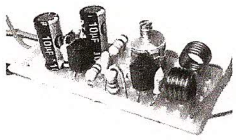
İZMİR UNIVERSITY OF ECONOMICS
Department of Electrical and Electronics Engineering
EEE 331 Analog Electronics
Fall 2015/2016



SOLUTIONS

MIDTERM EXAM I

Nov 21, 2015
110 min

INSTRUCTIONS

- Read all of the instructions and all of the questions before beginning the exam.
- There are 5 questions on this exam, totaling 100 points. The credit for each problem is given to help you allocate your time accordingly.
- Do not spend all your time on one problem and on one part and attempt to solve all of them.
- Calculators are allowed, but borrowing is not allowed.
- Your mobile phones must be turned off during the exam.
- Turn in the entire exam, including this cover sheet.
- You must show your work for all problems to receive full credit; simply providing answers will result in only partial credit, even if the answers are correct.
- Please indicate the number of page where your work is to be continued.
- Put your name on any additional material that you submit.
- Be sure to provide units where necessary.
- Please sign the honor pledge that is provided below.

Last Name :......................

Name :......................

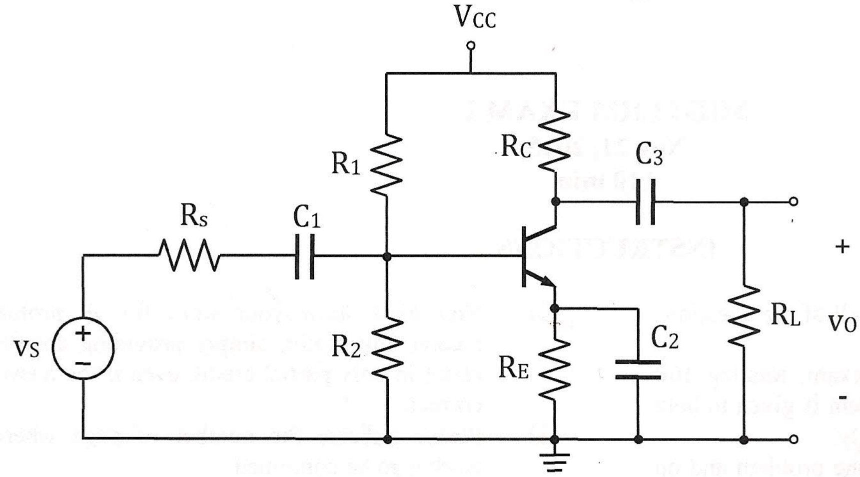
Section :......................

Student No :......................

Question	Points	Grade
1	25	
2	25	
3	30	
4	20	
TOTAL	100	

The basic equations of the output characteristics of an NMOS transistor		
V_{GS}	**V_{DS}**	**I_D**
i) $V_{GS} < V_{Tn}$	-	0
ii) $V_{GS} > V_{Tn}$	a) $V_{DS} < V_{GS} - V_{Tn}$	$K_n\left[2(V_{GS}-V_{Tn})V_{DS}-V_{DS}^2\right]$
	b) $V_{GS} - V_{Tn} \leq V_{DS}$	$K_n(V_{GS}-V_{Tn})^2$
where $K_n = \frac{K_n'}{2}\left(\frac{W}{L}\right)$ and $K_n' = \mu_n C_{ox}$		

Q1. Consider the common-emitter amplifier given below.

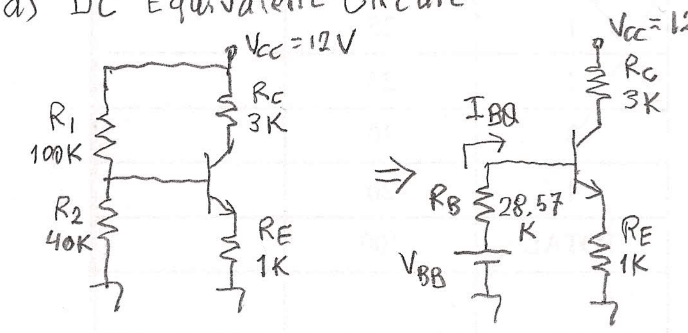


Circuit Parameters
V_{CC} = 12 V
R_S = 1 kΩ
R_1 = 100 kΩ
R_2 = 40 kΩ
R_C = 3 kΩ
R_E = 1 kΩ
R_L = 10 kΩ

Transistor Parameters
$V_{BE(ON)}$ = 0.7 V
β = 100
V_A = 50 V
$V_{CE(SAT)}$ = 0
V_T = 26 mV

(a) Determine the Q point values V_{CEQ} and I_{CQ}.
(b) Determine the AC small signal voltage gain $A_V = v_O/v_S$.
(c) Draw the DC and AC load lines.
(d) Determine maximum peak value of the undistorted swing at the output.

