

MIDTERM EXAM I

Nov 12, 2014

120 min

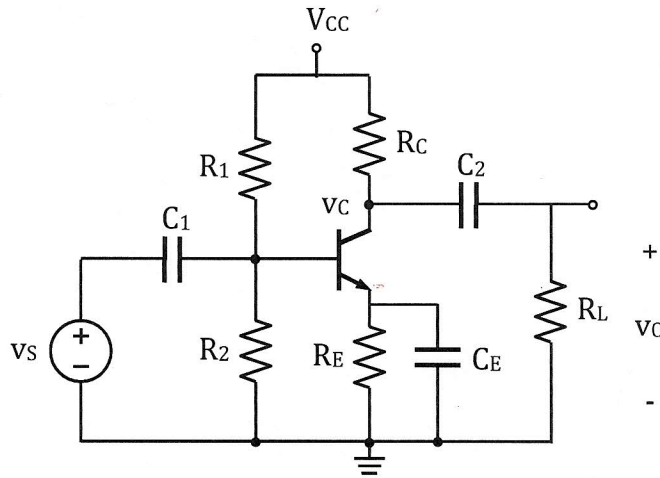
INSTRUCTIONS

- | | |
|---|--|
| <ul style="list-style-type: none"> Read all of the instructions and all of the questions before beginning the exam. There are 4 questions on this exam, totaling 100 points. The credit for each problem is given to help you allocate your time accordingly. Do not spend all your time on one problem and on one part and attempt to solve all of them. Calculators are allowed, but borrowing is not allowed. Your mobile phones must be turned off during the exam. | <ul style="list-style-type: none"> Turn in the entire exam, including this cover sheet. You must show your work for all problems to receive full credit; simply providing answers will result in only partial credit, even if the answers are correct. Please indicate the number of page where your work is to be continued. Put your name on any additional material that you submit. Be sure to provide units where necessary. Please sign the honor pledge that is provided below. |
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Last Name :	Question	Points	Grade
Name :	1	25	
Section :	2	25	
Student No :	3	25	
	4	25	
	TOTAL	100	

The basic equations of the output characteristics of an NMOS transistor		
V_{GS}	V_{DS}	I_D
i) $V_{GS} < V_{Tn}$	-	0
ii) $V_{GS} > V_{Tn}$	a) $V_{DS} < V_{GS} - V_{Tn}$	$K_n [2 (V_{GS} - V_{Tn}) V_{DS} - V_{DS}^2]$
	b) $V_{GS} - V_{Tn} \leq V_{DS}$	$K_n (V_{GS} - V_{Tn})^2$
where $K_n = \frac{K'_n}{2} \left(\frac{W}{L} \right)$ and $K'_n = \mu_n C_{ox}$		

Q1. (25 pts) Consider the following common emitter amplifier. The capacitors are large enough to be considered as short circuit at the frequencies of the input source.



Circuit Parameters

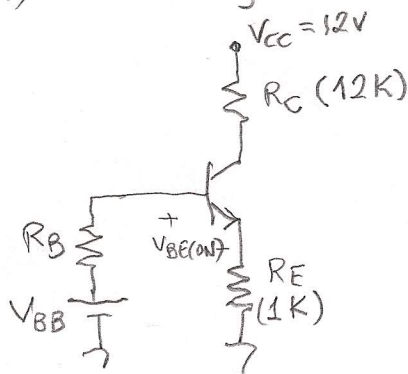
- $V_{CC} = 12\text{ V}$
- $R_1 = 100\text{ k}\Omega$
- $R_2 = 15\text{ k}\Omega$
- $R_C = 12\text{ k}\Omega$
- $R_E = 1\text{ k}\Omega$
- $R_L = 20\text{ k}\Omega$

Transistor Parameters

- $V_{BE(ON)} = 0.7\text{ V}$
- $\beta = 100$
- $V_A = \infty$

- (a) Draw the DC load line and locate Q point.
- (b) Determine the AC small signal voltage gain $A_v = v_o/v_s$.
- (c) Draw the AC load line. Determine the limits on the output voltage swing. Which limits the output signal amplitude, AC-SAT or AC-Cut-off?
- (d) Determine the total collector voltage v_c and the total output voltage v_o if $v_s = 2 \sin \omega t \text{ mV}$ (Be careful about the units!)

