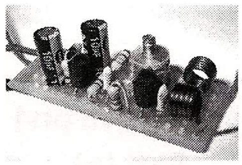


İZMİR UNIVERSITY OF ECONOMICS
Department of Electronics and Communication Engineering
ETE 331 Analog Electronics
Fall 2013/2014

SOLUTIONS

MIDTERM EXAM II

Dec 21, 2013
120 min

INSTRUCTIONS

- Read all of the instructions and all of the questions before beginning the exam.
- There are 4 questions on this exam, totaling 100 points. The credit for each problem is given to help you allocate your time accordingly.
- Do not spend all your time on one problem and on one part and attempt to solve all of them.
- Calculators are allowed, but borrowing is not allowed.
- Your mobile phones must be turned off during the exam.
- Turn in the entire exam, including this cover sheet.
- You must show your work for all problems to receive full credit; simply providing answers will result in only partial credit, even if the answers are correct.
- Please indicate the number of page where your work is to be continued.
- Put your name on any additional material that you submit.
- Be sure to provide units where necessary.
- Please sign the honor pledge that is provided below.

		Question	Points	Grade
Last Name	:........................	1	25	
Name	:........................	2	25	
Section	:........................	3	25	
Student No	:........................	4	25	
		TOTAL	100	

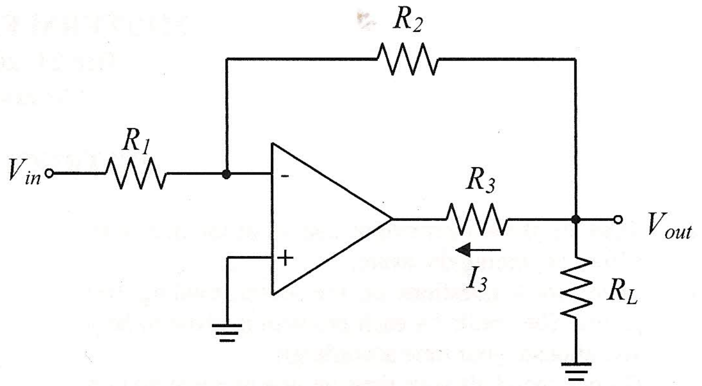
The basic equations of the output characteristics of an NMOS transistor		
V_{GS}	V_{DS}	I_D
i) $V_{GS} < V_{Tn}$	-	0
ii) $V_{GS} > V_{Tn}$	a) $V_{DS} < V_{GS} - V_{Tn}$	$K_n\left[2(V_{GS} - V_{Tn})V_{DS} - V_{DS}^2\right]$
	b) $V_{GS} - V_{Tn} \le V_{DS}$	$K_n(V_{GS} - V_{Tn})^2$
where $K_n = \frac{K'_n}{2}\left(\frac{W}{L}\right)$ and $K'_n = \mu_n C_{ox}$		

Q1. (25 pts) Consider the operational amplifier circuits given below.

Part (a)

The circuit given on the right is similar to the inverting amplifier except the resistor R_3 has been added. The circuit parameters are

$R_1 = 5\ k\Omega$, $R_2 = 25\ k\Omega$, $R_3 = 12.5\ k\Omega$, $R_L = 5\ k\Omega$



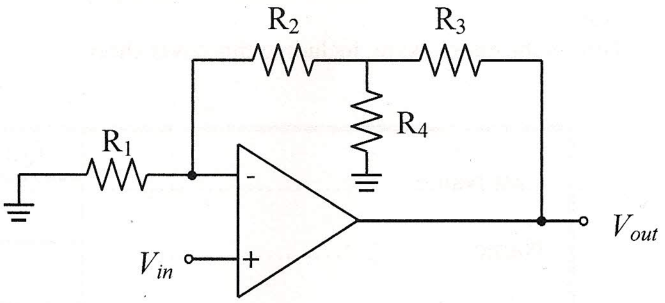
i) Derive the expression for V_{out} in terms of the input voltage V_{in}.
ii) Derive the expression for I_3 in terms of the input voltage V_{in}.
iii) What happens to I_3 if R_3 is doubled, i.e., $R_3 = 25\ k\Omega$

