

SOLUTIONS

MIDTERM EXAM I

Nov 5, 2016

110 min

INSTRUCTIONS

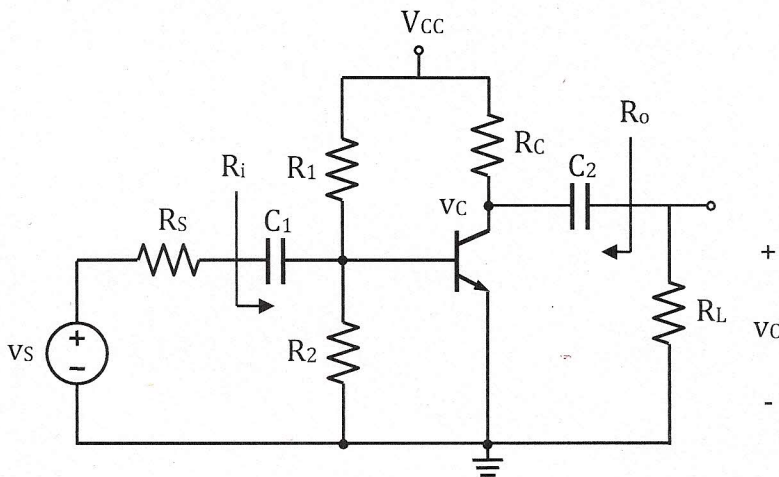
- Read all of the instructions and all of the questions before beginning the exam.
- There are 4 questions on this exam, totaling 100 points. The credit are equal for each problem.
- Do not spend all your time on one problem and on one part and attempt to solve all of them.
- Calculators are allowed, but borrowing is not allowed.
- Your mobile phones must be turned off during the exam.
- Turn in the entire exam, including this cover sheet.
- You must show your work for all problems to receive full credit; simply providing answers will result in no credit, even if the answers are correct.
- Please indicate the number of page where your work is to be continued.
- Put your name on any additional material that you submit.
- Be sure to provide units where necessary.
- Please sign the honor pledge that is provided below.

Honor Pledge: I have neither given nor received any aid at this exam.

Signed:.....

Last Name :.....	Question	Points	Grade
Name :.....	1	25	
Group :.....	2	25	
Student No :.....	3	25	
	4	25	
	TOTAL	100	

Q1. (25 pts) Consider the common emitter amplifier given below. The input is v_s and the output is v_o measured over R_L .



Circuit Parameters

$$V_{CC} = 9 \text{ V}$$

$$R_2 = 20 \text{ K}$$

$$R_L = 18 \text{ K}$$

$$R_S = 2 \text{ K}$$

BJT Parameters

$$\beta = 100$$

$$V_{BE(on)} = 0.6 \text{ V}$$

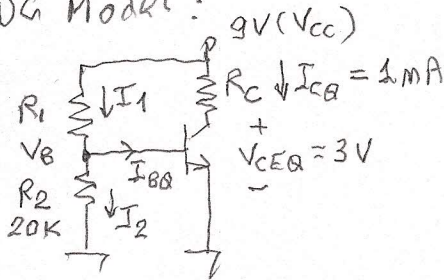
$$V_T = 26 \text{ mV}$$

$$V_{CE(sat)} = 0$$

$$V_A \rightarrow \infty$$

- Determine the R_1 and R_C to set $I_{CQ} = 1 \text{ mA}$, $V_{CEQ} = 3 \text{ V}$.
- Determine the voltage gain.
- Determine the input resistance R_i and output resistance R_o .
- Draw the DC and AC load lines.
- Determine the limitations on the output signal swing.
- If $v_s(t) = 20 \sin \omega_0 t \text{ mV}$, determine $v_c(t)$ and $v_o(t)$.

i. DC Model:



$$R_C = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{9 - 3}{1 \text{ mA}} = 6 \text{ K}$$

$$R_C = 6 \text{ K}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{1 \text{ mA}}{100} = 0.01 \text{ mA} = 10 \mu\text{A}$$

$$V_B = V_{BE} = 0.6 \text{ V} \Rightarrow I_2 = \frac{V_B}{R_2} = \frac{0.6 \text{ V}}{20 \text{ K}} = 30 \mu\text{A}$$