(a) DC Equivalent Circuit



$$R_B = 100K // 40K = \frac{(100)(40)}{140} = 28.57K$$

$$V_{BB} = \frac{40}{140} \cdot 12V = 3.43\ V$$

$$I_{BQ} = \frac{V_{BB} - V_{BE}}{R_B + (\beta+1)R_E} = \frac{3.43 - 0.7}{28.57 + 101} = \frac{2.73\ V}{129.5K}$$

$$I_{BQ} = 21.08\ \mu A$$

$$I_{CQ} = \beta I_{BQ} = 2.1\ mA \Rightarrow V_{CEQ} = V_{CC} - R_C I_{CQ} - R_E I_{EQ} = 12 - (3)(2.1) - (1)(2.12)$$

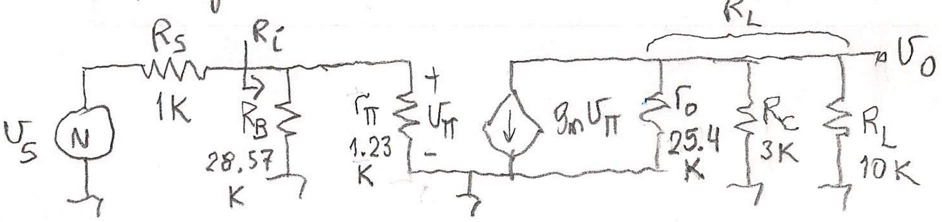
$$V_{CEQ} = 3.58\ V \quad (V_{CEQ} > V_{CE(SAT)}\text{; Forward active!})$$

(b) Small signal parameters :

$$r_\pi = \frac{V_T}{I_{BQ}} = \frac{26\ mV}{21.08\ \mu A} = 1.23K \Rightarrow g_m = \frac{\beta}{r_\pi} = 81.3\ m℧$$

$$r_o = \frac{V_A + V_{CEQ}}{I_{CQ}} = \frac{53.58V}{2.1\ mA} = 25.4\ K$$

Small signal AC model :



$$R_i = R_B // r_\pi = \frac{(28.57)(1.23)}{28.57 + 1.23} = 1.18K$$

$$R_L' = r_o // R_C // R_L = 2.12K$$

$$\left.\begin{aligned} v_\pi &= \frac{R_i}{R_S + R_i} v_s = \frac{1.18}{2.18} v_s = 0.54 v_s \\ v_o &= -g_m v_\pi R_L' = (-81.m25)(2.12K) = -171.7 v_\pi \end{aligned}\right\} A_v = \frac{v_o}{v_s} = -92.9$$

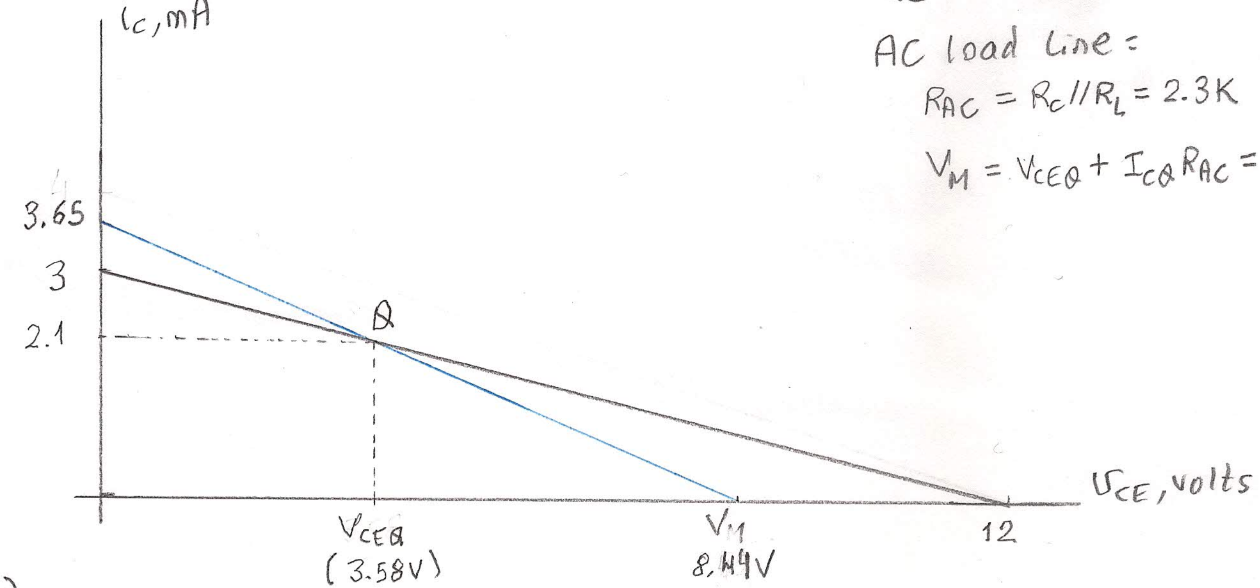
(c) DC Load Line equation : $V_{CC} = I_C R_C + V_{CE} + R_E I_E \cong V_{CE} + (R_C + R_E) I_C$

