İZMİR UNIVERSITY OF ECONOMICS
Department of Electronics and Communication Engineering
ETE 331 Analog Electronics
Fall 2012/2013

SOLUTIONS

MIDTERM EXAM I

Nov 17, 2012
120 min

INSTRUCTIONS

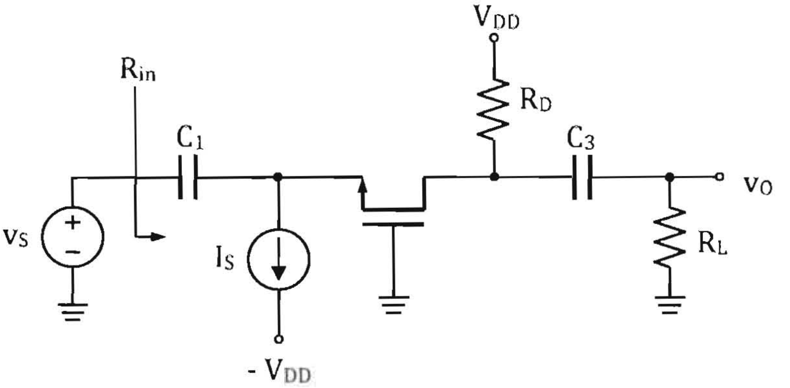
- Read all of the instructions and all of the questions before beginning the exam.
- There are 6 questions on this exam, totaling 100 points. The credit for each problem is given to help you allocate your time accordingly.
- Do not spend all your time on one problem and on one part and attempt to solve all of them.
- Calculators are allowed, but borrowing is not allowed.
- Your mobile phones must be turned off during the exam.
- Turn in the entire exam, including this cover sheet.
- You must show your work for all problems to receive full credit; simply providing answers will result in only partial credit, even if the answers are correct.
- Please indicate the number of page where your work is to be continued.
- Put your name on any additional material that you submit.
- Be sure to provide units where necessary.
- Please sign the honor pledge that is provided below.

		Question	Points	Grade
Last Name	:......................	1	25	
Name	:......................	2	25	
Section	:......................	3	25	
Student No	:......................	4	25	
		TOTAL	100	

The basic equations of the output characteristics of an NMOS transistor		
V_{GS}	V_{DS}	I_D
i) $V_{GS} < V_{Tn}$	-	0
ii) $V_{GS} > V_{Tn}$	a) $V_{DS} < V_{GS} - V_{Tn}$	$K_n\left[2(V_{GS} - V_{Tn})V_{DS} - V_{DS}^2\right]$
	b) $V_{GS} - V_{Tn} \le V_{DS}$	$K_n(V_{GS} - V_{Tn})^2$
where $K_n = \frac{K'_n}{2}\left(\frac{W}{L}\right)$ and $K'_n = \mu_n C_{ox}$		

Q1. (25 pts) Consider the common-gate amplifier given below. All capacitors are assumed short at the frequencies interest.

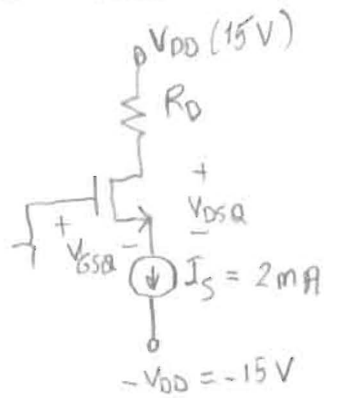
(a) Draw the DC model of the circuit.
(b) Determine the value of the resistor R_D to set $V_{DSQ} = 11$ V.
(c) Determine the small signal parameters g_m and r_o of the transistor.
(d) Draw the AC small signal equivalent circuit.
(e) Determine the overall small signal voltage gain $A_v = v_O/v_S$.
(f) Determine the small signal input resistance R_{in}.