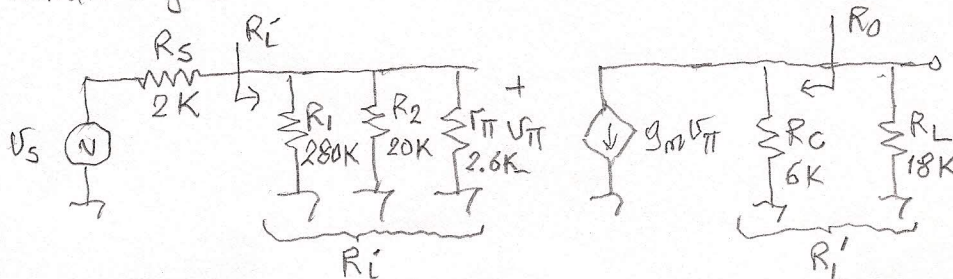
$$I_1 = I_B + I_2 = 40 \mu\text{A} \Rightarrow R_1 = \frac{V_{CC} - V_B}{I_1} = \frac{8.4 \text{ V}}{40 \mu\text{A}} = 0.21 \text{ M}\Omega$$

$$R_1 = 210 \text{ k}\Omega$$

ii. Small signal parameters:

$$r_{\pi} = \frac{26 \text{ mV}}{I_{BQ}} = \frac{26 \text{ mV}}{10 \mu\text{A}} = 2.6 \text{ K}; g_m = \frac{\beta}{r_{\pi}} = \frac{100}{2.6 \text{ K}} = 38.5 \text{ mS}, r_o = \infty$$

Small signal model:



$$R_i = R_1 // R_2 // r_{\pi}$$

$$R_i = 2.28 \text{ K}$$

$$R'_L = R_C // R_L = 4.5 \text{ K}$$

$$v_{\pi} = \frac{R_i}{R_s + R_i} v_s = \frac{2.28}{2 + 2.28} v_s = 0.53 v_s$$

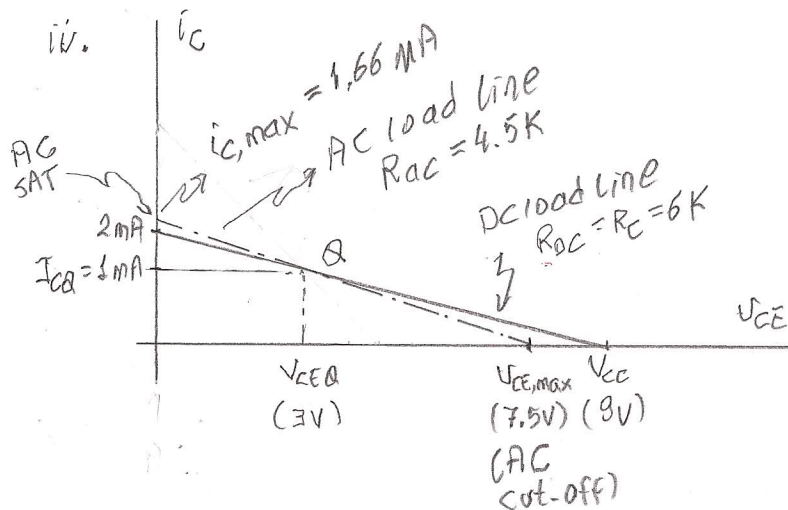
$$v_o = -g_m R'_L v_{\pi} = -(38.5 \text{ mS})(4.5 \text{ K}) v_{\pi} = -173.25 v_{\pi}$$

$$v_o = -92 v_s$$

$$A_v = -92$$

iii. $R_i = R_1 // R_2 // r_{\pi} = 2.28 K$

$R_D = R_C = 6 K$



$R_{ac} = R_C // R_L = 4.5 K$

$$V_{CE,max} = V_{CEQ} + R_{ac} I_{CQ}$$

$$= 3V + (4.5K)(1mA)$$

$$= 7.5V$$

$$i_{C,max} = I_{CQ} + \frac{V_{CEQ}}{R_{ac}} = 1.66mA$$

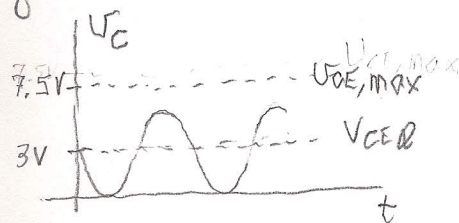
Output voltage swing around V_{CEQ} will be limited by AC-SAT.

