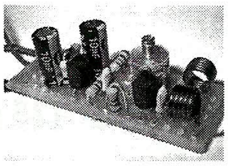
İZMİR UNIVERSITY OF ECONOMICS
Department of Electronics and Communication Engineering



EEE 331 Analog Electronics
Fall 2013/2014

SOLUTION SET

MIDTERM EXAM I

Nov 23, 2013
120 min

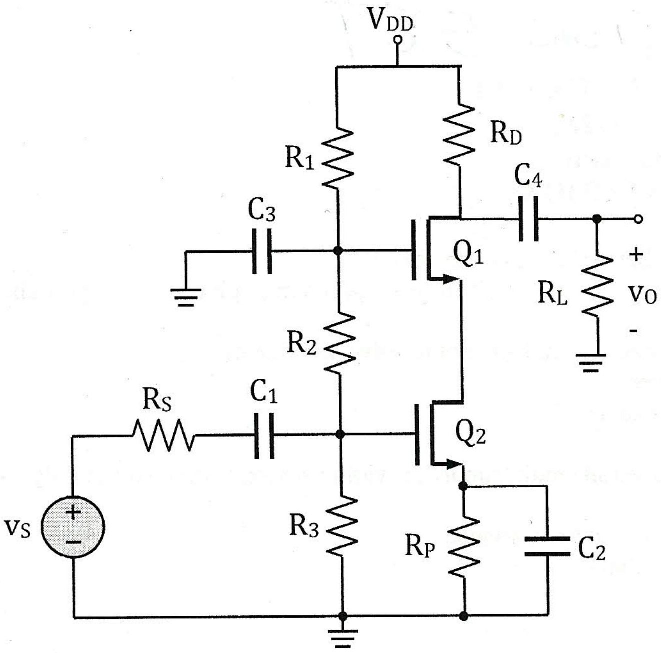
INSTRUCTIONS

- Read all of the instructions and all of the questions before beginning the exam.
- There are 6 questions on this exam, totaling 100 points. The credit for each problem is given to help you allocate your time accordingly.
- Do not spend all your time on one problem and on one part and attempt to solve all of them.
- Calculators are allowed, but borrowing is not allowed.
- Your mobile phones must be turned off during the exam.
- Turn in the entire exam, including this cover sheet.
- You must show your work for all problems to receive full credit; simply providing answers will result in only partial credit, even if the answers are correct.
- Please indicate the number of page where your work is to be continued.
- Put your name on any additional material that you submit.
- Be sure to provide units where necessary.
- Please sign the honor pledge that is provided below.

		Question	Points	Grade
Last Name	:......................	1	25	
Name	:......................	2	25	
Section	:......................	3	25	
Student No	:......................	4	25	
		TOTAL	100	

The basic equations of the output characteristics of an NMOS transistor		
V_{GS}	**V_{DS}**	**I_D**
i) $V_{GS} < V_{Tn}$	-	0
ii) $V_{GS} > V_{Tn}$	a) $V_{DS} < V_{GS} - V_{Tn}$	$K_n\left[2(V_{GS} - V_{Tn})V_{DS} - V_{DS}^2\right]$
	b) $V_{GS} - V_{Tn} \le V_{DS}$	$K_n(V_{GS} - V_{Tn})^2$
where $K_n = \frac{K'_n}{2}\left(\frac{W}{L}\right)$ and $K'_n = \mu_n C_{ox}$		

Q1. (25 pts) Consider the cascode amplifier given below.



Circuit Parameters

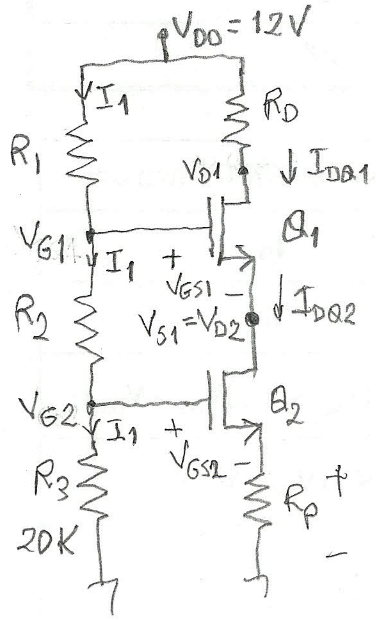
$V_{DD} = 12V$
$R_L = 10\ k\Omega$
$R_S = 1\ k\Omega$
$R_P = 0.5\ k\Omega.$
$R_3 = 20\ k\Omega.$
$I_{DQ2} = 2\ mA$
$V_{DS1} = 4\ V$
$V_{DS2} = 5\ V$