Part (b)

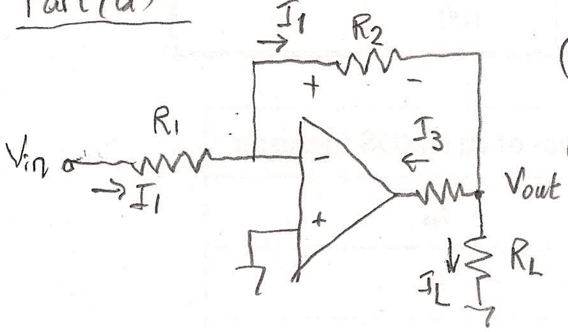
i) For the ideal op-amp circuit given on the right, determine the voltage gain $Av = V_{out} / V_{in}$
ii) Determine the range of V_{in} so that the output of the op-amp stays in the linear range?

The circuit parameters are

$R_1 = 5\ k\Omega$, $R_2 = 25\ k\Omega$, $R_3 = 5\ k\Omega$, $R_4 = 5\ k\Omega$



Part (a)



(i) $I_1 = \frac{V_{in}}{R_1}$; $V_{out} = -R_2 I_1 = -\frac{R_2}{R_1} V_{in} \Rightarrow V_{out} = -5V_{in}$

(ii) $I_L = \frac{V_{out}}{R_L} = -\frac{R_2}{R_1 R_L} V_{in}$

$I_3 = I_1 - I_L = \frac{V_{in}}{R_1} - \left(-\frac{R_2}{R_1 R_L}\right) V_{in} = \left(1 + \frac{R_2}{R_L}\right)\frac{V_{in}}{R_1}$

$I_3 = \left(1 + \frac{R_2}{R_L}\right)\frac{V_{in}}{R_1}$

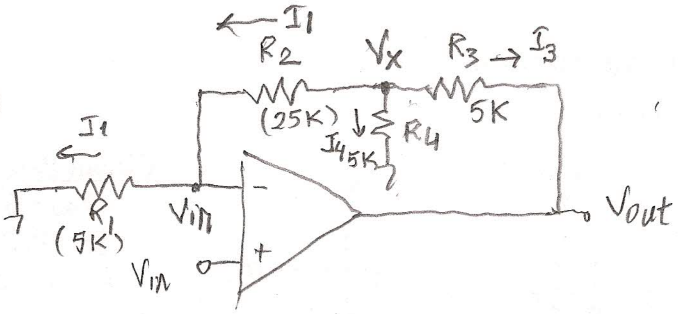
$R_1 = 5K$, $R_2 = 25K$, $R_3 = 12.5K$, $R_L = 5K$

$I_3 = \left(1 + \frac{25}{5}\right)\frac{V_{in}}{5} = (1+5)\frac{V_{in}}{5} = 1.2V_{in}$

(iii) Since I_3 is indepedent from R_3; when R_3 is doubled, I_3 stays as it is.

$I_3 = 1.2V_{in}$

(b) (i)



$I_1 = \frac{V_{in}}{R_1} = \frac{V_{in}}{5K} = 0.2\,V_{in}$ (V_{in} in mV)

$\frac{V_x - V_{in}}{R_2} = I_1 = \frac{V_{in}}{R_1}$

$R_1 (V_x - V_{in}) = R_2 V_{in}$

$\Rightarrow R_1 V_x = (R_1 + R_2) V_{in} \Rightarrow V_x = \frac{R_1 + R_2}{R_1} V_{in} = \frac{5+25}{5} V_{in} = 6\,V_{in}$

$I_4 = \frac{V_x}{R_4} = \frac{6\,V_{in}}{5} = 1.2\,V_{in}$ (V_{in} in mV)

$I_1 + I_3 + I_4 = 0 \Rightarrow I_3 = -(I_1 + I_4) = -(0.2\,V_{in} + 1.2\,V_{in}) = -1.4\,V_{in}$ (V_{in} in mV)