$R_{DC} = R_C + R_E = 4K$

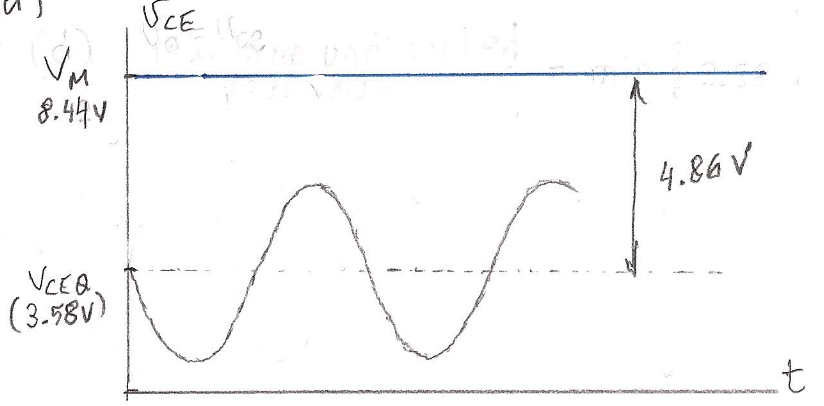
AC load Line :

$R_{AC} = R_C // R_L = 2.3K$

$V_M = V_{CEQ} + I_{CQ} R_{AC} = 8.44V$



(d)



Output signal is around V_{CEQ}.

Positive peaks will be limited by 8.42V (peak value = 8.44-3.58=4.86V

Negative peaks will be limited by 3.58V

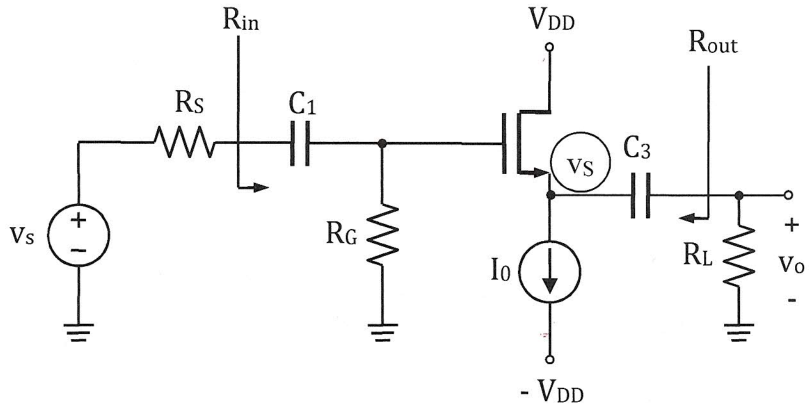
Then the maximum undistorted swing on both sides will be limited to 3.58 V.

The output signal and the corresponding input signal

$$v_{CE} = V_{CEQ} + v_{CE} = V_{CEQ} + v_o = 3.58 - 3.58 \sin\omega_0 t \text{ volts}$$

$$v_o = -3.48 \sin\omega_0 t \text{ volts} = A_v v_s \Rightarrow v_s = -\frac{3.58}{92.9} \sin\omega_0 t \text{ volts} = 38.5 \sin\omega_0 t \text{ mV}$$

Q2. (25 pts) Consider the common drain (source follower) circuit given below.

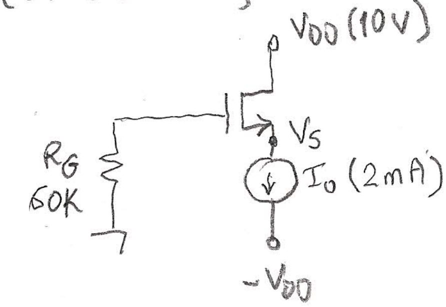


Circuit Parameters
V_{DD} = 10 V
I_o=2 mA
R_S = 5 kΩ
R_G = 60 kΩ
R_L = 5 kΩ

Transistor Parameters
V_{Tn} = 1 V
K_n = 8 mA/V^2
V_A = 100 V

a) Determine the quiescent point values V_{DSQ} and I_{DQ}. (Check SAT condition!)
b) Determine the small signal parameters g_m and r_o.
c) Draw the AC small signal model of the circuit.
d) Determine the small signal voltage gain $A_V = v_o/v_S$.
e) Determine the input resistance R_{in} and the output resistance R_{out}.
f) Determine the total source voltage v_S.

(a) DC Analysis



Assume SAT :