v. Therefore for undistorted signal swing

$$V_o = V_d = V_{CE} = V_{CEQ} + V_{op} \sin \omega t$$

and

$$|V_{op,max}| = 3V$$



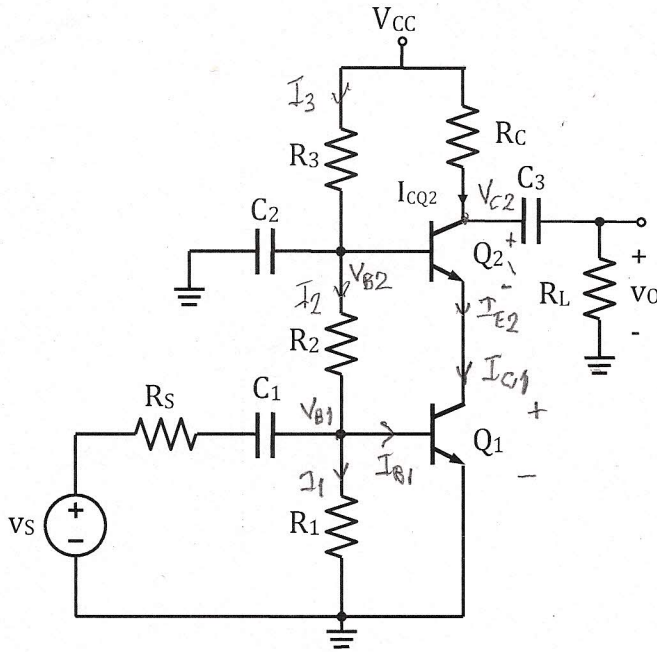
vi. $V_o = A_v V_s =$

$$V_s = 20 \sin \omega t \text{ mV} ; A_v = -92$$

$$V_o = -1840 \sin \omega t \text{ mV} = -1.84 \sin \omega t \text{ V}$$

$$V_C = V_{CEQ} + V_c = V_{CEQ} + V_o = 3 - 1.84 \sin \omega t \text{ V}$$

Q2. (25 pts) Consider the cascode amplifier given below.



Circuit Parameters

$$\begin{aligned} V_{CC} &= 12 \text{ V} \\ R_s &= 1 \text{ k}\Omega \\ R_1 &= 50 \text{ k}\Omega \\ R_L &= 14 \text{ k}\Omega \end{aligned}$$

Transistor Parameters Q₁ and Q₂

$$\begin{aligned} \beta &= 100, \\ V_{BE(ON)} &= 0.7 \text{ V}, \\ V_T &= 26 \text{ mV}, \\ V_A &= \infty \end{aligned}$$

- Determine R_C , R_2 , and R_3 to set $V_{CEQ1} = 1 \text{ V}$, $V_{CEQ2} = 4 \text{ V}$ and $I_{CQ2} = 1 \text{ mA}$.
- Determine the small-signal parameters for each transistor.
- Determine the overall AC small-signal voltage gain $A_v = v_o / v_s$.

(a)

$$\begin{aligned} I_{C2} &= 1 \text{ mA}, \beta = 100 \Rightarrow I_{E2} = 1.01 \text{ mA} \\ I_{B2} &= I_{C2} / \beta = 10 \mu\text{A} \\ I_{C1} &= I_{E2} = 1.01 \text{ mA} \Rightarrow I_{B1} = 10.1 \mu\text{A} \\ V_{B1} &= V_{BE1} = 0.7 \text{ V} \Rightarrow I_1 = \frac{V_{B1}}{R_1} = \frac{0.7 \text{ V}}{50 \text{ k}} = 14 \mu\text{A} \\ I_2 &= I_1 + I_{B1} = 24.1 \mu\text{A} \\ V_{B2} &= V_{BE2} + V_{CE1} = 0.7 + 1 \text{ V} = 1.7 \text{ V} \\ R_2 &= \frac{V_{B2} - V_{B1}}{I_2} = \frac{1.7 - 0.7 \text{ V}}{24.1 \mu\text{A}} = 4.15 \text{ k}\Omega \end{aligned}$$