Transistor Parameters

$K_{N1} = 1\ mA/V^2$
$K_{N2} = 2\ mA/V^2$
$V_{TN1} = 1\ V$
$V_{TN2} = 1\ V$
$\lambda_1 = 0$
$\lambda_2 = 0$

(a) Assume $I_{DQ2} = 2$ mA and $V_{DS1} = 4$ V, $V_{DS2} = 5$ V, and $R_3 = 20\ k\Omega$, find $\boldsymbol{R_D}$, $\boldsymbol{R_1}$, and $\boldsymbol{R_2}$.
(b) Determine the small-signal parameters (i.e., g_m and r_o) for each transistor.
(c) Determine the overall small-signal voltage gain $Av = vo / vs$.

(a) DC Model :



Same current I_1 flows through R_1, R_2 & R_3; since gate currents are zero

$I_{DQ1} = I_{DQ2} = 2\ mA$

$I_D = K_n (V_{GS} - V_{TN})^2 \Rightarrow V_{GS2} = \sqrt{\frac{I_{DQ2}}{K_{n2}}} + V_{TN2}$

$V_{GS2} = \sqrt{\frac{2}{2}} + 1 = 2V$

$V_{G2} = V_{GS2} + R_P I_{DQ2} = 2 + (0.5k)(2mA) = 3\ V$

$I_1 = \frac{V_{GQ1}}{R_3} = \frac{3V}{20\ k\Omega} = 0.15\ mA$

Then $V_{D2} = V_{S1} = V_{DS2} + I_{DQ} R_P = 5 + (0.5k)(2mA)$

$\Rightarrow V_{S1} = 6V$

$V_{GS1} = \sqrt{\frac{I_{DQ1}}{K_{N1}}} + V_{TN1} = \sqrt{\frac{2}{1}} + 1 = 2.41\ V$

$\Rightarrow V_{G1} = V_{GS1} + V_{S1} = 2.41 + 6 = 8.41\ V$

$R_2 = \frac{V_{G1} - V_{G2}}{I_1} \approx \frac{8.41 - 3}{0.15mA} = 36.09\ K$

$R_1 = \frac{V_{DD} - V_{G1}}{I_1} = \frac{12 - 8.41}{0.15\ mA} = 23.91K$

$V_{D1} = V_{DS1} + V_{S1} = 4 + 6 = 10V \Rightarrow R_D = \frac{V_{DD} - V_{D1}}{I_{D1}} = \frac{12-10}{2\ mA} = 1\ k\Omega$

Check for SAT

$Q_2: V_{DS2} \geq V_{GS2} - V_{TN2} \Rightarrow 5 \geq 2-1$ ✓

$V_{GS2} = 2V$

$g_{m2} = 2K_{n2}(V_{GS2} - V_{Tn2}) = 4(2-1)$

$= 4\ mA/V$

$r_{02} = \frac{1}{\lambda I_{DQ}} = \infty$

$Q_1: V_{DS1} \geq V_{GS1} - V_{TN1} \Rightarrow 4 \geq 2.41-1$ ✓

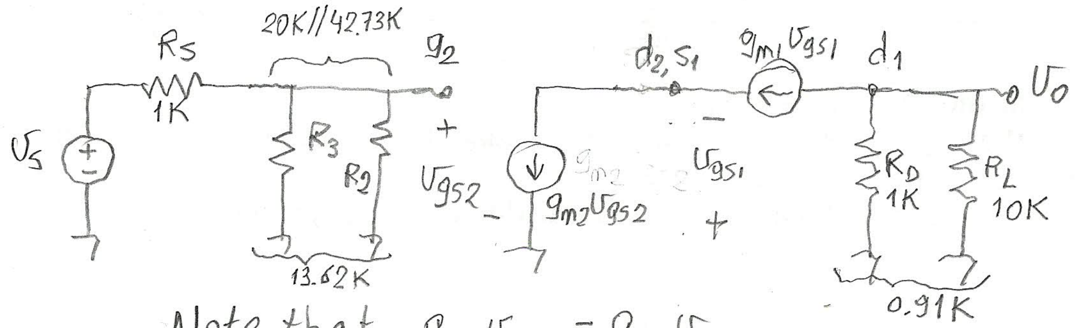
$V_{GS1} = 2.41V$

$g_{m1} = 2K_{N1}(V_{GS1} - V_{TN1}) = 2 \cdot 1 \cdot (1.41)$

$= 2.82\ mA/V$

$r_{01} = \infty$

Small signal AC Model :