Circuit Parameters
$V_{DD} = 15$ V
$I_S = 2$ mA
$R_L = 9$ kΩ

Transistor Parameters
$V_{TN} = 1$ V
$K_N = 2$ mA/V²
$\lambda = 0$

a) DC Model :



b) Assume SAT :

$I_D = K_n(V_{GSQ} - V_{Tn})^2 = I_S$

$2 = 2(V_{GSQ} - 1)^2$

$1 = V_{GSQ} - 1 \Rightarrow V_{GSQ} = 2V$

$V_{GSQ} = V_{GQ} - V_{SQ} = -V_{SQ} \Rightarrow V_{SQ} = -2V$

$V_{DSQ} = V_{DQ} - V_{SQ}$

$11 = V_{DQ} - (-2V) \Rightarrow V_{DQ} = 9V$

$V_{DQ} = V_{DD} - R_D I_{DQ}$

$9 = 15 - R_D(2) \Rightarrow R_D = \frac{15-9}{2} = 3K$

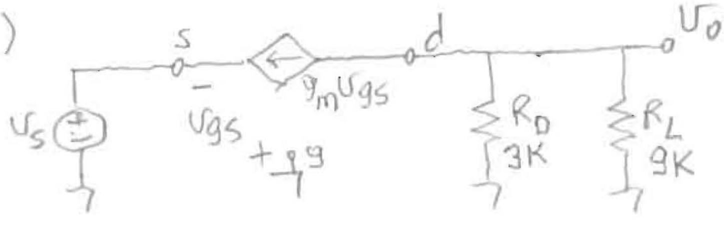
SAT check : $V_{DSQ} \geq V_{GSQ} - V_{Tn} \Rightarrow 11V \geq 2 - 1 = 1V$ ✓

c) Small signal parameters :

$g_m = 2K_n(V_{GS} - V_{Tn}) = (2)(2\ mA/V^2)(1V) = 4\ mA/V$

$r_o = \frac{1}{\lambda I_{DQ}} = \infty$

d)



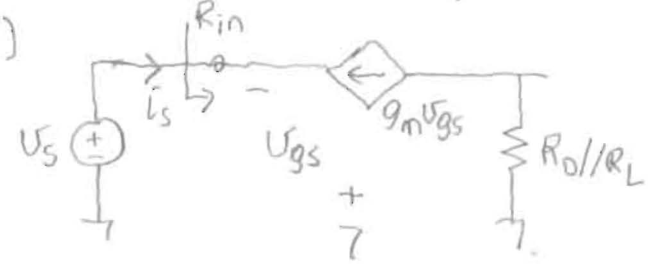
e) $v_o = -g_m v_{gs}(R_D // R_L)$

$v_s = -v_{gs} \Rightarrow v_o = g_m (R_D // R_L) v_s$

$$A_v = \frac{v_o}{v_s} = g_m (R_D // R_L) = (4\,mA/V)\frac{(3)(9)K}{12}$$

$$A_v = 9$$

f)



$$R_{in} = \frac{v_s}{i_s}$$

$$\left.\begin{aligned} v_s &= -v_{gs} \\ i_s &= -g_m v_{gs} \end{aligned}\right\} R_{in} = \frac{1}{g_m}$$

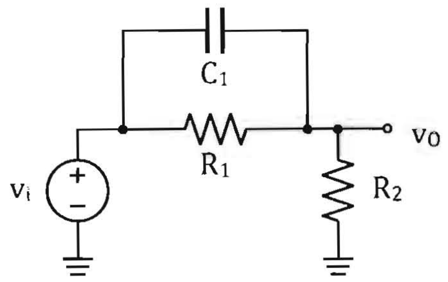
$$R_{in} = \frac{1}{4\,mA/V} = \frac{1\,V}{4 \times 10^{-3}\,A}$$

$$R_{in} = 250\,\Omega$$

Q2. (25 pts)

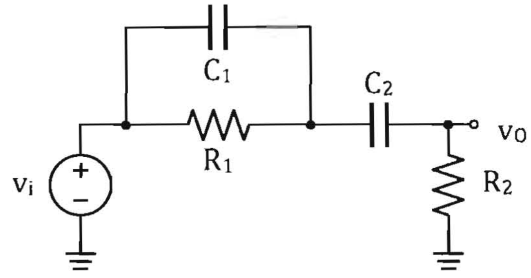
(a) Consider the RC-circuit given.
Assume R_1 =2.5 kΩ, R_2 =10kΩ, and C_1 =25 μF

i. Determine the voltage frequency transfer function $H(j\omega) = V_o(j\omega)/V_i(j\omega)$.
ii. Sketch the Bode magnitude plot and Bode phase plot of the voltage transfer function.