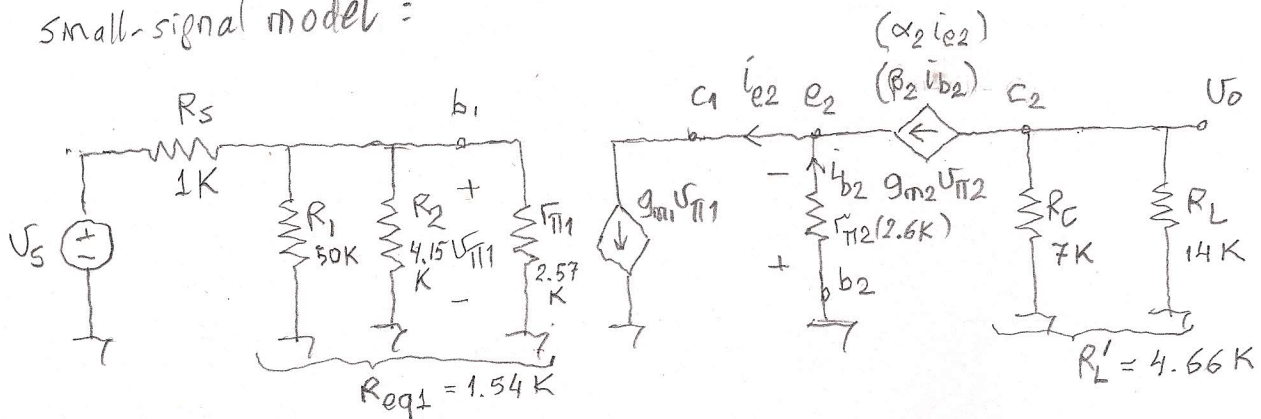
$$\begin{aligned} I_3 &= I_2 + I_{B2} = 24.1 \mu\text{A} + 10 \mu\text{A} = 34.1 \mu\text{A} \\ R_3 &= \frac{V_{CC} - V_{B2}}{I_3} = \frac{12 - 1.7 \text{ V}}{34.1 \mu\text{A}} = 331.4 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} V_{C2} &= V_{CE2} + V_{CE1} = 5 \text{ V} \\ R_C &= \frac{V_{CC} - V_{C2}}{I_{C2}} = \frac{12 - 5 \text{ V}}{1 \text{ mA}} = 7 \text{ k}\Omega \end{aligned}$$

(b)

$$\begin{aligned} \theta_1 &= r_{\pi 1} = \frac{V_T}{I_{B1}} = \frac{26 \text{ mV}}{10.1 \mu\text{A}} = 2.57 \text{ k}\Omega ; g_{m1} = \frac{\beta}{r_{\pi 1}} = 38.85 \text{ mS} ; r_{o1} = \infty \\ \theta_2 &= r_{\pi 2} = \frac{V_T}{I_{B2}} = \frac{26 \text{ mV}}{10 \mu\text{A}} = 2.6 \text{ k}\Omega ; g_{m2} = \frac{\beta}{r_{\pi 2}} = 38.46 \text{ mS} ; r_{o2} = \infty \end{aligned}$$

Small-signal model :



$$R_{eq1} = R_1 // R_2 // r_{\pi 1} = 1.54 K$$

$$R_L' = R_C // R_L = 4.66 K$$

$$V_{\pi 1} = \frac{R_{eq1}}{R_s + R_{eq1}} \cdot V_s = \frac{1.54}{2.54} V_s = 0.6 V_s$$

$$i_{e2} = g_{m1} V_{\pi 1} \Rightarrow i_{b2} = \frac{g_{m1}}{\beta + 1} V_{\pi 1}$$

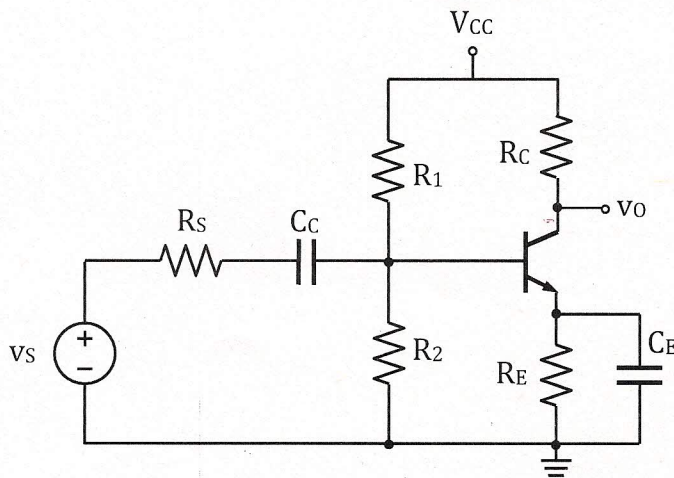
$$V_o = -\beta i_{b2} R_L' = -\beta \frac{g_{m1}}{\beta + 1} R_L' V_{\pi 1} = (-100) \cdot \frac{38.85 mS}{(100)} (4.66 K) V_{\pi 1}$$

$$\left. \begin{array}{l} V_o = -179.3 V_{\pi 1} \\ V_{\pi 1} = 0.6 V_s \end{array} \right\} \Rightarrow V_o = -107.6 V_s$$

$$A_v = -107.6$$

Q3. (25 pts) Consider the common emitter amplifier given below,

- Determine the small signal parameters.
- Write down the frequency gain expression.
- Determine the critical frequencies.
- Find the midband voltage gain.
- Plot the low frequency Bode magnitude and Bode phase responses.