$I_{DS} = I_0 = K_n (V_{GSQ} - V_{TN})^2$ $\quad I_{DQ} = I_0 = 2\,mA$

$2 = 8 (V_{GSQ} - 1)^2$

$\frac{1}{2} = V_{GSQ} - 1 \Rightarrow V_{GSQ} = 1.5\,V$

$V_{SQ} = -V_{GSQ} = -1.5\,V$
$V_{DQ} = V_{DD} = 10\,V$

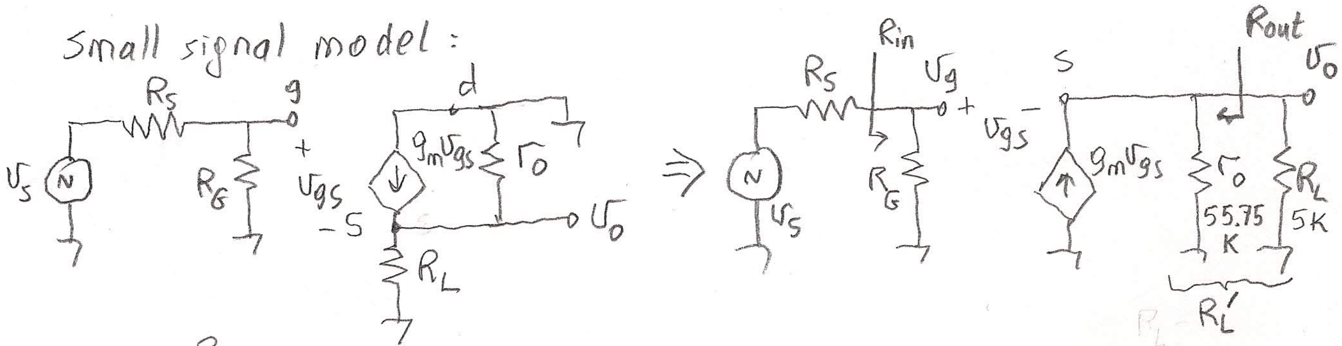
$V_{DSQ} = V_{DQ} - V_{SQ}$
$= 11.5\,V > V_{GSQ} - V_{TN} = 0.5\,V$

SAT assumption is satisfied.

(b) $g_m = 2K_N (V_{GSQ} - V_{TN}) = (2)(8\,mA/V^2)(1.5-1)V = 8\,mA/V$

$$r_0 = \frac{V_A + V_{DSQ}}{I_{DQ}} = \frac{100 + 11.5}{2} = 55.75\,K$$

(c) Small signal model :



$$v_g = \frac{R_G}{R_S + R_G} v_S = \frac{60}{5+60} v_S = 0.92\,v_S$$

$R_L' = r_0 // R_L = (55.75K)//(5K) = 4.59K$

(d) $v_o = g_m v_{gs} R_L' = g_m R_L'(v_g - v_s)$

$v_o = v_s \Rightarrow v_o = g_m R_L' v_g - g_m R_L' v_o$

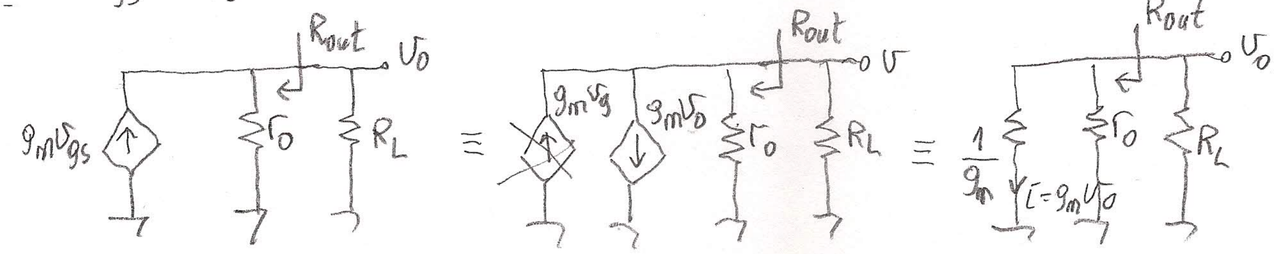
$(1 + g_m R_L')v_o = g_m R_L' v_g$

$$A_v = \frac{v_o}{v_g} = \frac{g_m R_L'}{1+g_m R_L'} = \frac{(8)(4.59)}{1+(8)(4.59)} = 0.97$$

(e) $R_{in} = R_G = 60K$

To determine R_o, let $v_s = 0$. Then $v_g = 0$

$v_{gs} = v_g - v_s = v_g - v_o$ since $v_o = v_s$



$$R_{out} = \frac{1}{g_m} \| r_o = \left(\frac{1}{8\,mA/V}\right) \| 55.75K = 0.125K \| 55.75K \cong 0.125K$$