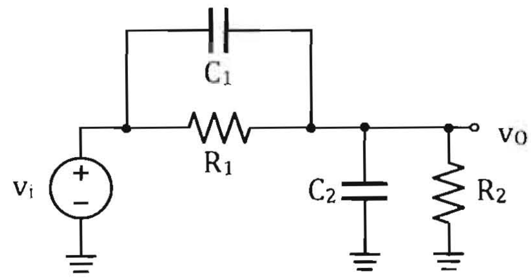
(b) For the RC-circuit given, assume
R_1 =4 kΩ, R_2 =1 kΩ, C_1 =1 μF and C_2 =2 μF

i. Using short circuit and open circuit time constants method, determine the break frequencies.
ii. Sketch the Bode magnitude plot and Bode phase plot of the voltage transfer function $H(j\omega) = V_o(j\omega)/V_i(j\omega)$.

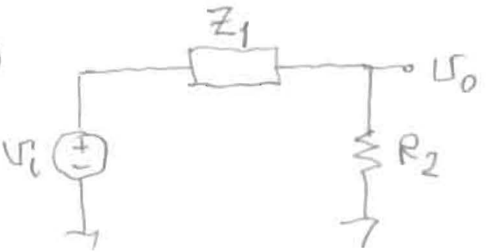


(c) For the RC-circuit given, the component values are chosen so that $\boldsymbol{R_1 C_1 = R_2 C_2}$

i. Determine the voltage transfer function $H(j\omega) = V_o(j\omega)/V_i(j\omega)$.
ii. Sketch the Bode magnitude plot and Bode phase plot.



(a)



$$Z_1 = R_1 // \frac{1}{j\omega C_1} = \frac{(R_1)\left(\frac{1}{j\omega C_1}\right)}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

$$\omega_1 = \frac{1}{R_1 C_1} \Rightarrow Z_1 = \frac{R_1}{1 + j\left(\frac{\omega}{\omega_1}\right)}$$

i. $$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_2}{R_2 + Z_1} = \frac{R_2}{R_2 + \frac{R_1}{1 + j\left(\frac{\omega}{\omega_1}\right)}} = \frac{R_2\left(1 + j\frac{\omega}{\omega_1}\right)}{R_1 + R_2 + j\left(\frac{\omega R_2}{\omega_1}\right)}$$

$$= \frac{R_2}{R_1 + R_2} \cdot \frac{1 + j\frac{\omega}{\omega_1}}{1 + j\left(\omega \frac{R_1 R_2 C}{R_1 + R_2}\right)} = \frac{R_2}{R_1 + R_2} \cdot \frac{1 + j\left(\frac{\omega}{\omega_1}\right)}{1 + j\left(\frac{\omega}{\omega_2}\right)}$$

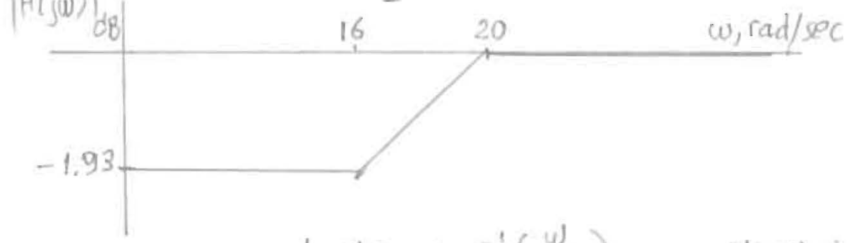
where $\omega_1 = \frac{1}{R_1 C_1}$; $\omega_2 = \frac{1}{(R_1 // R_2) C_1}$

$R_1 = 2.5\,k\Omega,\ C_1 = 25\,\mu F \Rightarrow \tau_1 = R_1 C_1 = 62.5$ msec

$$\omega_1 = \frac{1}{\tau_1} = \frac{1}{62.5 \times 10^{-3}} = 16 \text{ rad/sec}$$

$R_1 // R_2 = 2.5K // 10K = 2K \Rightarrow \tau_2 = C_1(R_1//R_2) = (25\mu F)(2K) = 50\,msec$

$$\omega_2 = \frac{1}{\tau_2} = 20 \text{ rad/sec}$$



$\omega << 16$ rad/sec

$|H(j\omega)| = \frac{R_2}{R_1+R_2} = \frac{10}{12.5} = 0.8$

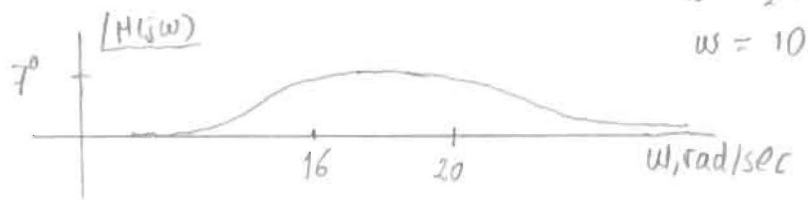
$20\log(0.8) = -1.93$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{\omega_1}\right) - \tan^{-1}\left(\frac{\omega}{\omega_2}\right)$$