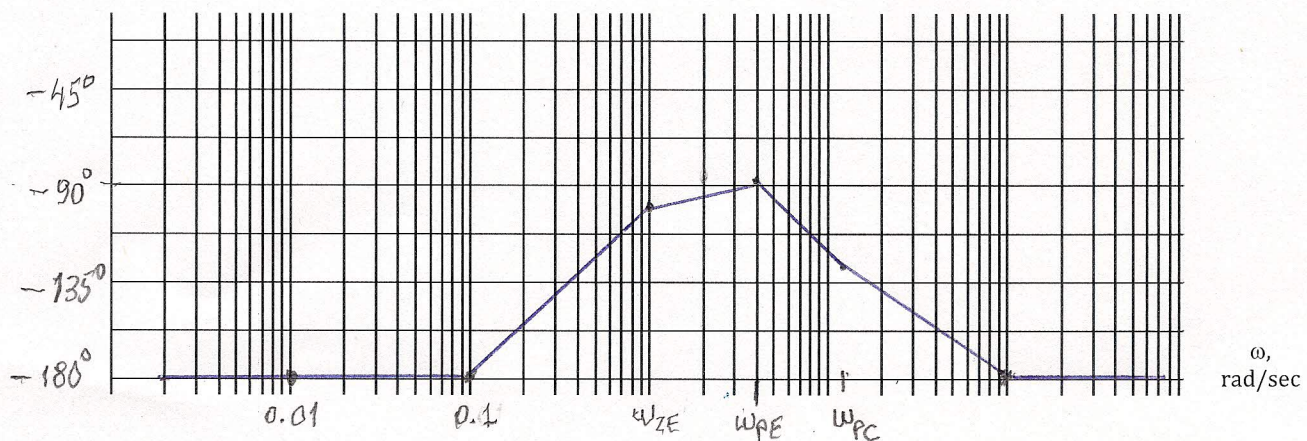
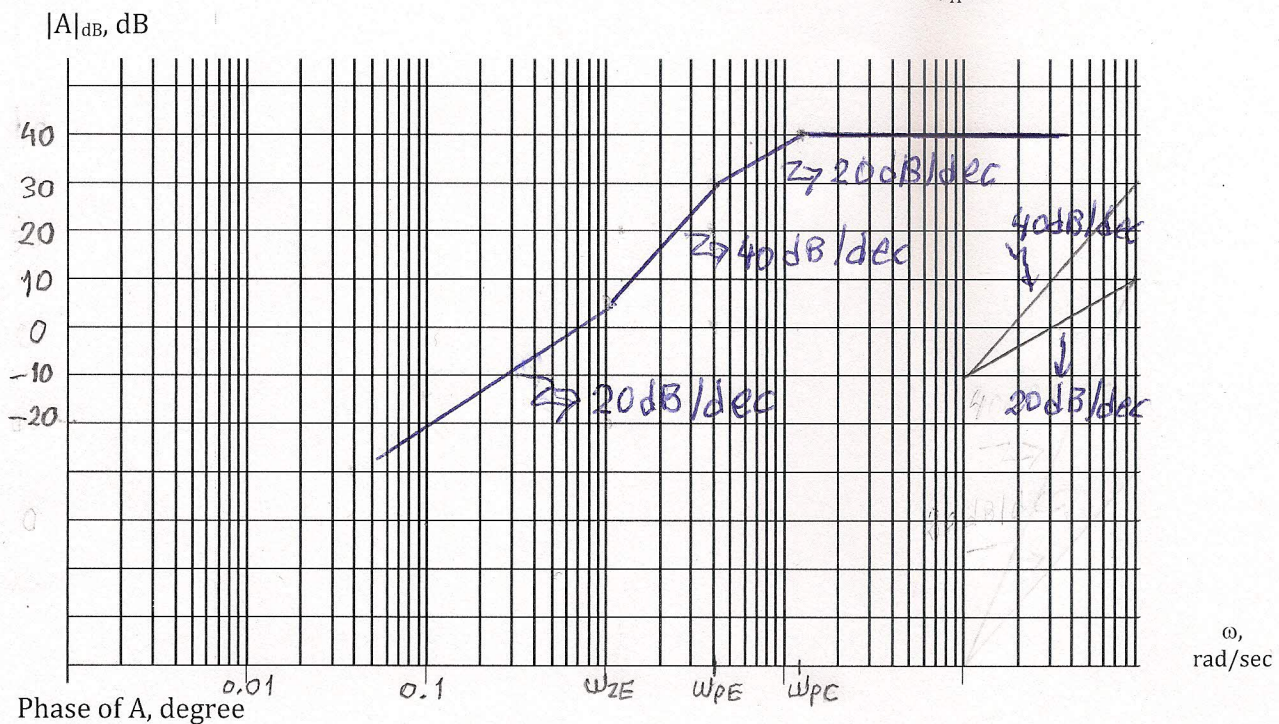