$\frac{V_x - V_{out}}{R_3} = I_3 \Rightarrow V_{out} = V_x - I_3 R_3 = 6\,V_{in} - (-1.4\,V_{in})(5K)$

$= 6\,V_{in} + 7\,V_{in} = 13\,V_{in}$

$V_{out} = 13\,V_{in}$

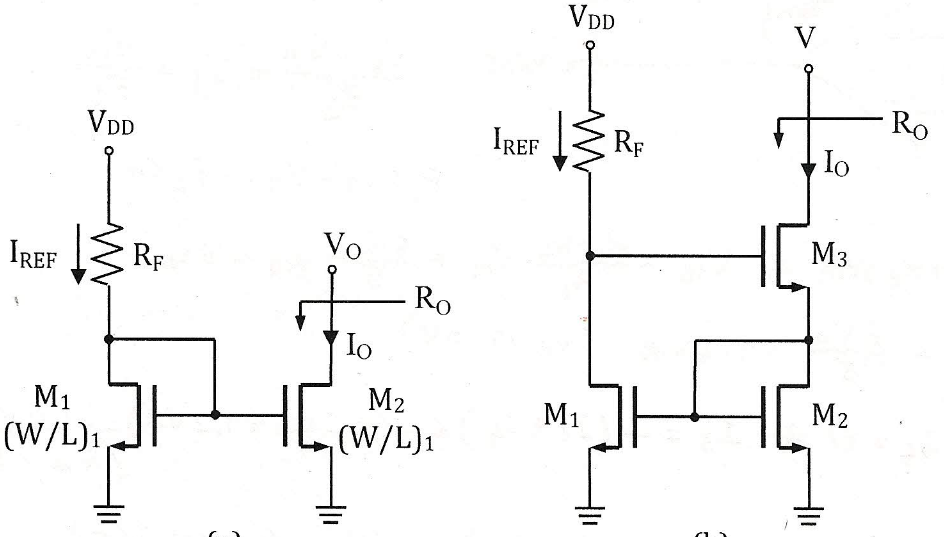
(ii) $-V_{cc} \leq V_{out} \leq V_{cc}$

$-V_{cc} \leq 13\,V_{in} \leq V_{cc}$

$-\frac{V_{cc}}{13} \leq V_{in} \leq \frac{V_{cc}}{13}$

Q2. (25 pts) Consider the current mirrors given below.

Part (a) Consider the circuit (a) given below.



(a) (b)

Circuit Parameters

$V_{DD} = 5$ V

$I_{REF} = 0.25$mA

$I_O = 0.1$ mA

$V_{GS1} = 2$ V

$V_{GS3} = 1.5$ V

Transistor Parameters

$K_n' = 50\ \mu A/V^2$

$V_{TN} = 1$ V

$\lambda = 0$

$\lambda_3 = \lambda_2 = 0.01\ V^{-1}$

a) Determine R_F.

b) Determine the ratio of the aspect ratios (W/L)s of the transistors M_1 and M_2. i.e.,

$$\left(\frac{W}{L}\right)_2 \Big/ \left(\frac{W}{L}\right)_1$$

c) Determine the output resistance R_o of the current miror.

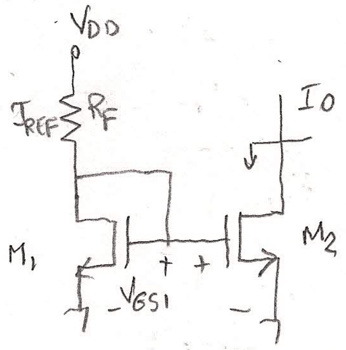
d) Determine the minimum value of V_O so that all transistors stay in SAT region?

Part (b) Consider the circuit (a) given above.

Repeat Part (a) for the circuit of Fig. (b) above.

