lecture3: probability

27 Şubat 2025 Perşembe   11:25

probability: the chance an uncertain event will occur. always between 0 and 1.

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0 ▭▭▭▭▭▭▭▭ 1
impossible        certain
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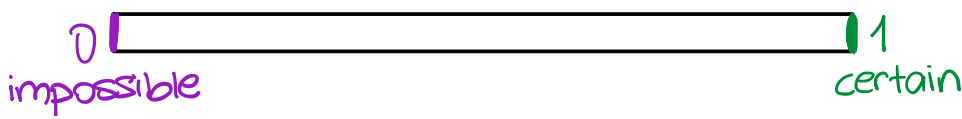
sample space: all possible outcomes of the experiment. Flipping a coin —> head and tail are in sample space

flipping twice: $S: \{HH, TT, HT, TH\}$
                         ↓   ↓   ↓   ↓
                         2   0   1   1

$P(Y=2)=\frac{1}{4}$, $P(Y=1)=\frac{1}{2}$, $P(Y=0)=\frac{1}{4}$

↳ probability of   ↳ prob of one   ↳ prob. of two
   two heads   head   tails
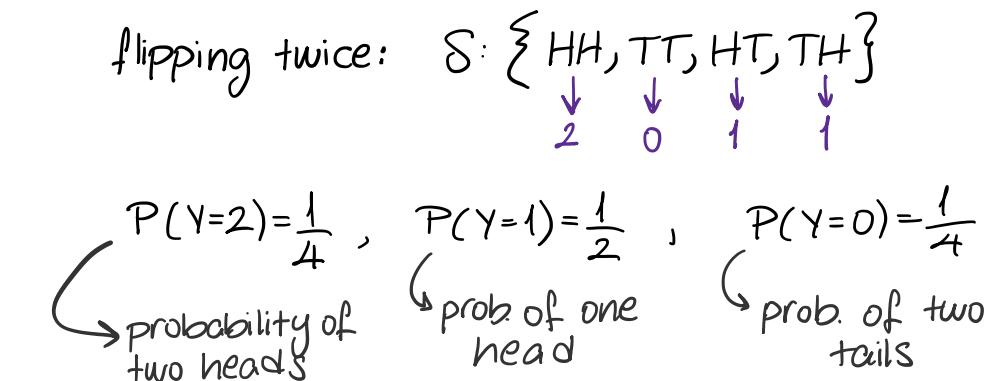
→ random variable assigns a numerical value to each outcome in a sample space.

↳ discrete or continuous variables

$\boxed{F(x) = P(X \leq x)}$ ← cumulative distribution function

↳ expresses the prob. that X does not exceed x.

Probability Density Function: $f(x)$

→ $f(x) \geq 0$ for all x.

→ total area under the PDF curve is equal to 1.

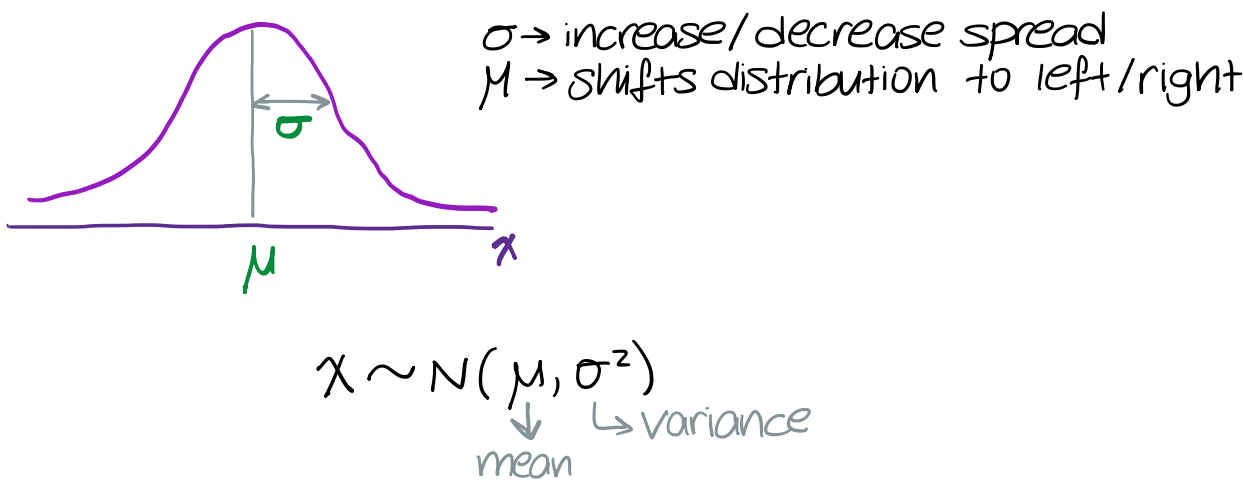→ the probability that x is between a and b is equal to area under curve bounded by $a-b$.