Circuit Parameters

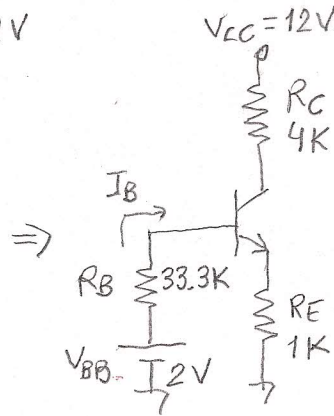
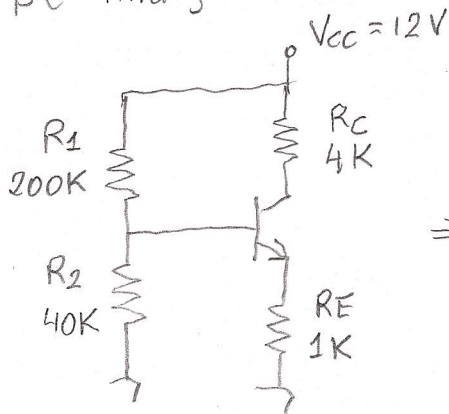
$$\begin{aligned} V_{CC} &= 12 \text{ V} \\ R_S &= 1 \text{ k}\Omega \\ R_1 &= 200 \text{ k}\Omega \\ R_2 &= 40 \text{ k}\Omega \\ R_E &= 1 \text{ k}\Omega \\ R_C &= 4 \text{ k}\Omega \\ C_C &= 25 \text{ }\mu\text{F} \\ C_E &= 1000 \text{ }\mu\text{F} \end{aligned}$$

The transistor Parameters

$$\begin{aligned} \beta &= 100, \\ V_{BE(ON)} &= 0.6 \text{ V}, \\ V_T &= 26 \text{ mV}, \\ V_A &= \infty \end{aligned}$$



i. DC Analysis



$$R_B = R_1 // R_2 = 33.3 \text{ K}$$

$$V_{BB} = \frac{R_2}{R_1 + R_2} V_{CC} = 2 \text{ V}$$

$$I_B = \frac{2 - 0.6}{33.3 \text{ K} + 101 \text{ K}} = 10.4 \mu\text{A}$$

$$I_C = 1.04 \text{ mA}$$

$$V_{CE} = 12 - (4 \text{ K})(1.04 \text{ mA}) - (1 \text{ K})(1.05 \text{ mA})$$

$$V_{CE} = 6.79 \text{ V} \quad (V_{CE} > V_{CE(\text{SAT})})$$

Forward active ✓

$$r_{\pi} = \frac{26 \text{ mV}}{I_B} = 2.5 \text{ K} ; g_m = \frac{\beta}{r_{\pi}} = 40 \text{ mS} ; r_o = \infty$$

$$ii. A(j\omega) = A_0 \underbrace{\left(\frac{j\omega}{j\omega + \omega_{PC}} \right)}_{C_C} \underbrace{\left(\frac{j\omega + \omega_{ZE}}{j\omega + \omega_{PE}} \right)}_{C_E} = -|A_0| \left(\frac{j\omega}{j\omega + \omega_{PC}} \right) \left(\frac{j\omega + \omega_{ZE}}{j\omega + \omega_{PE}} \right)$$

$$\omega_{PC} = \frac{1}{R_{eqC} C_C} , \omega_{PE} = \frac{1}{R_{eqE} C_E} ; \omega_{ZE} = \frac{1}{R_{CE}} = \frac{1}{(1 \text{ K})(1000 \mu\text{F})}$$

$$\omega_{ZE} = 1 \text{ rad/sec}$$

Assume C_E becomes short first; i.e

$$\omega_{PE} < \omega_{PC}$$

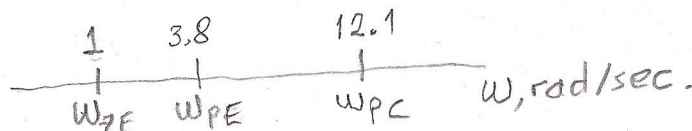
$$R_{eqE} = R_E // \frac{R_1 // R_2 + r_{\pi}}{\beta + 1} \quad (C_C \text{ is open})$$

$$= 1 \text{ K} // \left[\frac{200 \text{ K} // 40 \text{ K} + 2.5 \text{ K}}{101} \right] = 263 \Omega \Rightarrow \omega_{PE} = \frac{1}{R_{eqE} C_E} = 3.8 \text{ rad/sec}$$