Part (a): (a) $R_F = \frac{V_{DD} - V_{GS1}}{I_{REF}} = \frac{5-2\ V}{0.25mA} = 12K$

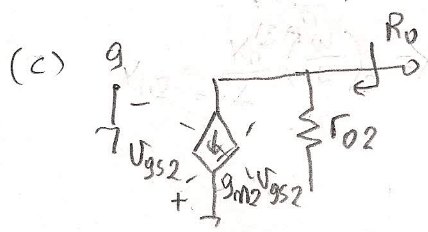
(b)



$$I_o = K_{n2}(V_{GS2} - V_{Tn})^2$$
$$I_{REF} = K_{n1}(V_{GS1} - V_{Tn})^2$$
$$\frac{I_o}{I_{REF}} = \frac{K_{n2}}{K_{n1}} = \frac{(W/L)_2}{(W/L)_1}$$

$$\Rightarrow \frac{(W/L)_2}{(W/L)_1} = \frac{I_o}{I_{REF}} = \frac{0.1}{0.25} = 0.4$$

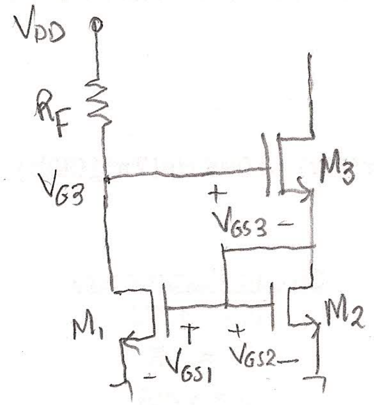
(c)



$$R_o = r_{o2} = \frac{1}{\lambda_2 I_o} = \frac{1}{(0.01)(0.1)10^{-3}} = 1\ M\Omega$$

(d) $V_o = V_{DS2} \geq V_{GS2} - V_{Tn} = 2 - 1\ V \Rightarrow V_o \geq 1V$

Part (b)

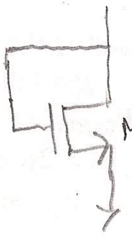


(a) $V_{G3} = V_{GS3} + V_{GS1} = 2 + 1.5 = 3.5\ V$

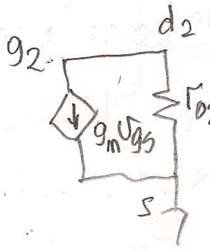
$$R_F = \frac{V_{DD} - V_{G3}}{I_{REF}} = \frac{5 - 3.5V}{0.25mA} = \frac{1.5\ V}{0.25\ mA} = \underline{6K}$$

(b) $$\frac{I_O}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1} = \frac{0.1}{0.25} = 0.4$$

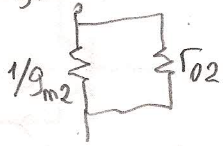
(c)



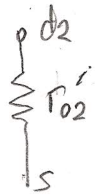
$\Rightarrow$ Small signal model



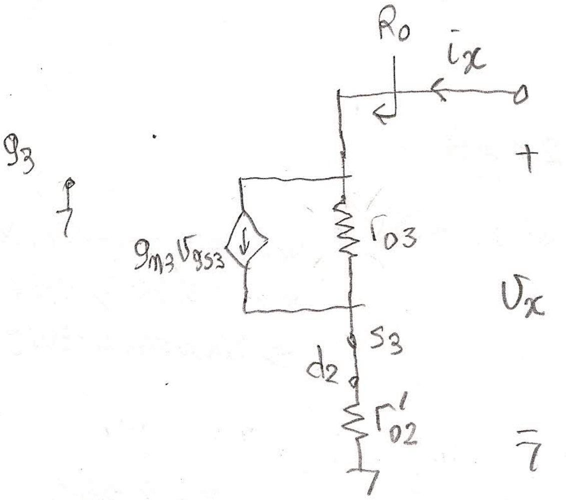
$v_{gs} = v_{ds}$



$\Rightarrow$



$$r_{o2}' = r_{o2} // \frac{1}{g_{m2}}$$



$$R_o = \frac{v_x}{i_x}$$

$$v_x = (i_x - g_{m3} v_{gs3}) r_{o3} + i_x r_{o2}'$$

$$= [i_x - g_{m3}(-v_{s3})] r_{o3} + i_x r_{o2}'$$

$$= ((i_x + g_{m3} r_{o2}' i_x) r_{o3} + i_x r_{o2}')$$

$$= \underbrace{(r_{o3} + g_{m3} r_{o2}' r_{o3} + r_{o2}')}_{R_o} i_x$$

$$g_{m3} = 2K_{n2}(V_{GS3} - V_{Tn}) = \frac{2K_{n3}(V_{GS3} - V_{Tn})^2}{V_{GS3} - V_{Tn}} = \frac{2 I_o}{V_{GS3} - V_{Tn}} = \frac{(2)(0.1)\ mA}{1.5 - 1\ V} = 0.4 mA/V$$