(a) DC Analysis :



$$R_B = R_1 // R_2 = 13\text{ K}$$

$$V_{BB} = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{15}{115} \cdot 12 = 1.565\text{ V}$$

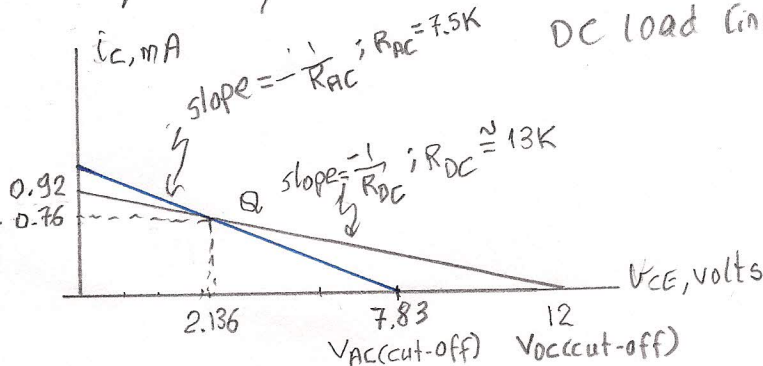
$$I_B = \frac{V_{BB} - V_{BE(ON)}}{R_B + (\beta + 1)R_E} = \frac{1.565 - 0.7}{13 + 101} = 7.59\text{ }\mu\text{A}$$

$$I_C = \beta I_B = 0.759\text{ mA}$$

$$V_{CEQ} = V_{CC} - \left[R_C + \frac{(\beta + 1)}{\beta} R_E \right] I_C = 2.136\text{ V}$$

$$\text{DC load line: } V_{CC} = V_{CE} + \left[R_C + \frac{\beta + 1}{\beta} R_E \right] I_C$$

$$R_{DC} \approx R_C + R_E = 13\text{ K}$$

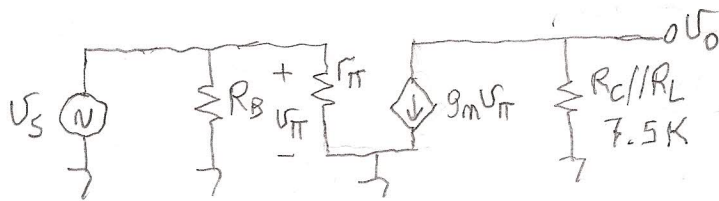


(c) AC load line = $R_{AC} = R_C // R_L = 12\text{ K} // 20\text{ K} = 7.5\text{ K}$

$$V_{AC(cut-off)} = V_{CEQ} + R_{AC} I_{CQ} = 2.136 + (7.5)(0.759) = 7.83\text{ V}$$

(b) AC small signal model: $r_{\pi} = \frac{25 \text{ mV}}{I_{BQ}} = \frac{25 \text{ mV}}{7.59 \mu\text{A}} = 3.3 \text{ K}$

$$g_m = \frac{\beta}{r_{\pi}} = 30 \text{ mS}$$

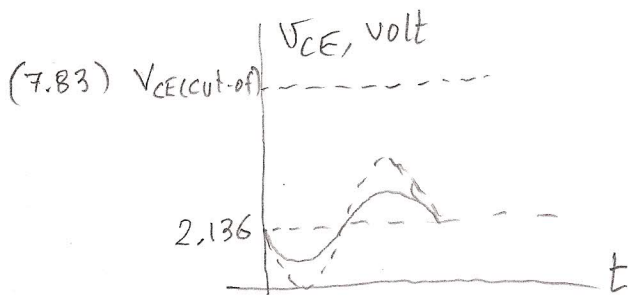


$$U_{\pi} = U_s$$

$$U_o = -g_m (R_C || R_L) U_{\pi} \\ = (-30 \text{ mS})(7.5) U_s$$

$$A_v = -225$$

(c) AC-SAT limits the output swing.