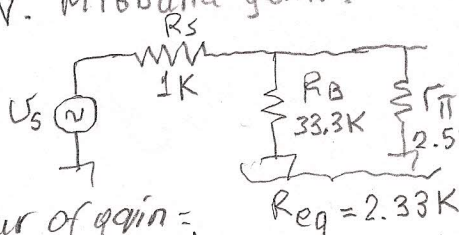
$$R_{eqC} = R_S + [R_1 // R_2 // r_{\pi}] \quad (C_E \text{ is short})$$

$$= 1 \text{ K} + (200 \text{ K} // 40 \text{ K} // 2.5 \text{ K}) = 3.33 \text{ K}$$

$$\omega_{PC} = \frac{1}{R_{eqC} C_C} = 12.1 \text{ rad/s}$$



iv. Midband gain:



$$V_{\pi} = \frac{2.33}{3.33} V_S = 0.7 V_S$$

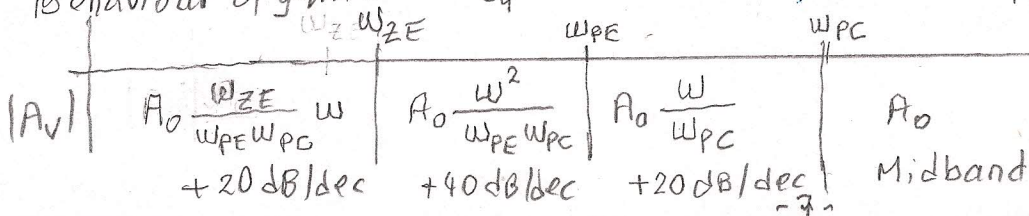
$$V_O = -g_m R_C V_{\pi} = (-40 \text{ mS})(4 \text{ K}) V_{\pi} = -160 V_{\pi} = -111 V_S$$

$$A_V = -111 \Rightarrow A_{V, \text{dB}} = 41 \text{ dB}$$

Phase behaviour

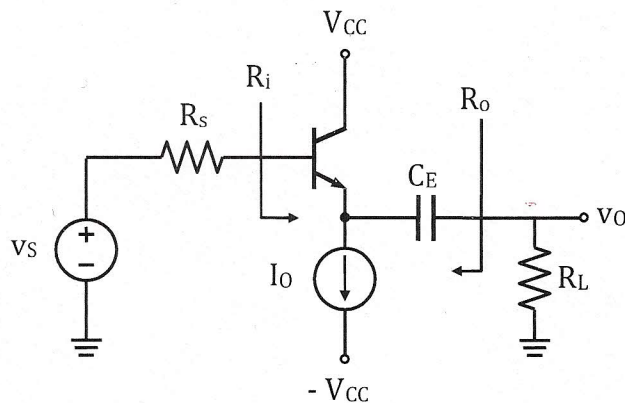
$$\phi = \pi + \tan^{-1} \omega + \tan^{-1} \frac{\omega}{\omega_{ZE}} - \tan^{-1} \left(\frac{\omega}{\omega_{PC}} \right) - \tan^{-1} \left(\frac{\omega}{\omega_{PE}} \right)$$

Behaviour of gain =



Q4. (25 pts) Consider the common emitter amplifier biased with a current source given below.

- Draw the DC equivalent circuit and determine quiescent point values I_{CQ} and V_{CEQ} .
- Determine the small signal parameters.
- Draw the small signal equivalent circuit and calculate the small signal voltage gain $A_V = v_o/v_s$.
- Determine the input resistance R_i and output resistance R_o .
- Determine the lower corner frequency.



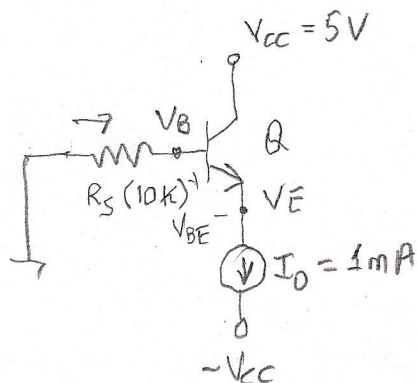
Circuit Parameters

$$\begin{aligned} V_{CC} &= 5 \text{ V} \\ R_S &= 10 \text{ k}\Omega \\ R_L &= 10 \text{ k}\Omega \\ I_o &= 1 \text{ mA} \\ C_E &= 10 \text{ }\mu\text{F} \end{aligned}$$

The transistor Parameters

$$\begin{aligned} \beta &= 100, \\ V_{BE(ON)} &= 0.7 \text{ V}, \\ V_T &= 26 \text{ mV}, \\ V_A &= 100 \text{ V} \end{aligned}$$

i. AC sources are short, c's are open!