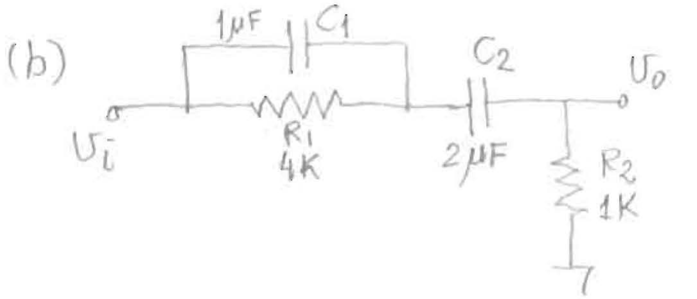
$\omega = \omega_1 = 16$ rad/sec $\Rightarrow \angle H(j\omega) = 7^\circ$

$\omega = \omega_2 = 20$ rad/sec $\Rightarrow \angle H(j\omega) = 6^\circ$

$\omega = 10$ rad/sec $\Rightarrow \angle H(j\omega) = 6^\circ$



(b)



$$H(j\omega) = \frac{R_2}{Z_1 + \frac{1}{j\omega C_2} + R_2}$$

$$= \frac{R_2}{Z_1 + \frac{1+j\omega R_2 C_2}{j\omega C_2}}$$

$$= \frac{R_2}{\frac{R_1}{1+j\omega R_1 C_1} + \frac{1+j\omega R_2 C_2}{j\omega C_2}}$$

$$H(j\omega) = \frac{(j\omega R_2 C_2)(1+j\omega R_1 C_1)}{1+j\omega(R_1C_1 + R_2C_2 + R_1C_2) - \omega^2 R_1 C_1 R_2 C_2} \qquad (B1)$$

$$= \frac{(j\omega\tau_2)(1+j\omega\tau_1)}{(1+j\omega\tau_3)(1+j\omega\tau_4)} \quad \text{where } \tau_2 = R_2C_2,\ \tau_1 = R_1C_1 \quad (B2)$$

The time constants τ_3 and τ_4 may be obtained using short circuit and open circuit time constants.

- At very low frequencies where C_2 is becoming active, but C_1 is open; τ_3 is obtained as the open circuit time constant for C_2, (C_1 is open) :
$$\tau_3 = C_2(R_1+R_2) = (2\mu F)(4K+1K) = 10 \text{ msec}$$
- At the frequencies where C_1 is becoming active (C_2 is already short); τ_4 is obtained as the short circuit time constant for C_1 (C_2 is short) :
$$\tau_4 = C_1(R_1//R_2) = (1\mu F)(4K//1K) = 0.8 \text{ msec}$$

From Eq. B1 and Eq. B2;

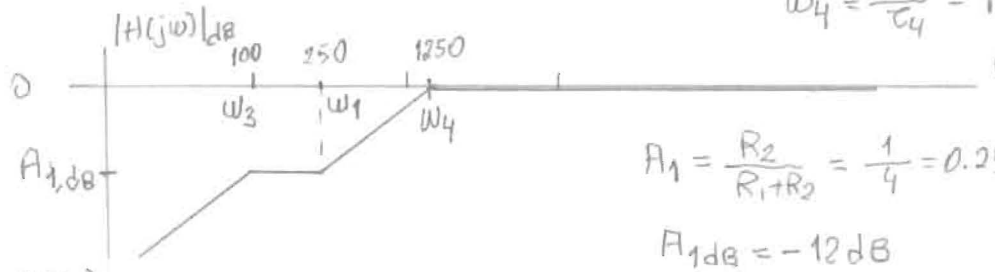
$$\tau_3 \tau_4 = R_1 C_1 R_2 C_2$$

Since $\tau_3 = C_2(R_1+R_2)$; $\tau_4 = C_1(R_1 // R_2) = C_1 \frac{R_1 R_2}{R_1+R_2}$; above condition is satisfied.