$$g_{m2} = \frac{2 I_o}{V_{GS2} - V_{Tn}} = \frac{(2)(0.1\ mA)}{2 - 1} = 0.2\ mA/V \Rightarrow \frac{1}{g_{m2}} = 5\ k\Omega$$

$$r_{o2}' = \frac{1}{g_{m2}} // r_{o2} = 5K // 1M\Omega = 5\ k\Omega \ ; \ r_{o3} = \frac{1}{\lambda_3 I_o} = 1 M\Omega$$

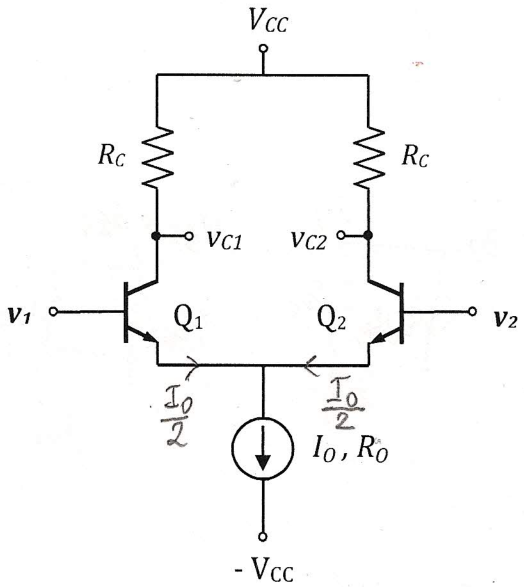
$$R_o = 1M\Omega + (0.4 mA/V)(5k)(1M) + 5k = 1M + 2M = 3M$$

(d) $V_o = V_{DS3} + V_{GS1} \geq (V_{GS3} - V_{Tn}) + V_{GS1} = (1.5 - 1) + 2 = 2.5\ V$

$\underline{V_o \geq 2.5\ V}$

Q3. (25 pts) Consider the differential amplifier given below. R_O indicates the small signal resistance of the current source I_O.

(a) Determine the DC values of the transistors when $v_1 = v_2 = 0$. Check whether the transistors are in forward active region or not.
(b) Find the differential-mode gain.
(c) Find the common-mode gain.
(d) Find CMRR
(e) If $v_1 = 2\sin(2\pi*50t) + 0.005\sin(2\pi*1000t)$ volts, and $v_2 = 2\sin(2\pi*50t) - 0.005\sin(2\pi*1000t)$ volts, then find $v_O = v_{C1} - v_{C2}$



Circuit Parameters
$V_{CC} = 12$ V
$R_C = 3$ K
$I_O = 5$ mA
$R_O = 40$ K

Transistor Parameters
$\beta = 100$
$V_{BE(ON)} = 0.7$ V
$V_T = 26$ mV
$V_A = \infty$

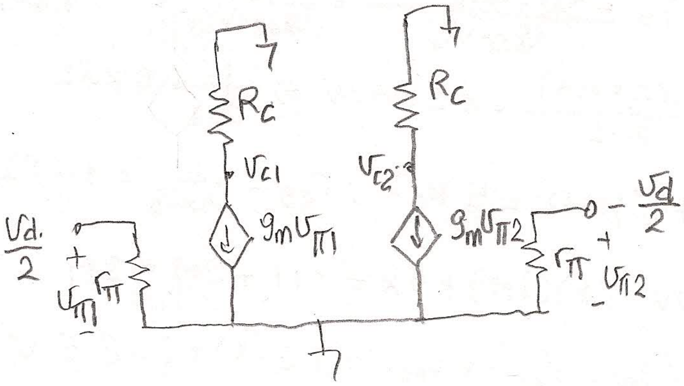
(a) $I_{E1} = \frac{I_0}{2} = 2.5mA \Rightarrow I_{C1} \cong I_{E1} = 2.5\,mA$