$$V_o(\text{peak-to-peak}) = 2 \times (2.136) \\ = 4.272 \text{ V}$$

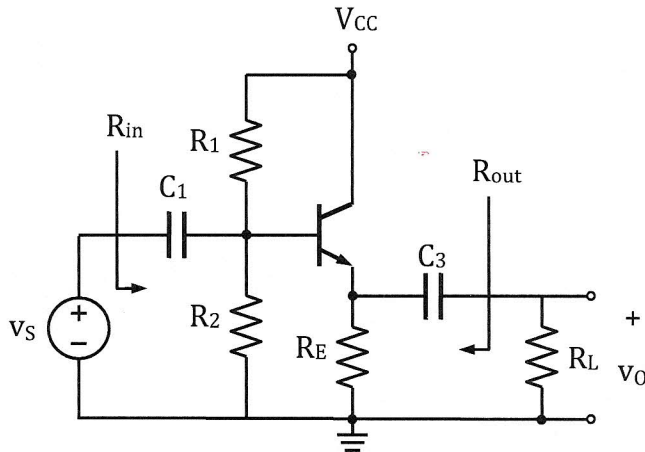
(d) $A_v = -225$

$$U_o = V_o(\text{DC}) + U_o = 0 + A_v U_s$$

$$= -450 \sin \omega t \text{ mV} = -0.45 \sin \omega t \text{ volts}$$

Q2. (25 pts) Consider the common-collector (emitter follower) amplifier given below. All capacitors are assumed short at the frequencies of interest.

- (a) Determine R_1 to set $I_{CQ} = 1 \text{ mA}$.
 (b) Now assume $R_1 = 45 \text{ k}\Omega$, Draw the AC small signal equivalent circuit.
 (c) Determine the AC small signal gain $A_v = v_o/v_s$, input resistance R_{in} and output resistance R_{out} .

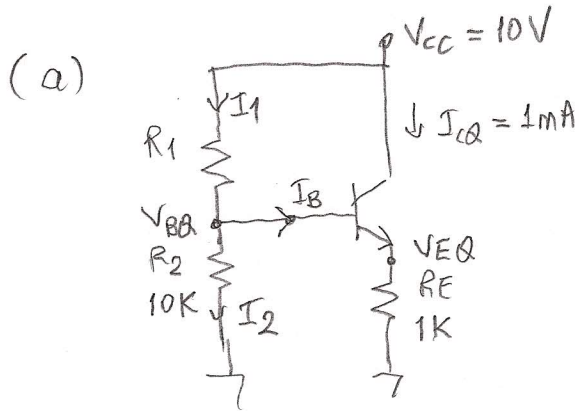


Circuit Parameters

- $V_{CC} = 10 \text{ V}$
 $R_2 = 10 \text{ k}\Omega$
 $R_E = 1 \text{ k}\Omega$
 $R_L = 10 \text{ k}\Omega$

Transistor Parameters

- $\beta = 100$
 $V_{BE(ON)} = 0.7 \text{ V}$
 $V_A = \infty$



$$V_{EQ} = I_E R_E \approx (1 \text{ mA})(1 \text{ k}) = 1 \text{ V}$$

$$V_{BB} = V_{BE(ON)} + V_{EQ} = 1.7 \text{ V}$$

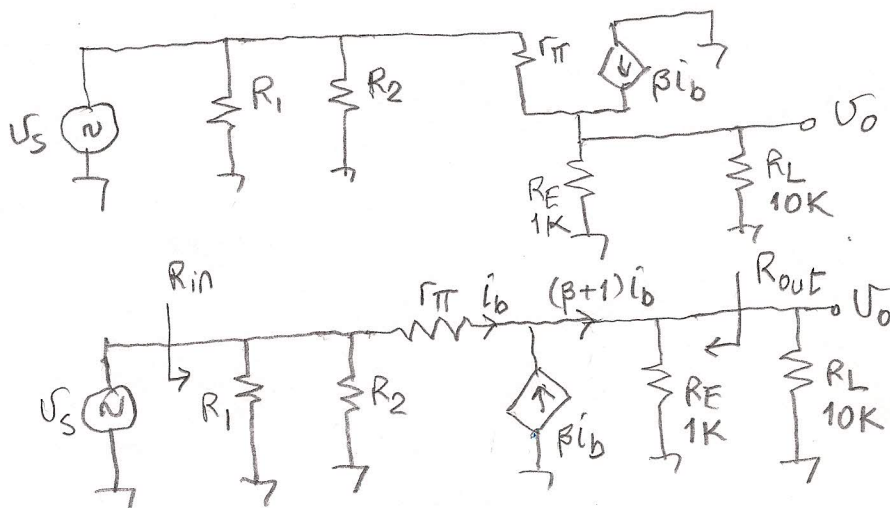
$$I_2 = \frac{V_B}{R_2} = \frac{1.7 \text{ V}}{10 \text{ k}} = 0.17 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1 \text{ mA}}{100} = 0.01 \text{ mA}$$