Note that $g_{m1}U_{gs1} = g_{m2}U_{gs2}$

$U_O = -g_{m1}U_{gs1}(R_D // R_L) = -g_{m2}U_{gs2}(R_D // R_L)$

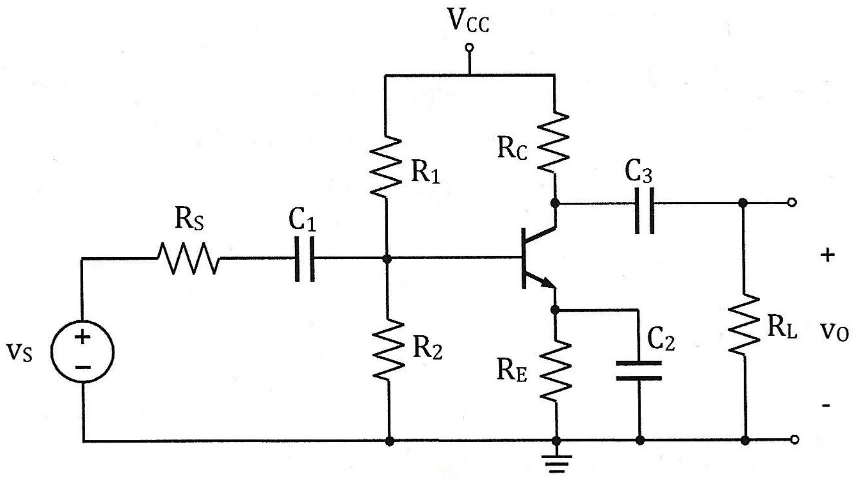
$= -g_{m2}\left(\frac{R_3 // R_2}{R_S + R_3 // R_2}\right) U_S (R_D // R_L)$

$= -g_{m2}\left(\frac{R_3 // R_2}{R_S + R_3 // R_2}\right)(R_D // R_L) U_S$

$= -(4)\left(\frac{13.62}{1 + 13.62}\right)(0.91K) U_S = -3.39 U_S$

$A_V = -3.39$

Q2. (25 pts) Consider the common emitter amplifier given below.



Circuit Parameters

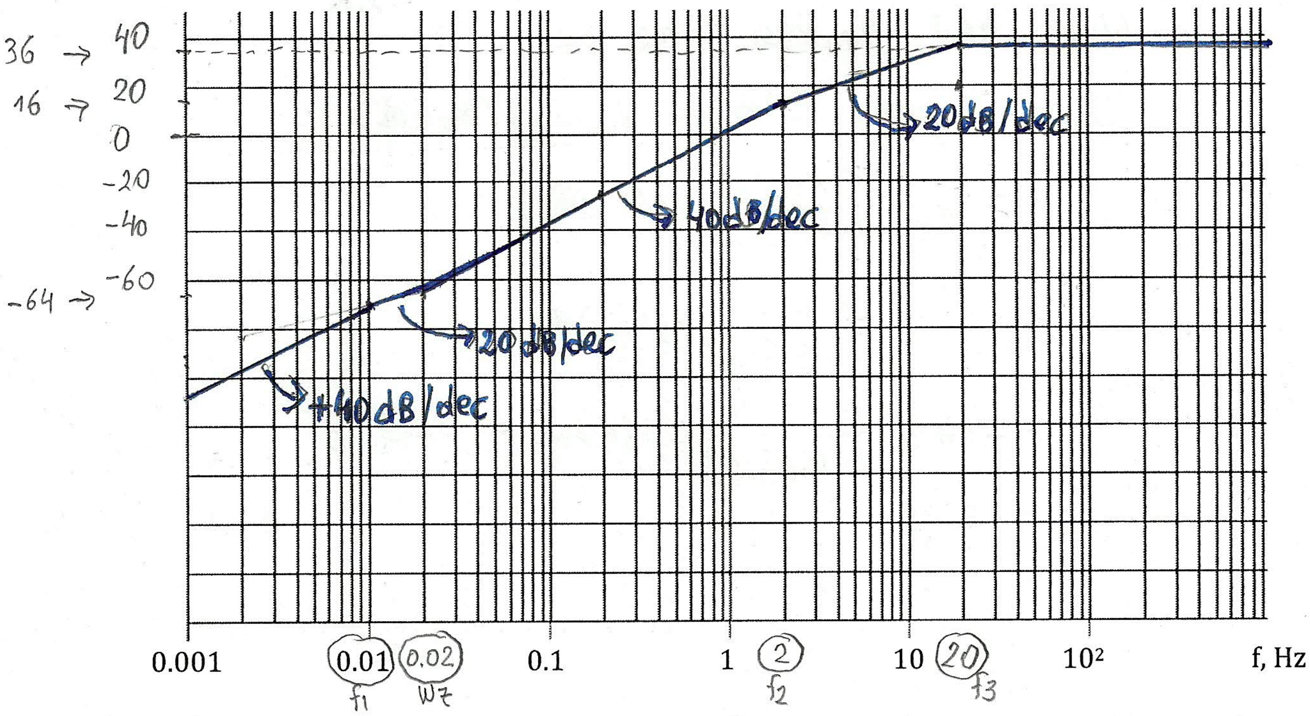
$V_{CC} = 12$ V
$R_1 = 100$ K
$R_2 = 15$ K
$R_E = 1$ K
$R_C = 4$ K
$R_S = 1$K
$R_L = 12$ K

