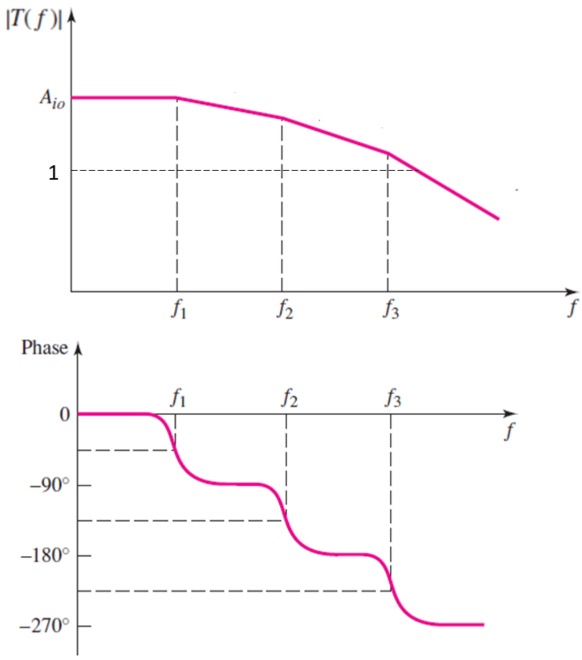
Question 3 (30 points).

a. (10 points)



Consider the Nyquist diagram on the left for the loop gain of a three-pole feedback amplifier. Is this system stable or not? Explain how you check it from the diagram.

b. (10 points)

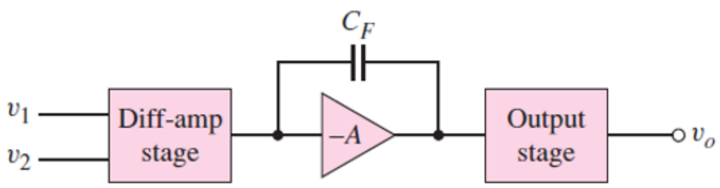


Consider the magnitude and phase plots on the left for the loop gain of a three-pole feedback amplifier. Is this system stable or not? Explain how you check it from these plots.

c. (10 points)

Consider below three-stage amplifier with a compensation capacitor which is used to move the gain stage pole to 10 Hz in order to make the circuit stable. Design the circuit, i.e., find the value of C_F for a second stage gain of 10^4 and effective resistance between the amplifier input node and ground of 100 kΩ.

Hint: The effective input Miller capacitance is $C_M = C_F(1 + A)$.



Answer 3.

a. System is **stable** because when the phase of the loop gain is -180º (where the plot crosses the negative real axis), the magnitude of the loop gain is less than 1.

b. System is **unstable** because at the frequency where phase plot is -180º, the magnitude plot is greater than 1.

c.

$$f_p = \frac{1}{2\pi R_{ef} C_M}$$

$$C_M = \frac{1}{2\pi f_p R_{ef}} = \frac{1}{2\pi(10)(100\text{ k})} = \frac{10^{-6}}{2\pi}$$

$$C_M = C_F(1 + A)$$

$$C_F = \frac{C_M}{1+A} = \frac{10^{-6}}{2\pi(1+10^4)} \cong \frac{10^{-6}}{2\pi(10^4)} = \frac{100\times10^{-12}}{2\pi} = \frac{50}{\pi}\ \text{pF}$$