$$\Rightarrow I_1 = I_2 + I_B = 0.18 \text{ mA}$$

$$R_1 = \frac{V_{CC} - V_B}{I_1} = \frac{10 - 1.7 \text{ V}}{0.18 \text{ mA}} = 46.1 \text{ k}$$

(b) $R_1 = 45 \text{ k}$; $R_1 \parallel R_2 = 45 \text{ k} \parallel 10 \text{ k} = 8.18 \text{ k}$



$$r_{\pi} = \frac{25 \text{ mV}}{I_{BQ}}$$

$$= \frac{25 \text{ mV}}{0.01 \text{ mA}}$$

$$r_{\pi} = 2.5 \text{ k}$$

$$R_C \parallel R_L = 0.91 \text{ k}$$

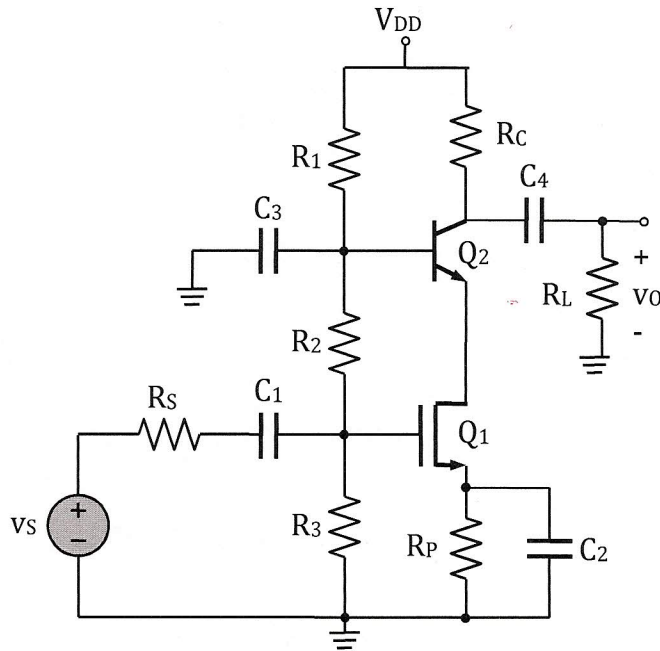
$$\begin{aligned}
 & \left. \begin{aligned}
 V_S &= [\tau_{\pi} + (\beta+1)(R_E \parallel R_L)] \hat{I}_b \\
 V_O &= (\beta+1)(R_C \parallel R_L) \hat{I}_b
 \end{aligned} \right\} \frac{V_O}{V_S} = \frac{(\beta+1)(R_C \parallel R_L)}{\tau_{\pi} + (\beta+1)(R_C \parallel R_L)} \\
 & = \frac{(101)(0.91)}{2.5 + (101)(0.91)} = \frac{92}{2.5 + 92}
 \end{aligned}$$

$$A_V = \frac{V_O}{V_S} = 0.973$$

$$\begin{aligned}
 R_{in} &= \underbrace{R_1 \parallel R_2}_{8.12} \parallel [\tau_{\pi} + (\beta+1)(R_E \parallel R_L)] \\
 &= 8.12 \parallel 94.5 \cong 7.5 \text{K}
 \end{aligned}$$

$$R_{out} = \frac{\tau_{\pi}}{\beta+1} = 24.75 \Omega$$

Q3. (25 pts) Consider the cascode amplifier given below.