Transistor Parameters

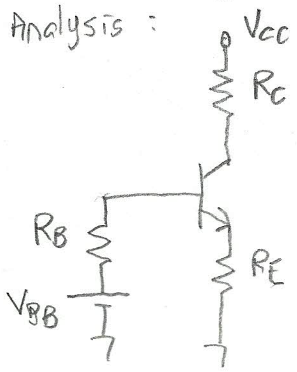
$\beta = 100$
$V_{BE(ON)} = 0.7$ V
$V_A = \infty$

a) Determine the midband voltage gain.
b) Choose the capacitors C_1, C_2, and C_3 such that the corresponding break frequencies f_1=0.01Hz, f_2=2 Hz, f_3=20 Hz.
c) Sketch the magnitude Bode plot on the logarithmic graph paper given plot.

$|A|_{dB}$, dB



(a) DC Analysis :



$$V_{BB} = \frac{R_2}{R_1+R_2} V_{CC} = \frac{15}{100+15} \cdot 12 = 1.565 \text{ V} \checkmark$$

$$R_B = R_1 // R_2 \simeq 13K$$

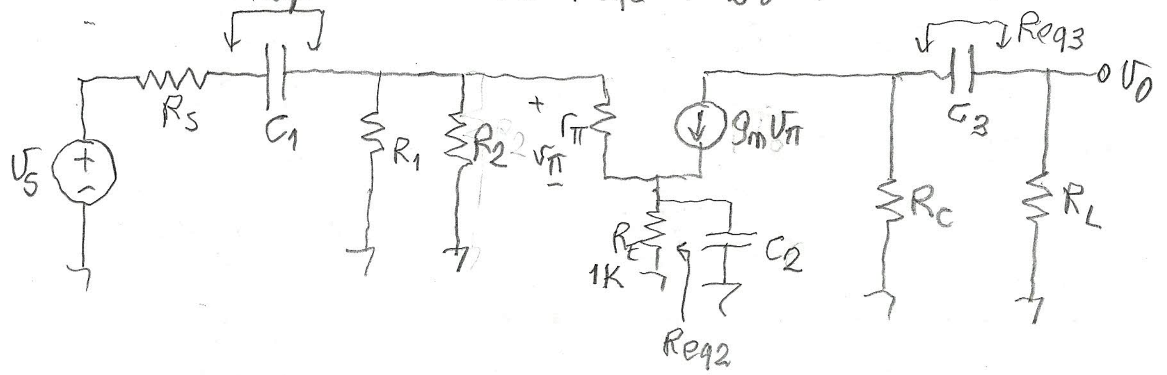
$$I_B = \frac{V_{BB} - V_{BE(ON)}}{R_B + (\beta+1)R_E} = \frac{1.565 - 0.7}{13 + (101)(1)} \simeq 7.6 \mu A$$

Small signal parameters :

$$r_\pi = \frac{V_T}{I_{BQ}} = \frac{26\,mV}{7.6\,\mu A} = 3.42\,K \Rightarrow g_m = \frac{\beta}{r_\pi} = 29.24\ mA/V$$

$$r_0 = \infty$$

Small signal model : Note that f_1 from C_1 is smaller than f_2 of C_2, then for R_{eq1} calculations C_2 will be assumed open. For R_{eq2} of C_2 ; C_1 is short.