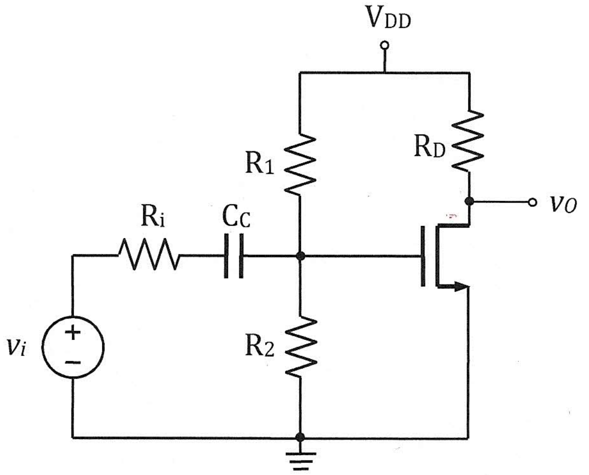
since $55.75\,K \gg 0.125K$

(f) $V_{SQ} = -V_{GSQ} = -1.5\,V$

$v_S = V_{SQ} + v_s = V_{SQ} + v_o = -1.5\,V + A_v v_s = -1.5\,V + 0.97\,v_s$

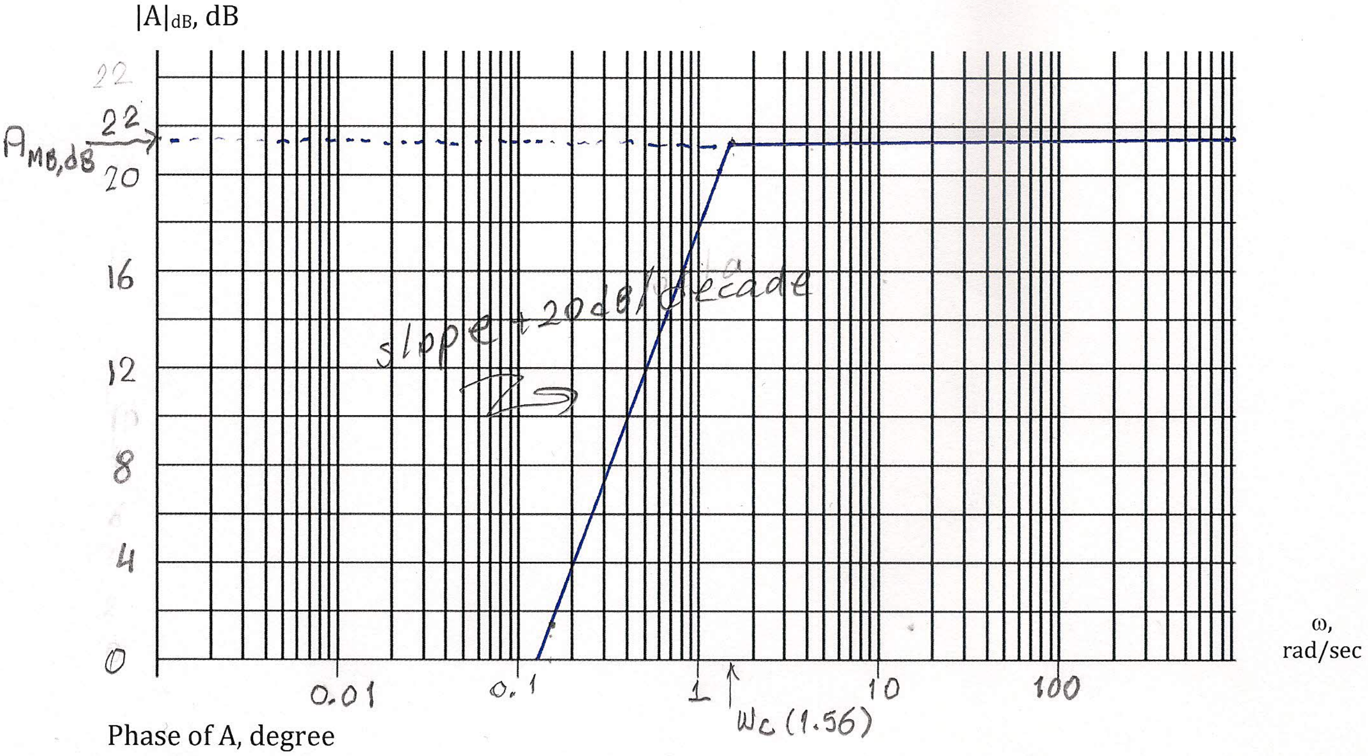
Q3. (30 pts) For the MOSFET circuit given below,

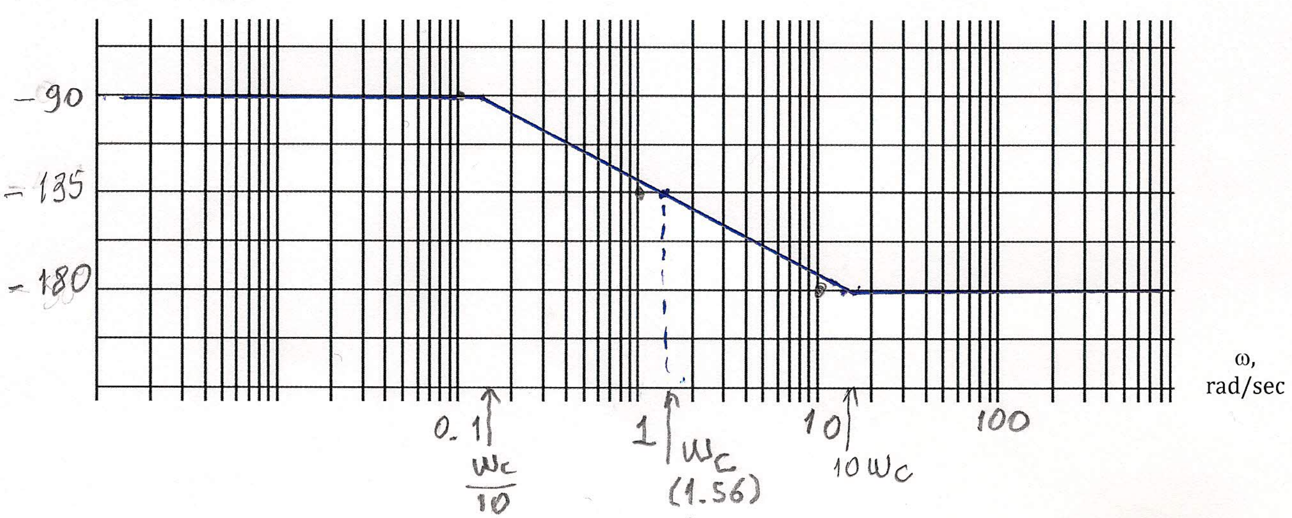
i. Determine the small signal parameters and draw the low frequency small signal model.
ii. Determine the lower corner frequency.
iii. Find the midband voltage gain.
iv. Plot the low frequency magnitude response.