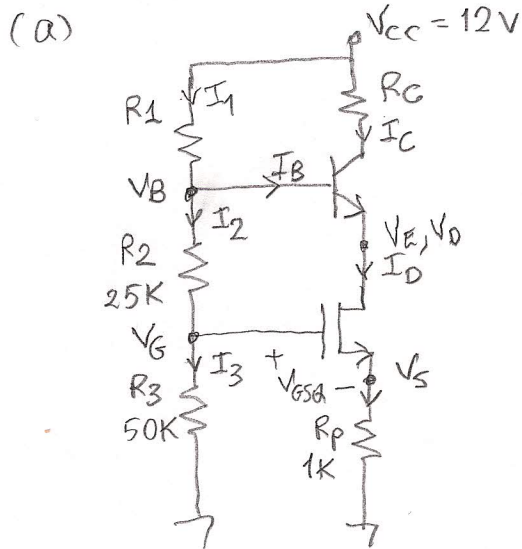
Circuit Parameters

- $V_{DD} = 12V$
- $R_2 = 25\text{ k}\Omega$
- $R_3 = 50\text{ k}\Omega$
- $R_L = 10\text{ k}\Omega$
- $R_S = 1\text{ k}\Omega$
- $R_P = 1\text{ k}\Omega$

Transistor Parameters

- Q1: $K_N = 2\text{ mA/V}^2$, $V_{TN} = 1\text{ V}$, $V_A = \infty$
- Q2: $\beta = 100$, $V_{BE(ON)} = 0.7\text{ V}$, $V_A = \infty$

- (a) Determine R_1 to set $V_{GSQ1} = 2\text{ V}$.
- (b) Check the SAT conditions for the transistors.
- (c) Determine the small-signal parameters for each transistor.
- (d) Determine the overall AC small-signal voltage gain $A_v = v_o / v_s$.



$$V_{GSQ} = 2\text{ V} \Rightarrow I_{DQ} = K_N (V_{GS} - V_{TN})^2 = (2\text{ mA/V}^2) (2 - 1)^2$$

$$\Rightarrow I_{DQ} = 2\text{ mA}$$

$$V_S = I_D R_P = (2\text{ mA})(1\text{ k}) = 2\text{ V}$$

$$V_G = V_{GSQ} + V_S = 2 + 2 = 4\text{ V}$$

$$\Rightarrow I_3 = \frac{V_G}{R_3} = \frac{4\text{ V}}{50\text{ k}} = 0.08\text{ mA}$$

$$\Rightarrow I_2 = I_3 = 0.08\text{ mA}$$

$$V_B = R_2 I_2 + V_G = (25\text{ k})(0.08\text{ mA}) + 4$$

$$V_B = 6\text{ V}$$

$$I_B = \frac{I_D}{\beta + 1} \approx \frac{2\text{ mA}}{100} = 0.02\text{ mA} \Rightarrow I_1 = I_2 + I_B = 0.1\text{ mA}$$

$$R_1 = \frac{V_{CC} - V_B}{I_1} = \frac{12 - 6\text{ V}}{0.1\text{ mA}} = \frac{6\text{ V}}{0.1\text{ mA}} = 60\text{ k}$$

(b) NMOS

$$V_D = V_E = V_B - V_{BE(ON)} = 6 - 0.7 = 5.3V$$

$$V_S = 2V$$

$$V_{DS} = V_D - V_S = 3.3V$$

$$V_{DS} \geq V_{GS} - V_{TN}$$

$$3.3 \geq 2 - 1 = 1V \quad \checkmark \text{ SAT}$$

BJT

$$V_E = 5.3V; V_C = V_{CC} - R_C I_C = 12 - 2R_C \quad (I_C \cong I_B = 2mA)$$

$$V_{CE} \geq V_{CE(SAT)} \Rightarrow (12 - 2R_C) - 5.3 \geq 0.2V$$

$$\frac{12 - 5.3 - 0.2}{6.5} \geq 2R_C \Rightarrow R_C \leq 3.25K \text{ satisfies SAT,}$$