- $R_{eq1} = R_S + R_1 // R_2 // [r_\pi + (\beta+1) R_E] = 1K + [100K // 15K // 104.42] = 12.56\,K$

$$\omega_1 = 2\pi f_1 = \frac{1}{R_{eq1} C_1} \Rightarrow C_1 = \frac{1}{2\pi (0.01)(12.56 \times 10^3)} = 1.27 \times 10^{-3} F = 1.27\ mF$$

- $$R_{eq2} = \left(\frac{R_S // R_1 // R_2 + r_\pi}{\beta+1} \right) // R_E = \frac{1//100//15 + 3.42}{101} // 1K = 43\Omega // 1K = 41.2\Omega$$

$$C_2 = \frac{1}{\omega_2 R_{eq2}} = \frac{1}{(2\pi)(2)(41.2)} = 1.93 \times 10^{-3} F = 1.93\ mF$$

- $R_{eq3} = R_C + R_L = 4 + 12 = 16\,K$

$$C_3 = \frac{1}{\omega_3 R_{eq3}} = \frac{1}{(2\pi)(20)(16 \times 10^3)} = 0.49 \times 10^{-6} F = 0.49\ \mu F$$

- Zero because of C_2 :

$$\omega_z = \frac{1}{R_E C_2} = \frac{1}{(1K)(1.93 \times 10^{-3})} = 0.52\ rad/sec \Rightarrow f_z = 0.02\ Hz$$

Sequence of break frequencies :

0.01	0.02	2	20
f_1	f_z	f_2	f_3

Midband gain : All capacitors are short

$$V_\pi = \frac{R_1 // R_2 // r_\pi}{R_S + R_1 // R_2 // r_\pi} V_S = \frac{2.7}{3.7} V_S = 0.73\, V_S \ ; \ V_0 = -g_m V_\pi (R_C // R_L) =$$

$$= -(29.24)(3)(0.73) V_S =$$

$$A_{MB} = -64$$

$$A_{MB,dB} = 20 \log |-64| = 36\ dB$$

Plot is on the above page.

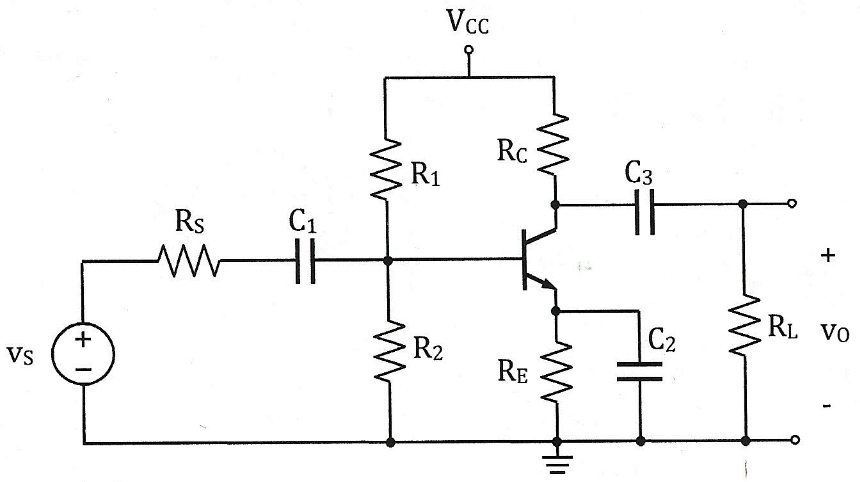
Q3. (25 pts) Consider the common source amplifier given below.

(a) Determine the beta corner (cut-off) frequency of the transistor.

(b) Determine the small signal parameters r_π, g_m and r_o of the transistor.

(c) Determine the high frequency corner frequency of the amplifier.

(d) Sketch the Bode magnitude plot of the high frequency behaviour of the amplifier over the logarithmic paper given below.