$$I_C \approx I_E = I_o = 1 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = 10 \text{ }\mu\text{A}$$

$$V_B = -R_S I_B = 0.1 \text{ V}$$

$$V_B = V_{BE} + V_E \Rightarrow V_E = V_B - V_{BE} = -0.8 \text{ V}$$

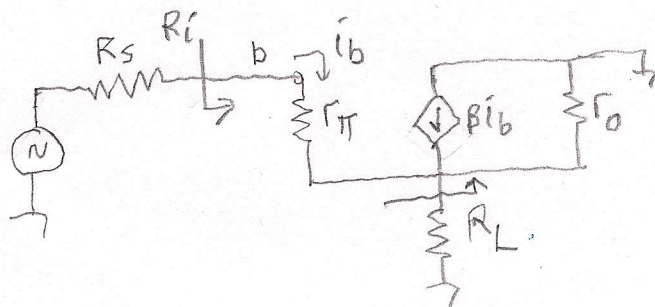
$$\left. \begin{aligned} V_E &= -0.8 \text{ V} \\ V_C &= 5 \text{ V} \end{aligned} \right\} V_{CE} = V_C - V_E = 5.8 \text{ V}$$

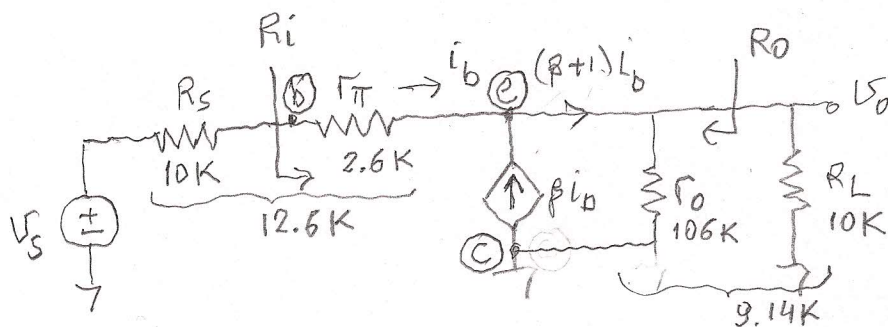
($V_{CE} > V_{CE(SAT)}$; Q is forward active)

$$ii. r_{\pi} = \frac{V_T}{I_B} = \frac{26 \text{ mV}}{10 \text{ }\mu\text{A}} = 2.6 \text{ k}\Omega; g_m = 0.44 \text{ mS};$$

$$r_o = \frac{V_A + V_{CEQ}}{I_{CQ}} = \frac{105.8 \text{ V}}{1 \text{ mA}} = 106 \text{ k}\Omega$$

iii Small signal model (Midband)





$$i_b = \frac{V_S - V_o}{R_S + r_{\pi}} \quad ; \quad V_o = (\beta+1) i_b (r_o \parallel R_L)$$

$$V_o = (\beta+1) \left(\frac{V_S - V_o}{R_S + r_{\pi}} \right) (r_o \parallel R_L)$$

$$(R_S + r_{\pi}) V_o = (V_S - V_o) (\beta+1) (r_o \parallel R_L)$$

$$[(R_S + r_{\pi}) + (\beta+1) (r_o \parallel R_L)] V_o = (\beta+1) (r_o \parallel R_L) V_S$$

$$A_v = \frac{V_o}{V_S} = \frac{(\beta+1) (r_o \parallel R_L)}{(R_S + r_{\pi}) + (\beta+1) (r_o \parallel R_L)}$$

$$= \frac{(101)(9.14K)}{12.6K + (101)(9.14K)} = \frac{923.14K}{935.74K} = 0.987$$

$$iV. \quad R_i = r_{\pi} + (\beta+1) (r_o \parallel R_L)$$

$$= 2.6K + 923.14K = 925.74K$$

$$R_o = r_o \parallel \left(\frac{R_S + r_{\pi}}{\beta+1} \right) = 106K \parallel \frac{12.6K}{101} = 106K \parallel 125\Omega = 125\Omega$$

$$x. \quad A_v(j\omega) = A_o \left(\frac{j\omega}{j\omega + \omega_{PE}} \right)$$

$$\omega_{PE} = \frac{1}{R_{eqE} C_E} \quad ; \quad R_{eqE} = \left(r_o \parallel \frac{R_S + r_{\pi}}{\beta+1} \right) + R_L$$

$$= 125\Omega + 10K = 10.1K$$

$$C_E = 10\mu F$$

$$\omega_{PE} = \frac{1}{(10.1K)(10\mu F)} = 9.9 \text{ rad/sec} \quad (f_{PE} = 1.58 \text{ Hz})$$