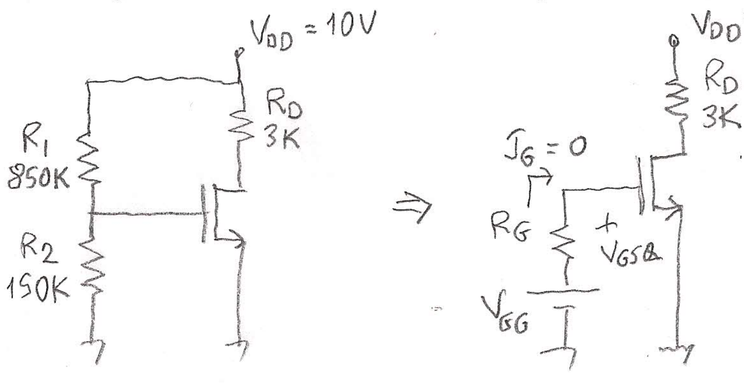
The transistor Parameters
$V_{TN} = 0.5$ V
$K_N = 2$ mA/V^2
$\lambda = 0$

Circuit Parameters
$V_{DD} = 10$ V
$R_i = 1$ kΩ
$R_1 = 850$ kΩ
$R_2 = 150$ kΩ
$R_D = 3$ kΩ
$C_C = 5$ μF





(i) DC Analysis



$V_{GG} = \frac{R_2}{R_1 + R_2} V_{DD} = \frac{150}{1000} \cdot 10V$

$= 1.5V$

$R_G = 850K // 150K$

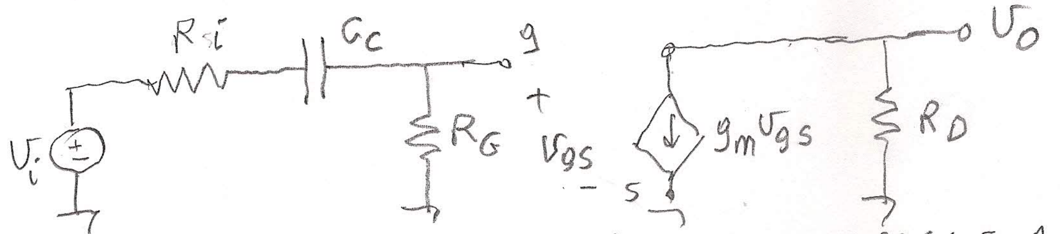
$= 127.5K$

$V_{GSQ} = V_{GG} = 1.5V$ (since $I_G = 0$)

$I_{DQ} = K_N (V_{GSQ} - V_{TN})^2 = (2\,mA/V^2)(1.5 - 0.5)^2 = 2\,mA$

$V_{DSQ} = V_{DD} - I_{DQ} R_D = 10 - (3K)(2MA) = 4V > V_{GSQ} - V_{TN} = 1V$ (SAT ✓)

Low frequency small signal model:



$g_m = 2K_N(V_{GSQ} - V_{TN}) = (2)(2\,mA/V^2)(1.5 - 0.5V) = 4\,mA/V$

$r_o = \frac{1}{\lambda I_{DQ}} = \infty$

(ii) $\omega_c = \frac{1}{C_C R_{eq}}$; $R_{eq} = R_i + R_G = 1 + 127.5 = 128.5\,K$

$\omega_c = \frac{1}{(5 \times 10^{-6})(128.5 \times 10^{3})} = 1.56$ rad/sec

(iii) $v_{gs} = \frac{R_G}{R_i + R_G} v_i$; $v_o = -g_m v_{gs} R_D = -g_m R_D \frac{R_G}{R_i + R_G} v_i$