Break frequencies:

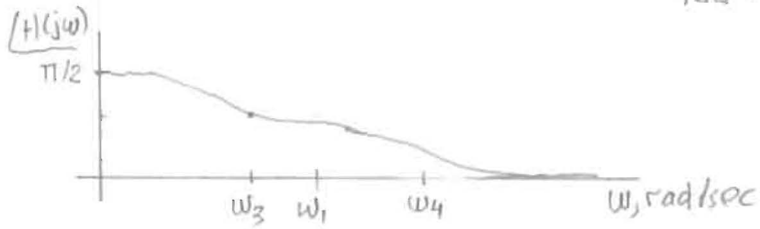
$$\omega_1 = \frac{1}{\tau_1} = \frac{1}{(1\mu F)(4K)} = 250 \text{ msec}; \quad \omega_3 = \frac{1}{\tau_3} = 100 \text{ rad/sec}$$

$$\omega_4 = \frac{1}{\tau_4} = 1250 \text{ rad/sec}$$



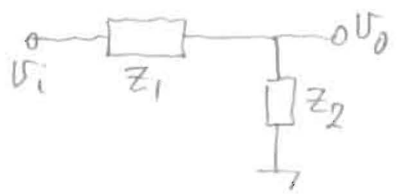
$$A_1 = \frac{R_2}{R_1+R_2} = \frac{1}{4} = 0.25$$

$$A_{1dB} = -12 \text{ dB}$$



$$\angle H(j\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{\omega_3}\right) + \tan^{-1}\left(\frac{\omega}{\omega_1}\right) - \tan^{-1}\left(\frac{\omega}{\omega_4}\right)$$

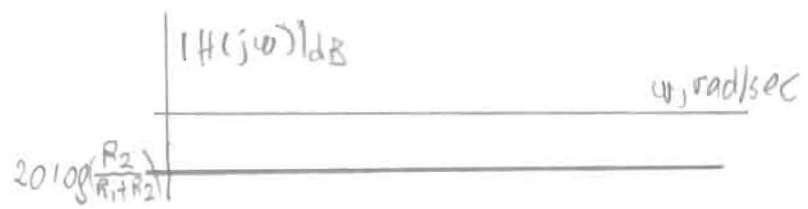
(c)



$$Z_1 = \frac{R_1}{1+j\omega R_1 C_1}, \quad Z_2 = \frac{R_2}{1+j\omega R_2 C_2}$$

$$H(j\omega) = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{R_2}{1+j\omega R_2 C_2}}{\frac{R_1}{1+j\omega R_1 C_1} + \frac{R_2}{1+j\omega R_2 C_2}}$$

Substitute $R_2 C_2 = R_1 C_1 \Rightarrow H(j\omega) = \frac{\frac{R_2}{1+j\omega R_2 C_2}}{\frac{R_1+R_2}{1+j\omega R_2 C_2}} = \frac{R_2}{R_1+R_2}$



$|H(j\omega)|_{dB} = 20\log\left(\frac{R_2}{R_1+R_2}\right)$ constant!

$\angle H(j\omega) = 0$

Q3. (25 pts) Consider the common source amplifier given below.

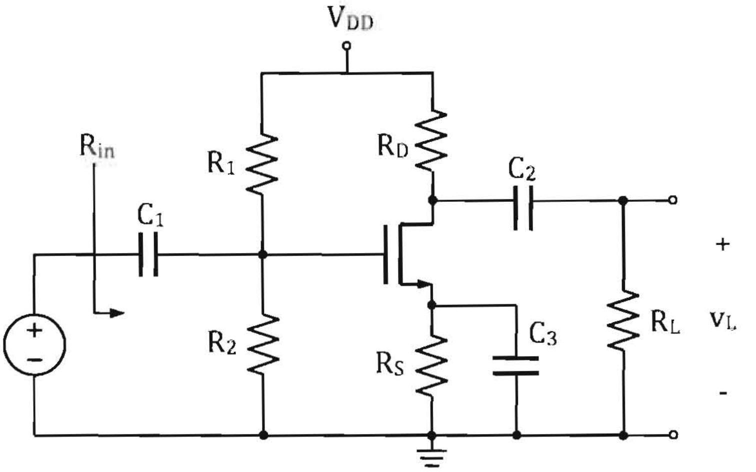
(a) Determine R_S and R_D so that $V_{GSQ} = 2$ V and $V_{DSQ} = 4$ V.