Circuit Parameters

$V_{CC} = 12$ V

$R_1 = 100$ K

$R_2 = 15$ K

$R_E = 1$ K

$R_C = 4$ K

$R_S = 1$K

$R_L = 12$ K

Transistor Parameters

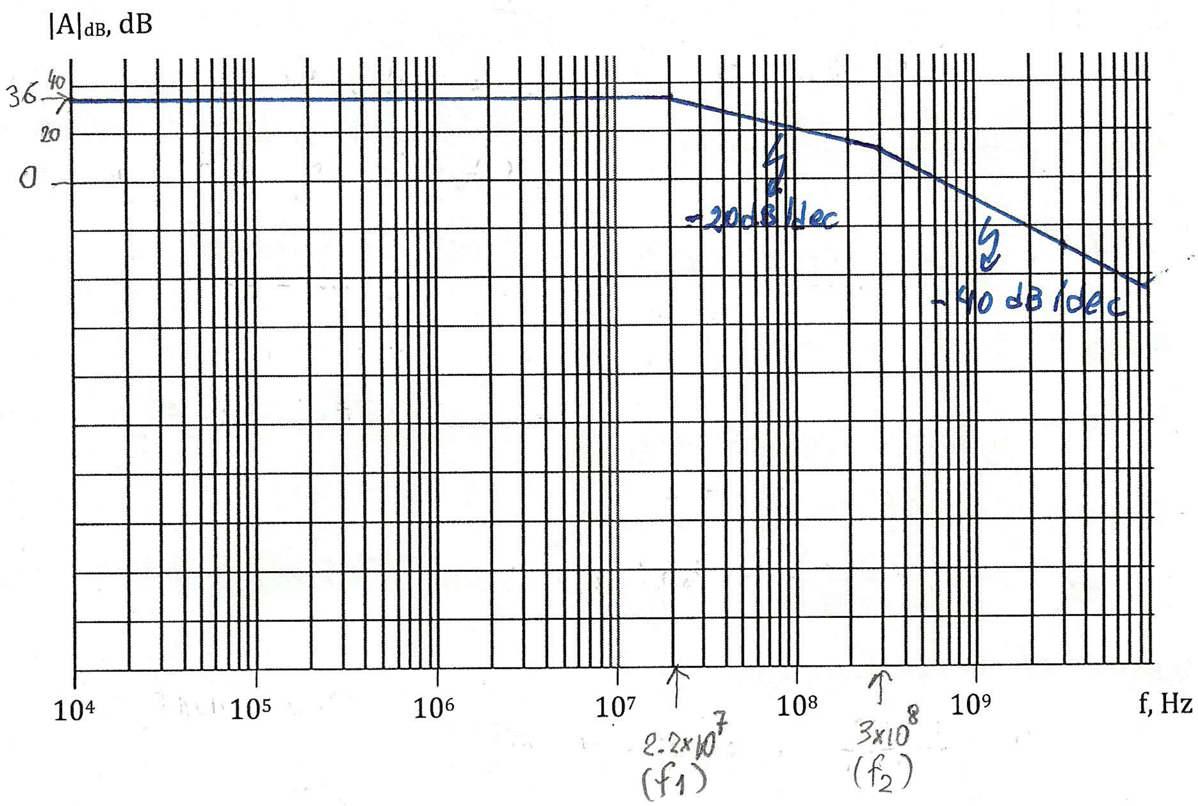
$\beta = 100$

$V_{BE(ON)} = 0.7$ V

$V_A = \infty$

$C_\pi =$ pF 1pF

$C_\mu =$ pF 0.1 pF



(a) $\beta(j\omega) = \dfrac{\beta_{MB}}{1 + j\frac{\omega}{\omega_c}}$ $\quad \omega_c = \dfrac{1}{r_\pi(C_\pi + C_\mu)}$

The DC conditions and AC parameters are exactly the same as in Q_2:

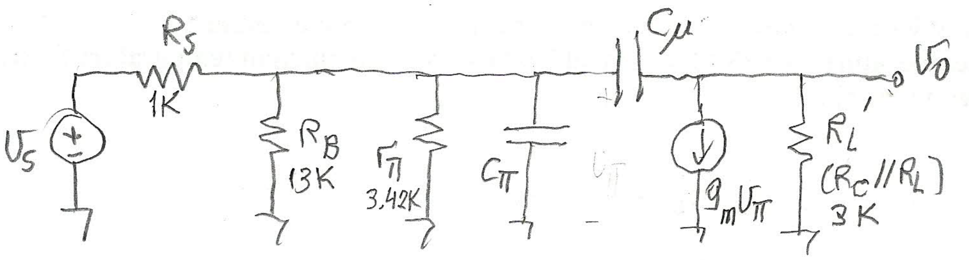
$$r_\pi = 3.42K \Rightarrow \omega_c = \frac{1}{(3.42\times10^3)(1.1\times10^{-12})} = \frac{1}{3.762\times10^{-9}} = 265\times10^6 \text{ rad/sec}$$