$V_{CQ1} = V_{CC} - R_C I_{C1} = 12 - (3k)(2.5\,mA) = 4.5V$
$V_{EQ1} = -V_{BE(ON)} = -0.7V$

$V_{CE1} = 4.5 - (-0.7)\,V$
$V_{CE1} = 5.2V \geq V_{CE(SAT)}$
$\Rightarrow$ Forward active !

(b) $v_1 = \frac{v_{dm}}{2}$; $v_2 = -\frac{v_{dm}}{2}$;

$r_\pi = \frac{26\,mV}{I_{CQ}/\beta} = \frac{(26mV)(100)}{2.5\,mA} = 1.04\,k\Omega \Rightarrow g_m = \frac{\beta}{r_\pi} = 96.2\,mA/V$



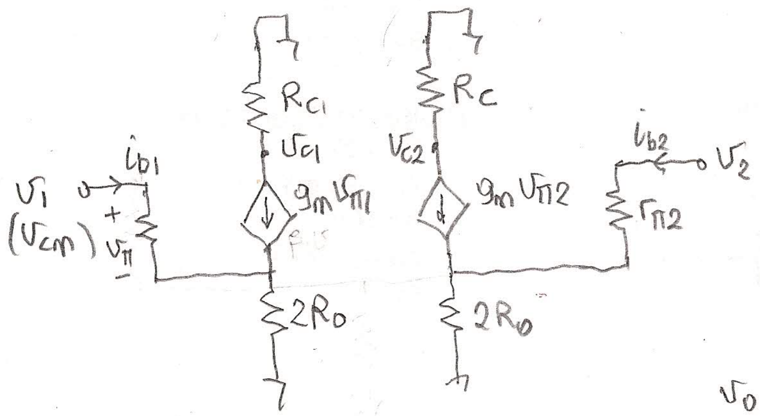
$v_{\pi1} = \frac{v_d}{2} \Rightarrow v_{C1} = -R_C g_m \frac{v_d}{2}$

$v_{\pi2} = -\frac{v_d}{2} \Rightarrow v_{C2} = R_C g_m \frac{v_d}{2}$

$v_o = v_{C1} - v_{C2} = -R_C g_m v_d$

$A_{dm} = -R_C g_m = (3k)(96.2\,mA/V)$
$= -288.5$

(c) $v_1 = v_{cm}$; $v_2 = v_{cm}$



$$i_{b1} = \frac{v_{cm}}{r_\pi + 2(\beta+1) r_o}$$

$$i_{b2} = \frac{v_{cm}}{r_\pi + 2(\beta+1) r_o}$$

$$v_{c1} = -R_c i_{b1}$$

$$v_{c2} = -R_c i_{b2} = v_{c1}$$

$$v_o = v_{c1} - v_{c2} = 0 \Rightarrow A_{cm} = 0$$

(d) $$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \frac{288,5}{0} = \infty$$

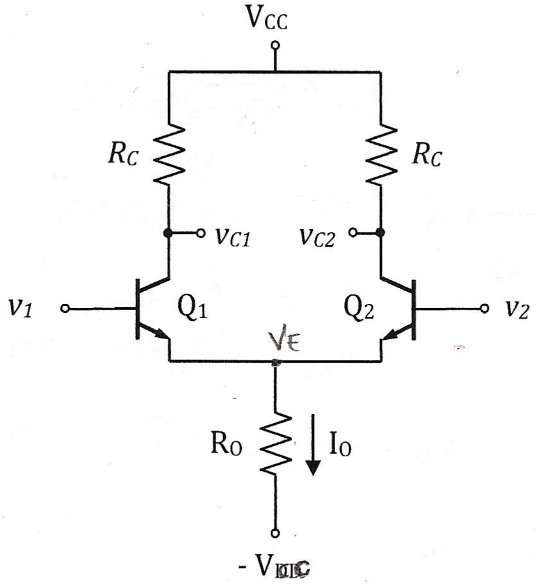
(e) $$v_d = v_1 - v_2 = 0.01 \sin 2\pi \cdot 1000\, t \text{ volts}$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2 \sin 2\pi \times 50 t \text{ volts}$$