Assume $R_C = 1K$ $\Rightarrow V_{CE} = 10V \geq V_{CE(SAT)}$

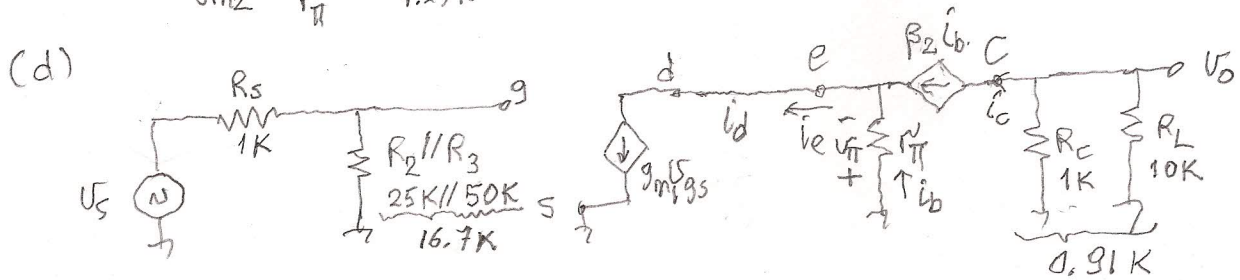
(c) NMOS

$$g_{m1} = 2K_n (V_{GS} - V_{TN}) = (2)(2)(2 - 1) = 4mS; r_o = \infty$$

BJT

$$r_{\pi 2} = \frac{25mV}{I_B} = \frac{25mV}{0.02mA} = 1.25K$$

$$g_{m2} = \frac{\beta}{r_{\pi}} = \frac{10}{1.25K} = 8mS; r_o = \infty$$



$$V_{GS} = \frac{16.7}{1 + 16.7} U_s = 0.94 U_s$$

$$i_d = g_m V_{GS} = (4)(0.94) U_s = 3.77 U_s = i_e \Rightarrow i_b = \frac{3.77}{\beta + 1} U_s$$

$$i_c = \beta i_b = \frac{\beta}{\beta + 1} (3.77) U_s = 3.72 U_s$$

$$V_o = -i_c (R_C // R_L) = -(3.72)(0.91) U_s = -3.38 U_s$$

$$\Rightarrow A_v = -3.38$$

Q4. (25 pts) Consider the circuit given below.

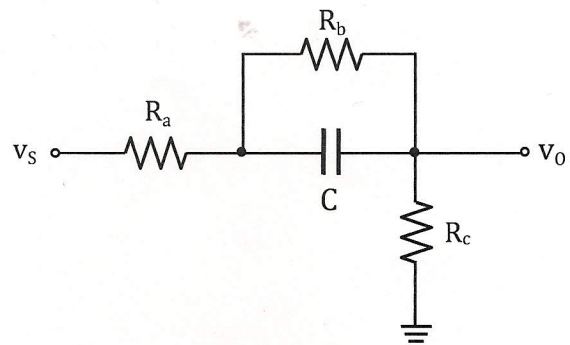
i. Show that the transfer function is given as

$$H(j\omega) = A_0 \frac{1 + j\frac{\omega}{\omega_1}}{1 + j\frac{\omega}{\omega_2}}$$