(b) Determine the small signal parameters g_m and r_o of the transistor.

(c) Determine the break frequencies and 3-dB corner frequency.

(d) Sketch the Bode magnitude plot of the voltage frequency transfer function.

(e) If the resistor R_S is replaced with a constant-current source producing the same I_{DQ} quiescent current, determine the break frequencies and the 3-dB corner frequency and sketch the Bode magnitude plot.



Circuit Parameters

$V_{DD} = 12$ V

$R_1 = 200$ kΩ

$R_2 = 100$ kΩ

$R_L = 9$ kΩ

$C_1 = 100$ μF

$C_2 = 50$ μF

$C_3 = 100$ μF

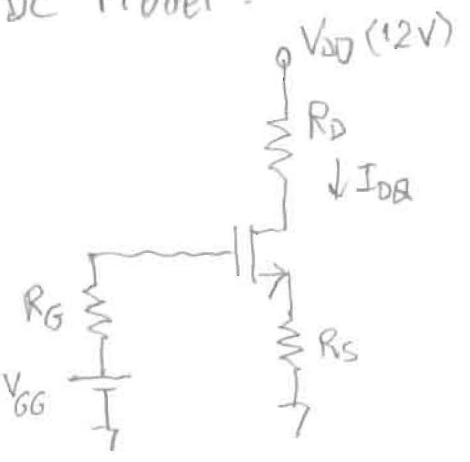
Transistor Parameters

$V_{TN} = 1$ V

$K_N = 2$ mA/V²

$\lambda = 0$

(a) DC Model :



$R_G = R_1 // R_2 = 66$ kΩ

$V_{GG} = \frac{R_2}{R_1 + R_2} V_{DD} = \frac{100}{300} \cdot 12 = 4V$

$V_{GSQ} = 2V \Rightarrow I_{DQ} = K_n (V_{GSQ} - V_{Tn})^2$

$I_{DQ} = 2mA/V^2 (2-1)^2 = 2mA$

$V_{GG} = V_{GSQ} + R_S I_{DQ}$

$\Rightarrow R_S = \frac{V_{GG} - V_{GSQ}}{I_{DQ}} = \frac{4-2V}{2mA} = 1K$

$R_D + R_S = \frac{V_{DD} - V_{DSQ}}{I_{DQ}} = \frac{12-4V}{2mA} = 4K \Rightarrow R_D = 4K - R_S = 3K$

Check SAT : $V_{DSQ} \geq V_{GSQ} - V_{Tn} \Rightarrow 4V \geq 2 - 1V = 1V$ ✓

(b) $g_m = 2K_n(V_{GSQ} - V_{Tn}) = (2)(2mA/V^2)(2-1) = 4$ mA/V

$r_o = \frac{1}{\lambda I_{DQ}} = \infty$

$\frac{1}{g_m} = \frac{1}{4mA/V} = 0.25K$

(c) $\tau_1 = C_1(R_1 // R_2) = (100\mu F)(66K) = 6.6\,sec \Rightarrow \omega_1 = \frac{1}{\tau_1} = 0{,}15\ rad/sec$

$\tau_2 = C_2(R_D + R_L) = (50\mu F)(3K + 3K) = 0.6\,sec \Rightarrow \omega_2 = \frac{1}{\tau_2} = 1.66\ rad/sec$

There are two break frequencies due to C_3:

$\tau_3 = C_3 R_S = (100\mu F)(1K) = 0.1\,sec \Rightarrow \omega_3 = \frac{1}{\tau_3} = 10\ rad/sec$

$\tau_4 = C_3(R_S // \frac{1}{g_m}) = (100\mu F)(1K // 0.25K) = 20\,msec \Rightarrow \omega_4 = \frac{1}{\tau_4} = 50\ rad/sec$