$$v_o = A_{dm} v_d + \cancel{A_{cm}} v_{cm} = (288.5)(0.01) \sin 2\pi \times 1000 t \text{ volts}$$

$$= 2.885 \sin 2\pi * 1000\, t \text{ volts}$$

Q4. (25 pts) Consider the differential amplifier given, with mismatched V_T voltages of the transistor, so that the g_m of the transistors are not identical. Output is measured as $v_O = v_{C1} - v_{C2}$



Circuit Parameters

V_{CC} = 12 V

R_C = 3 kΩ

Transistors' Parameters

V_{T1} = 27 mV

V_{T2} = 25 mV

β = 100

$V_{BE(on)}$ = 0.7 V

$V_A = \infty$

i. Determine R_O to set $I_{C1} = I_{C2}$ = 2 mA. (Let $v_1 = v_2 = 0$ in the DC analysis)

For the remaining of the problem assume R_O **=3 kΩ.**

ii. Determine g_{m1} and g_{m2} for the transistors Q_1 and Q_2. Also determine $\Delta g_m = g_{m1} - g_{m2}$ and $g_m = (g_{m1} + g_{m2})/2$

iii. Determine the differential mode gain.

iv. Determine the common mode gain.

v. Determine $CMRR_{dB}$.

i. $I_O = I_{E1} + I_{E2} \cong I_{C1} + I_{C2} = 4\ mA$

$V_E = -V_{BE(ON)} = -0.7V$

$R_O = \frac{V_E - (-V_{CC})}{I_O} = \frac{-0.7 + 12\ V}{4\ mA} = 2.825\ k\Omega$

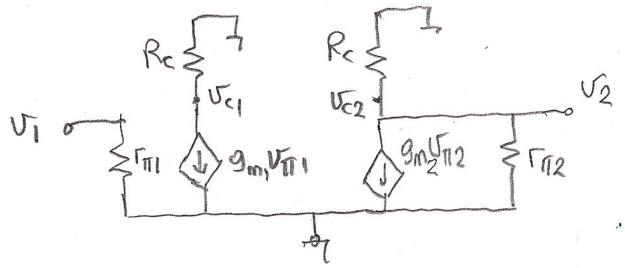
ii. $R_o = 3\ k\Omega$

$r_{\pi 1} = \frac{V_{T1}\beta}{I_{C1}} = \frac{(27\ mV)(100)}{2\ mA} = 1.35\ k\Omega \Rightarrow g_{m1} = \frac{\beta}{r_{\pi 1}} = 74.07\ mA/V$

$r_{\pi 2} = \frac{V_{T2}\beta}{I_{C2}} = \frac{(25 mV)(100)}{2\ mA} = 1.25\ k\Omega \Rightarrow g_{m2} = \frac{\beta}{r_{\pi 2}} = 80\ mA/V$