$$f_c = \frac{\omega_c}{2\pi} = 42.32 \text{ MHz}$$

(b) From Q_2: $r_\pi = 3.42K$

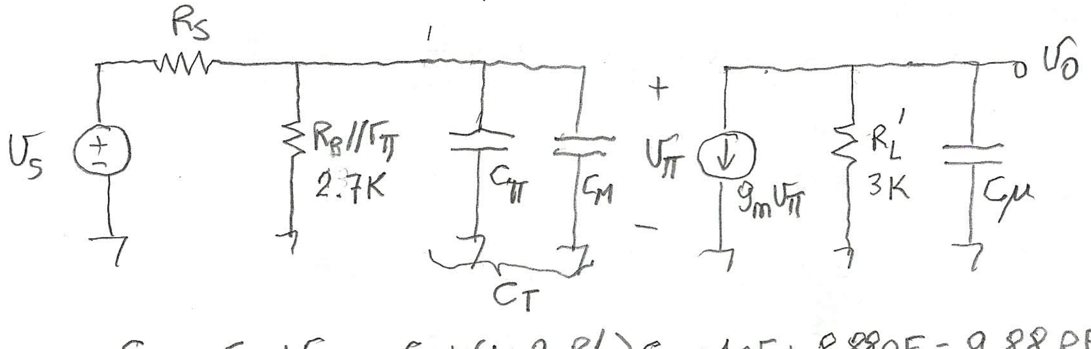
$r_o = \infty$, $g_m = 29.24$ mA/V

(c) High frequency model of the amplifier (all external capacitors are short already)



$g_m = 29.24$ mA/V

Using Miller effect, C_μ is reflected to the input and output.



$$C_T = C_\pi + C_M = C_\pi + (1 + g_m R_L')C_\mu = 1\text{pF} + 8.88\text{pF} = 9.88 \text{ pF}$$

$$R_{eq,T} = R_S // R_B // r_\pi = 0.73K$$

$$\Rightarrow \omega_1 = \frac{1}{R_{eq,T}C_T} = \frac{1}{(0.73\times10^3)(9.88\times10^{-12})} = 138\times10^6 \text{ rad/sec}$$

$$f_1 = 22 \text{ MHz} = 2.2\times10^7 \text{ Hz}$$

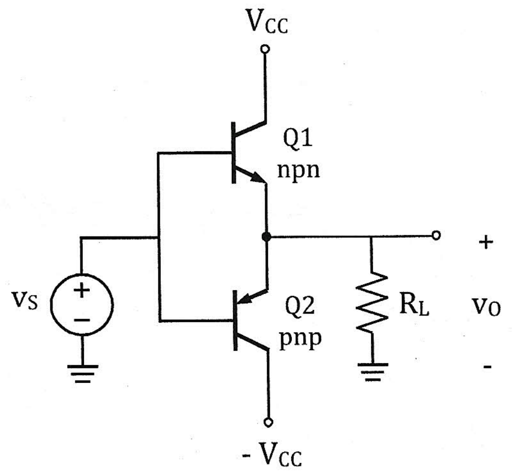
$$\omega_2 = \frac{1}{R_L' C_\mu} = \frac{1}{(3K)(0.1\times10^{-12})} = 3.33\times10^9 \text{ rad/sec} \Rightarrow f_2 = 3\times10^8 \text{ Hz}$$

From Q2: Midband gain:

$$A_{MB} = -64 \Rightarrow A_{MB,dB} = 20\log|-64| = 36 \text{ dB}$$

Plot is on the above page.

Q4. (25 pts) Consider the class-B amplifier given below.



Circuit Parameters
V_{CC} = 12 V
R_L = 10 Ω

Transistors' Parameters
β = 100
θ_{JC} = 10 ^{0}C/W (Thermal resistance between junction and case)
θ_{CA} = 80 ^{0}C/W (Thermal resistance between case and ambient)

Heat Sink
θ_{CH} = 6 ^{0}C/W (Thermal resistance between case and heat-sink)
$\theta_{\cancel{CA}\,HA}$ = 8 ^{0}C/W (Thermal resistance between heat-sink and ambient)

Assume the transistors are ideal with $V_{BE1(ON)}$ = 0 and $V_{EB2(ON)}$ = 0, and the input is a sinusoidal

$v_S = V_P \sin \omega_0 t$ volts

a) Determine the amplifier efficiency η when V_P = 6 V.
b) What is the value of V_P to get maximum efficiency? What is the efficiency η then?
c) For which value of V_P, the power dissipation over any transistor is maximum?
d) If the ambient temperature T_A = 25 ^{0}C, what will be the maximum junction temperature (i) without heat sink, (ii) with heat sink?