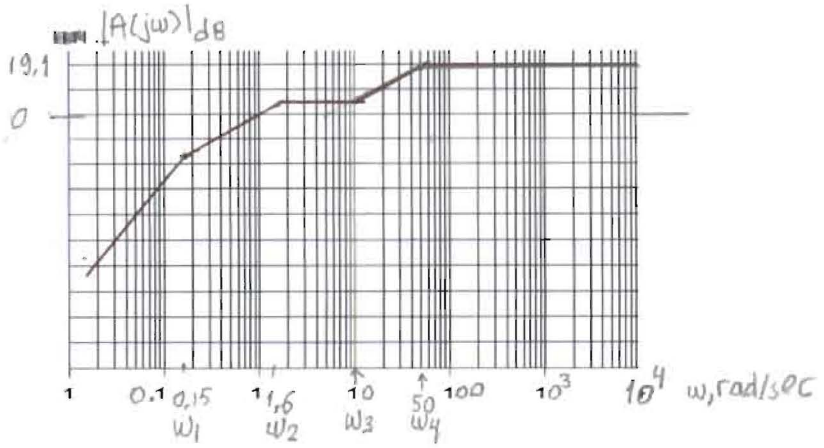
Then

$$A_v(j\omega) = A_{MB}\frac{(j\frac{\omega}{\omega_1})(j\frac{\omega}{\omega_2})(1 + j\frac{\omega}{\omega_3})\omega_3}{(1 + j\frac{\omega}{\omega_1})(1 + j\frac{\omega}{\omega_2})(1 + j\frac{\omega}{\omega_4})\omega_4}$$

$$= A_{MB}\frac{(j\omega)(j\omega)(\omega_3 + j\omega)}{(\omega_1 + j\omega)(\omega_2 + j\omega)(\omega_4 + j\omega)}$$

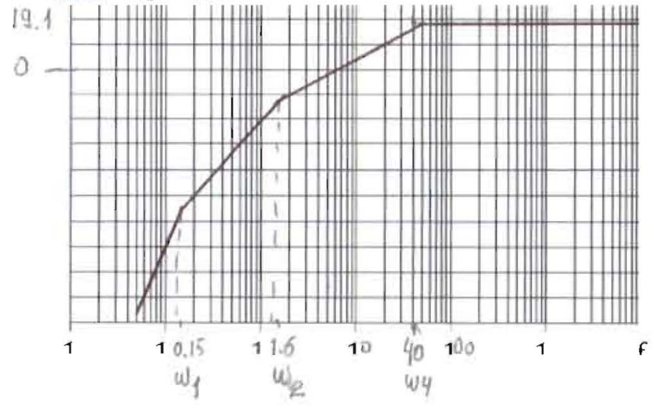
where $A_{MB} = -g_m(R_D // R_L) = -(4mA/V)(3K // 9K) = -9$

$A_{MB,dB} = 20\log 9 \cong 19.1\,dB$



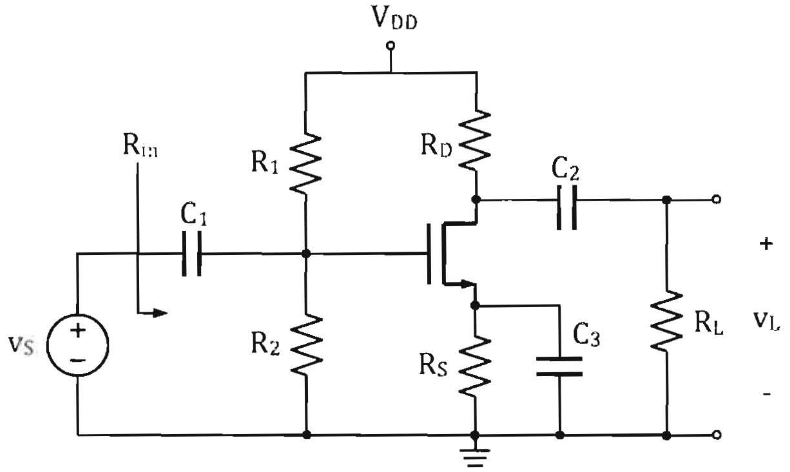
(e) If a current source is used then $R_S = \infty$. ω_3 and ω_4 are recalculated as

$$\left.\begin{aligned} \omega_3 &= \frac{1}{C_3 R_S} = 0 \\ \omega_4 &= \frac{1}{C_3(\frac{1}{g_m})} = \frac{1}{(100\mu F)(0.25K)} = 40\ rad/sec \end{aligned}\right\} A(j\omega) = \frac{A_{MB}(j\omega)(j\omega)(j\omega)}{(\omega_1 + j\omega)(\omega_2 + j\omega)(\omega_4 + j\omega)}$$



Q4. (25 pts) Consider the common source amplifier given below. The Q point is located at $V_{GSQ} = 2$ V.