$\Delta g_m = -6\ mA/V$; $g_m = 77\ mA/V$

iii. $v_1 = \frac{v_d}{2}$; $v_2 = -\frac{v_d}{2}$



$v_{c1} = -g_{m1} R_C \frac{v_d}{2}$; $v_{c2} = g_m R_C \frac{v_d}{2}$

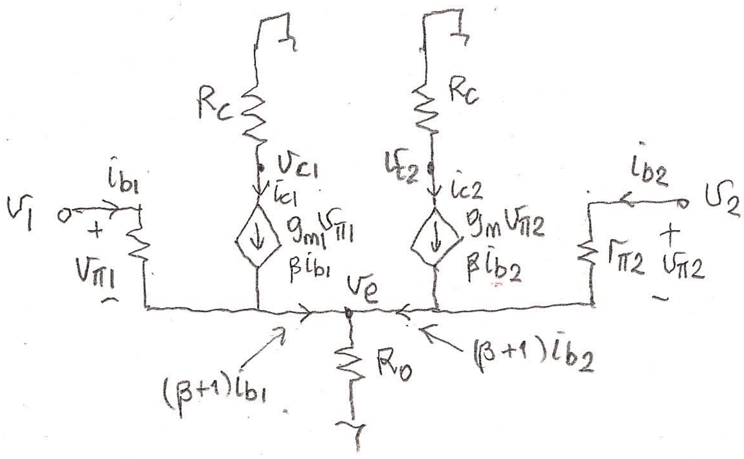
$v_o = v_{c1} - v_{c2} = -(g_{m1} + g_{m2}) R_C \frac{v_d}{2}$

$= -g_m R_C v_d$

$A_{dm} = (-g_m)(R_C) = -(77 mA/V)(3k)$

$A_{dm} = -231$

iv) $v_1 = v_{cm}$, $v_2 = v_{cm}$



$$v_e = [(\beta+1)i_{b1} + (\beta+1)i_{b2}]R_o$$

$$i_{b1} = \frac{v_1 - v_e}{r_{\pi1}} = \frac{v_c - v_e}{r_{\pi1}}$$

$$i_{b2} = \frac{v_2 - v_e}{r_{\pi2}} = \frac{v_c - v_e}{r_{\pi2}}$$

$$v_e = R_o\left[(\beta+1)\frac{v_c - v_e}{r_{\pi1}} + (\beta+1)\frac{v_c - v_e}{r_{\pi2}}\right]$$

$$= R_o(\beta+1)(v_c - v_e)\left(\frac{1}{r_{\pi1}} + \frac{1}{r_{\pi2}}\right)$$

$$= R_o(\beta+1)(v_c - v_e)\left(\frac{g_{m1}}{\beta} + \frac{g_{m2}}{\beta}\right)$$

$$\Rightarrow v_e = R_o(\beta+1)\left(\frac{g_{m1}+g_{m2}}{\beta}\right)(v_c - v_e) \qquad \beta \gg 1,\ \beta+1 \cong \beta$$

$$\cong 2R_o g_m (v_c - v_e)$$

$$(1+2R_o g_m)v_e = 2R_o g_m v_c \Rightarrow v_e = \frac{2R_o g_m}{1+2R_o g_m} v_c$$

$$i_{b1} = \frac{v_c - v_e}{r_{\pi1}} = \frac{1}{r_{\pi1}}\left(v_c - \frac{2R_o g_m}{1+2R_o g_m} v_c\right) = \frac{1}{r_{\pi1}} \frac{1}{1+2R_o g_m} v_c$$

$$i_{c1} = \beta i_{b1} = \frac{\beta}{r_{\pi1}} \cdot \frac{1}{1+2R_o g_m} v_c = \frac{g_{m1}}{1+2R_o g_m} v_c$$

$$\Rightarrow v_{c1} = -R_c i_{c1} = \frac{-R_c g_{m1}}{1+2R_o g_m} v_c \cong -\frac{R_c}{2R_o} \frac{g_{m1}}{g_m} v_c$$

$$v_{c2} = -R_c i_{c2} \cong -\frac{R_c}{2R_o} \frac{g_{m2}}{g_m} v_c$$

Then $v_o = v_{c1} - v_{c2} = -\frac{R_c}{2R_o g_m}(g_{m1} - g_{m2})v_c = \underbrace{-\frac{R_c}{2R_o}\frac{\Delta g_m}{g_m}}_{A_{CM}} v_c$

$$A_{cm} = -\frac{R_c}{2R_o} \cdot \frac{\Delta g_m}{g_m} = -\frac{3k}{2(3K)} \cdot \frac{6}{77} = -0.039$$

(v) $$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \frac{231}{0.039} = 5.93 \times 10^3$$

$$CMRR_{dB} = 20\log(5.93\times10^3) = 75.46\ dB$$