(a) $V_S = 6 \sin \omega_0 t$ volts $\Rightarrow V_o = 6 \sin \omega_0 t$ volts ($V_P = 6$ V

$P_{L,AC} = \frac{V_P^2}{2R_L} = \frac{36}{(2)(10)} = 1.8$ W

Power from V_{CC}:

$P_{CC} = V_{CC} \cdot \frac{I_{CP}}{\pi} = V_{CC} \cdot \frac{V_P}{\pi R_L} = (12)\frac{(6)}{\pi (10)} = 2.29$ W

Total power from the two supplies:

$P_S = 2P_{CC} = 4.58$ W

Then $\eta = \frac{P_{L,AC}}{P_S} = \frac{1.8\text{ W}}{4.58\text{ W}} = 0.39 = 39\%$

(b) Maximum efficiency is reached when

$V_P = 12$V

Then $P_{L,AC} = \frac{V_P^2}{2R_L} = 7.2$ W

$P_S = 2 \cdot V_{CC} \cdot \frac{V_P}{R_L} \cdot \frac{1}{\pi} = \frac{2V_{CC}^2}{\pi R_L} = \frac{(2)(144)}{(3.14)(10)} = 9.17$W

Hence efficiency

$\eta = \frac{P_{L,AC}}{P_S} = \frac{7.2}{9.17} = 0.785 = 78.5\%$

(c) Power dissipation over the transistors will not be maximum when $V_p = V_{CC}$. To determine $P_{T,max}$:

$$P_{LAC} = \frac{V_p^2}{2R_L}$$

$$P_S = 2P_{CC} = 2 \cdot V_{CC} I_p = 2V_{CC} \cdot \frac{V_p}{R_L} = \frac{2V_{CC}V_p}{\pi R_L}$$

So the power dissipation over one transistor:

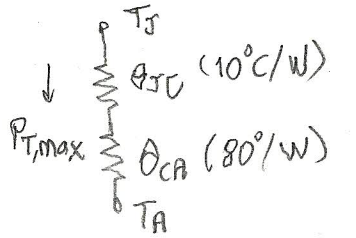
$$P_T = \frac{1}{2}(P_S - P_{LAC}) = \frac{1}{2}\left(\frac{2V_{CC}V_p}{\pi R_L} - \frac{V_p^2}{2R_L}\right) = \frac{1}{2R_L}\left(\frac{2V_{CC}}{\pi}V_p - \frac{V_p^2}{2}\right)$$

$$\frac{dP_T}{dV_p} = 0 \Rightarrow V_p = \frac{2}{\pi}V_{CC} \Rightarrow P_{T,max} = \frac{V_{CC}^2}{\pi^2 R_L}$$

So for the above power amplifier, power over one transistor will be maximum if $V_p = \frac{2 \cdot 12V}{\pi} = 7.64V$

and the corresponding $P_{T,max} = \frac{V_{CC}^2}{\pi^2 R_L} = \frac{144}{\pi^2 \cdot 10} = 1.46\ W$

(d) Without heat sink:



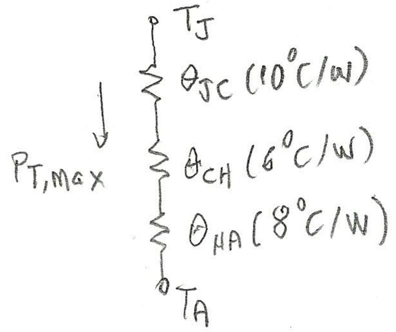
$$T_{J,max} - T_A = P_{T,max} \cdot (\theta_{JC} + \theta_{CA})$$

$$= (1.46)(10+80) = 131.4°C$$

$$T_{J,max} = 131.4 + 25 = 156.4°C$$

(probably it is burned out)

with heat sink:



$$T_{J,max} = P_{T,max}(\theta_{JC} + \theta_{CH} + \theta_{HA}) + T_A$$

$$= (1.46)(24) + 25$$

$$\simeq 60°C \text{ (safe !)}$$