$A_{MB} = -(4)(3) \frac{128.5}{129.5} = -11.9$

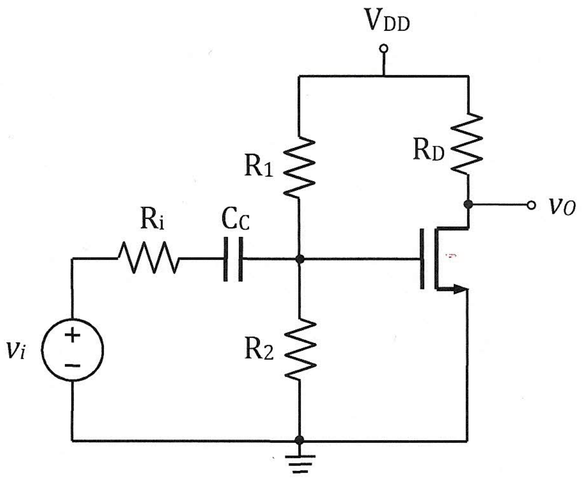
$A_{MB,dB} = \overbrace{(-11.9)}^{A_{MB}} \frac{j\omega/\omega_L}{1 + j(\frac{\omega}{\omega_L})}$ $\left(A_{MB,dB} = 20\log(11.9) = 21.5\text{ dB}\right)$

$\theta = -\underset{\substack{\uparrow \\ (-)\text{sign}}}{180^\circ} + \underset{\substack{\downarrow \\ (j\text{ term})}}{90^\circ} - \tan^{-1}\underset{\substack{\uparrow \\ \text{Denominator}}}{\left(\frac{\omega}{\omega_L}\right)} = -90^\circ - \tan^{-1}\left(\frac{\omega}{\omega_L}\right)$

Plots are on the previous page.

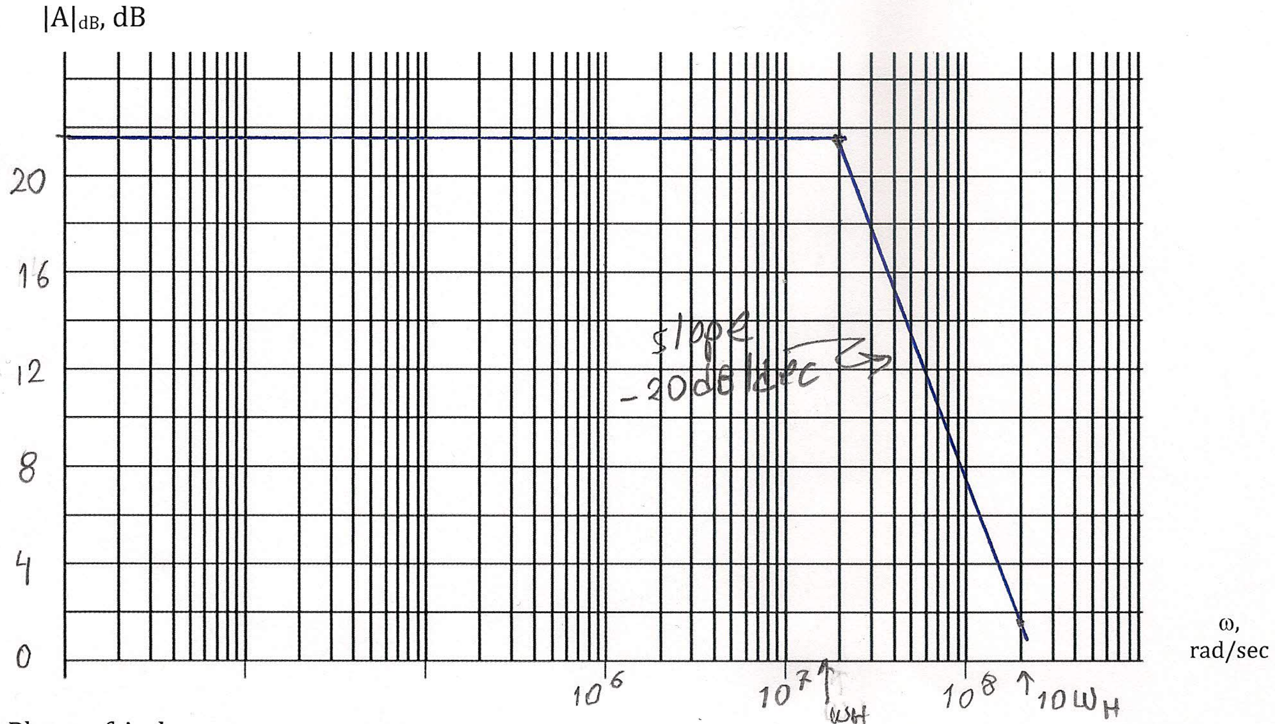
Q4. (20 pts) Consider the MOSFET circuit given in Question 3 above.