where

$$\omega_1 = \frac{1}{R_b C} \quad \text{and}$$

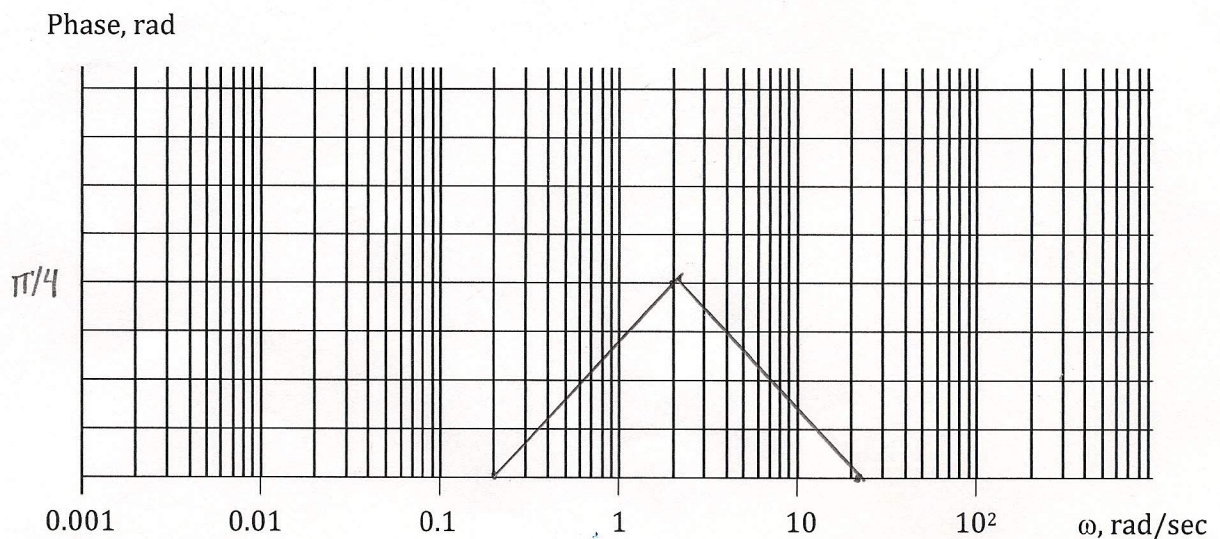
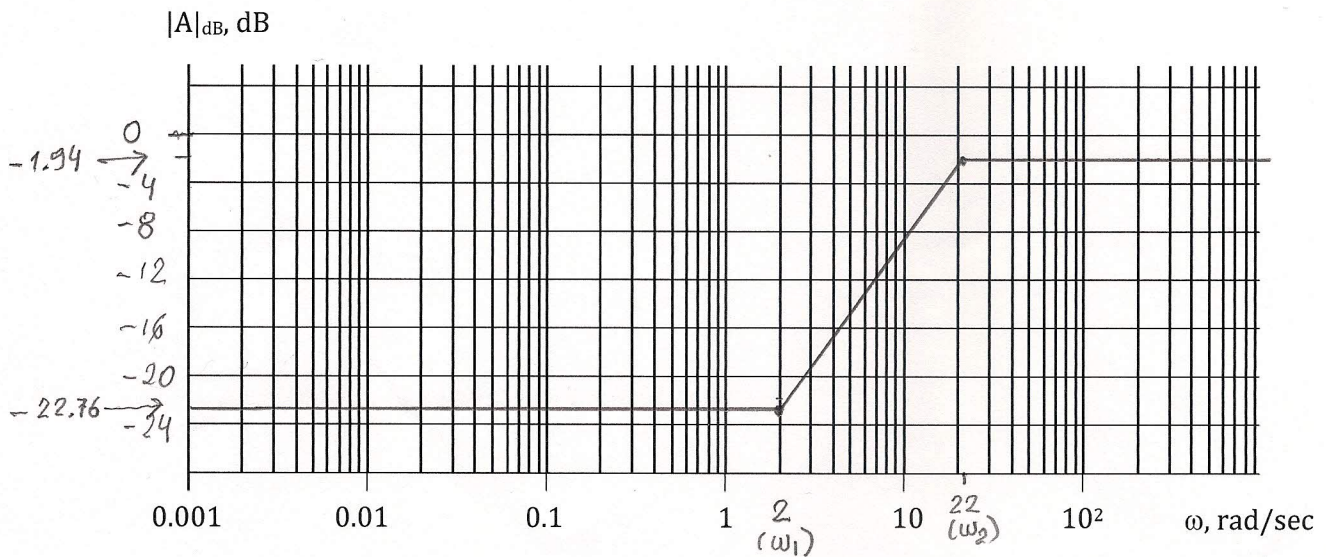
$$\omega_2 = \frac{1}{R_{eq2} C}; \quad R_{eq2} = (R_a + R_c) // R_b$$

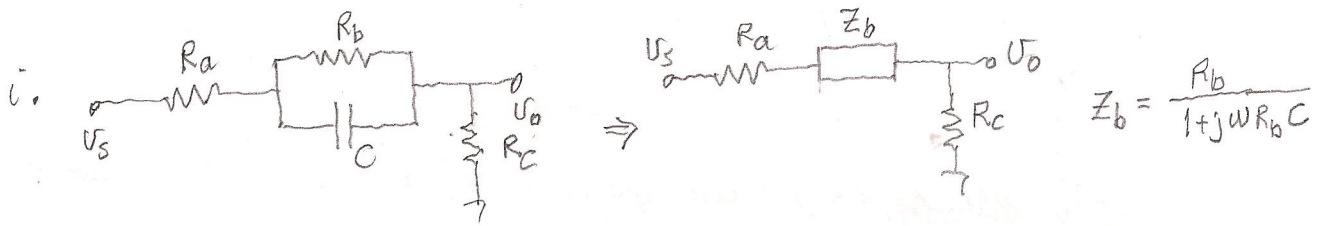


Determine A_0

ii. If $R_a = 1 \text{ K}$, $R_b = 50 \text{ K}$, $R_c = 4 \text{ K}$, and $C = 10 \mu\text{F}$, determine and plot