(a) Draw the high frequency small signal equivalent circuit.
(b) Determine the Miller capacitance.
(c) Determine the break frequencies and the upper 3-dB frequency.
(d) Sketch the Bode magnitude plot over the high frequency range.



Circuit Parameters

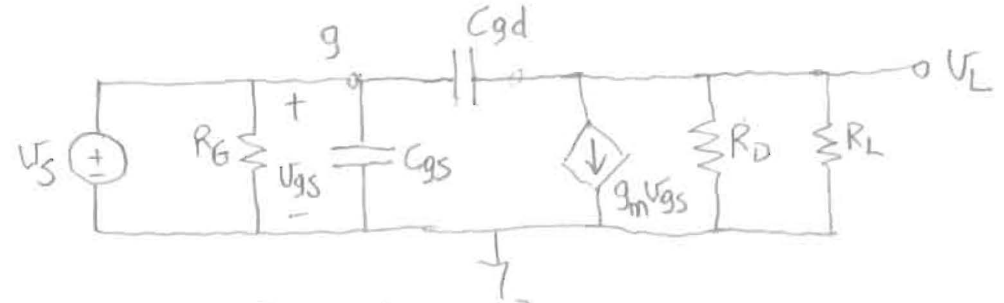
$V_{DD} = 12$ V
$R_1 = 200$ kΩ
$R_2 = 100$ kΩ
$R_D = 3$ kΩ
$R_L = 9$ kΩ

Transistor Parameters

$V_{Tn} =$ V
$K_n =$ mA/V²
$\lambda = 0$
$C_{gs} = 2$ pF
$C_{gd} = 0.2$ pF

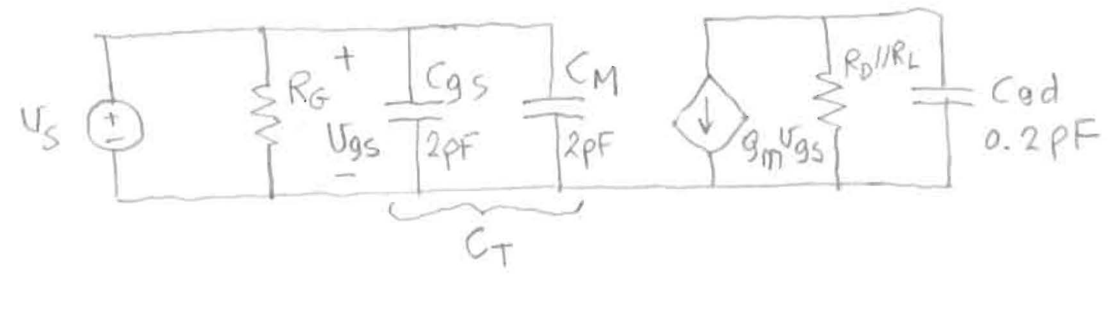
(a) $g_m = 2K_n(V_{GSQ} - V_{Tn}) = (2)(2\,\text{mA/V}^2)(2 - 1\,\text{V}) = 4\,\text{mA/V}$

High frequency small signal model (all external capacitors are short circuited.)



(b) $C_M = C_{gd}\left[1 + g_m(R_D \| R_L)\right]$

$= (0.2\,\text{pF})\left[1 + 4\cdot(3K \| 9K)\right] = (0.2\,\text{pF})(10) = 2\,\text{pF}$



(c) $\omega_1 = \dfrac{1}{C_T R_{eq,T}}$

$R_{eq,T} = 0 \quad (v_s = 0\,!) \Rightarrow \omega_1 = \infty$

$$\omega_2 = \frac{1}{C_{gd}(R_D // R_L)} = \frac{1}{(0.2\times10^{-12})(2.25\times10^{3})} = 2.2\times10^{9}\ \text{rad/sec}$$

$$A(j\omega) = \frac{A_{MB}}{\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_2}\right)} = \frac{A_{MB}}{1+j\frac{\omega}{\omega_2}}$$

$$A_{MB} = -g_m(R_D // R_L) = -(4\ \text{mA/V})(2.25\text{K}) \qquad 9$$

$$A_{MB,dB} = 20\log 9 = 19.1\ \text{dB}$$

(d)