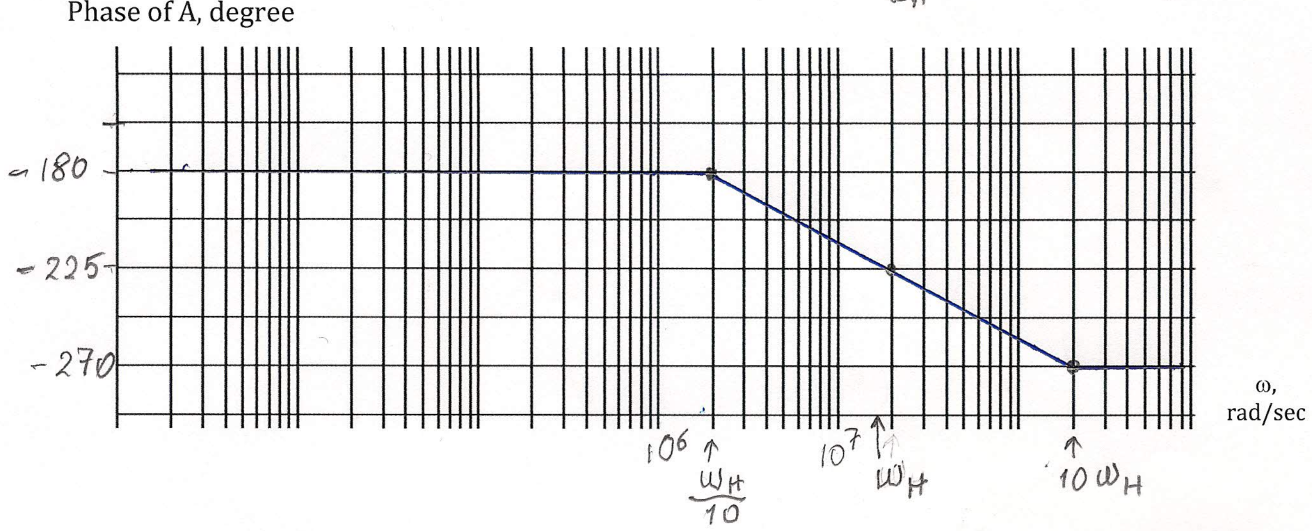
i. Draw the simplified high-frequency equivalent circuit.
ii. Determine the higher corner frequency.
iii. Plot the overall frequency response.



The transistor Parameters
(As given in Q3.)

Circuit Parameters
(As given in Q3)
C_{gs} = 10 pF
C_{gd} = 4 pF





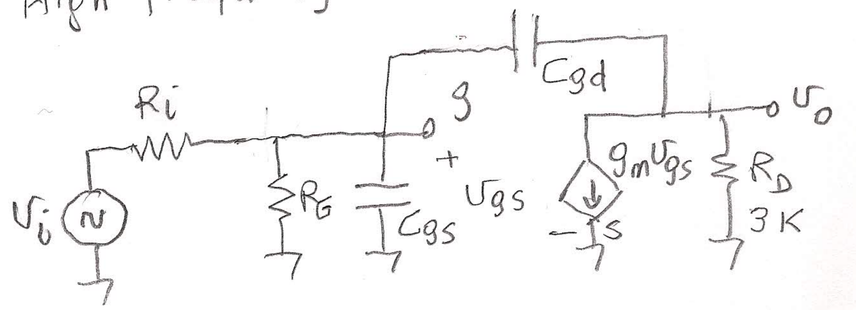
i) AC Analysis : (From Q3)

$V_{GSQ} = 1.5V$, $I_{DQ} = 2mA$ $V_{DSQ} = 4V$ (SAT ✓)

AC Analysis

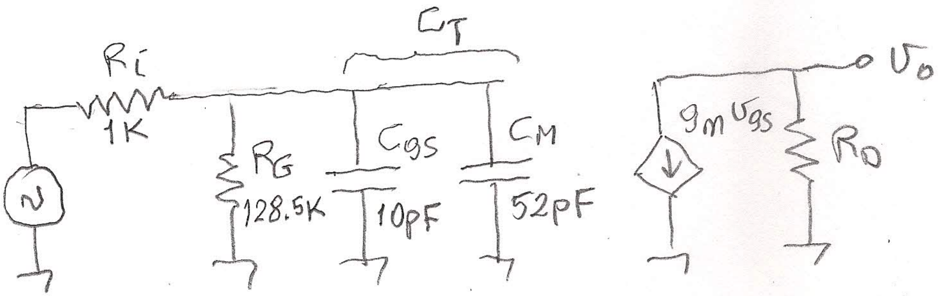
$g_m = 4\ mA/V$

High frequency model : (C_c is short)



$C_{gs} = 10pF$

$C_{gd} = 4pF$



$$C_M = C_{gd}(1+g_m R_D) = (4pF)(1+4\cdot3) = 52pF$$

$$C_T = C_{gs} + C_M = 62pF$$

ii) $\omega_H = \frac{1}{C_T R_{eq}}$ $R_{eq} = R_G // R_i \cong 1K$ (Corner frequency)

$$\omega_H = \frac{1}{(52\times10^{-12})(10^3)} = \frac{10^9}{62} \approx 16.26\ \text{Mrad/sec}$$

iii. $A_v = \frac{A_{MB}}{1+j\left(\frac{\omega}{\omega_H}\right)} = \frac{-11.9}{1+j\left(\frac{\omega}{\omega_H}\right)}$

$A_{MB} = 11.9 \Rightarrow A_{MB,dB} = 22.5\ dB$ Midban

$$\theta = \underbrace{-180^\circ}_{(-)\text{ sign}} \underbrace{-\tan^{-1}\left(\frac{\omega}{\omega_H}\right)}_{\text{Denominator}}$$ (Phase)

Plots are on the left.