- The Bode magnitude plot
- The Bode phase plot





$$H(j\omega) = \frac{V_o(j\omega)}{V_s(j\omega)} = \frac{R_c}{R_a + Z_b + R_c} = \frac{R_c}{R_a + \frac{R_b}{1 + j\omega R_b C} + R_c} = \frac{R_c}{R_a + R_c + \frac{R_b}{1 + j\omega R_b C}}$$

$$= \frac{R_c(1 + j\omega R_b C)}{R_a + R_b + R_c + j\omega R_b(R_a + R_c)C} = \frac{R_c(1 + j\omega R_b C)}{R_a + R_b + R_c \left[1 + j\omega \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} C \right]}$$

$$= \underbrace{\frac{R_c}{R_a + R_b + R_c}}_{A_0} \frac{1 + j\omega R_b C}{1 + j\omega R_{eq2} C} \quad ; \quad R_{eq2} = \frac{R_c(R_a + R_c)}{R_a + R_b + R_c} = (R_a + R_c) \parallel R_b$$

$$= A_0 \frac{1 + j\frac{\omega}{\omega_1}}{1 + j\frac{\omega}{\omega_2}} \quad ; \quad A_0 = \frac{R_c}{R_a + R_b + R_c} \quad ; \quad \omega_1 = \frac{1}{R_b C} \quad ; \quad \omega_2 = \frac{1}{R_{eq2} C}$$

ii. $R_a = 1K, R_b = 50K, R_c = 4K, C = 10\mu F$

$$A_0 = \frac{R_c}{R_a + R_b + R_c} = \frac{4}{1 + 50 + 4} = \frac{4}{55} = 0.073 \Rightarrow A_{0, dB} = -22.76 \text{ dB}$$

$$\omega_1 = \frac{1}{R_b C} = \frac{1}{50 \times 10^3 \times 10 \times 10^{-6}} = \frac{10^3}{500} = 2 \text{ rad/sec}$$

$$R_{eq2} = (R_a + R_c) \parallel R_b = 5K \parallel 50K = 4.55K$$

$$\omega_2 = \frac{1}{R_{eq2} C} = \frac{1}{4.55 \times 10^3 \times 10 \times 10^{-6}} = \frac{10^3}{45.5} = 22 \text{ rad/sec}$$

$$\omega < \omega_1: |H(j\omega)|_{dB} = 20 \log A_0 = -22.76 \text{ dB}$$

$$\omega_1 < \omega < \omega_2: |H(j\omega)|_{dB} = 20 \log A_0 \frac{\omega}{\omega_1} = 20 \log \omega + 20 \log \left(\frac{A_0}{\omega_1} \right) \quad (20 \text{ dB/dec})$$

$$\omega_2 < \omega: |H(j\omega)|_{dB} = 20 \log A_0 \frac{\omega/\omega_1}{\omega/\omega_2} = 20 \log A_0 \cdot \frac{\omega_2}{\omega_1} = 20 \log \left(\frac{4}{55} \cdot \frac{22}{2} \right)$$

$$= 20 \log \left(\frac{4}{5} \right) = -1.94 \text{ dB}$$

Magnitude plot is above.

Phase $\phi = \tan^{-1} \left(\frac{\omega}{\omega_1} \right) - \tan^{-1} \left(\frac{\omega}{\omega_2} \right)$

ω (rad/sec)	$\tan^{-1}(\omega/2)$	$\tan^{-1}(\omega/22)$	ϕ (rad)
$\omega = 0.2$	0	0	0
$\omega = 2$	$\pi/4$	0	$\pi/4$
$\omega = 22$	$\pi/2$	$\pi/4$	0
$\omega = 220$	$\pi/2$	$\pi/2$	0

Phase plot is above.