Cumulative Density Func. is area under the PDF curve from x=min to x_0.

$F(b) = P(X < b)$

$F(a) = P(X < a)$

since distr. is symmetrical
$P(x<a) = P(x \geq -a)$

$P(a < X < b) = F(b) - F(a)$

Normal Distribution (Gaussian)

→ bell shaped, symmetrical.

(Mean = Median = Mode)
(μ) → location is based

Spread is determined by σ (standard devi)

σ → increase/decrease spread
μ → shifts distribution to left/right

$X \sim N(\mu, \sigma^2)$
       ↓   ↳ variance
      mean

a. Two Normal Distributions with Same Variance but Different Means
b. Two Normal Distributions with Different Variances and Mean = 5

z-score: $\boxed{z = \frac{x - \mu}{\sigma}}$ generally → $\frac{\text{measurement} - \text{mean}}{\text{standard deviation}}$

$z_1 = \frac{x_1 - \mu}{\sigma}$

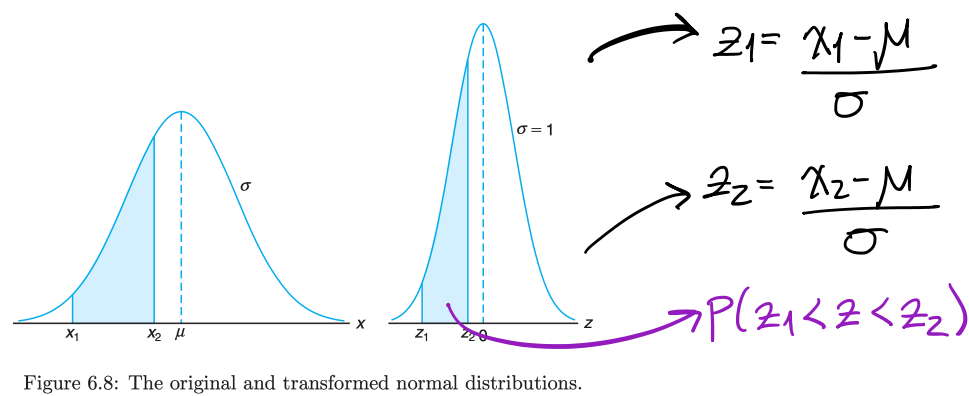$z_2 = \frac{x_2 - \mu}{\sigma}$

→ $P(z_1 < z < z_2)$

Figure 6.8: The original and transformed normal distributions.

Example

Aluminum sheets used to make beverage cans have thicknesses (in thousandths of an inch) that are normally distributed with mean 10 and standard deviation 1.3.

a) A particular sheet is 10.8 thousandths of an inch thick. Find the z-score.

b) The thickness of a certain sheet has a z-score of −1.7. Find the thickness of the sheet in the original units of thousandths of inches.

$F(x) = P(X \leq x)$, $z = \frac{x - \mu}{\sigma}$

$\mu = 10$, $\sigma = 1.3$

a) $z = \frac{10.8 - 10}{1.3} = 0.615$

b) $z = -1.7 = \frac{x - 10}{1.3}$ ⤳ $-2.21 = x - 10$
                                        $x = 7.79$

Example:

A random variable has a Normal distribution with variance 100. Find its mean if the probability that it will take on a value less than 77.5 is 0.8264.

σ^2
$\sigma = 10$

$F(x) = P(X < x)$ ⤳ $\boxed{P(X \leq 77.5) = 0.8264}$
                              then convert to z-score.
↳ $P(Z < z)$
$= F(z$-score$)$ $P\left(Z \leq \frac{77.5 - \mu}{10}\right) = 0.8264$
                                                        find it in
                          from table   table
                          0.94 ⤳ $\mu = 68.1$

Example:

Example 6.6: Given a normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of x that has
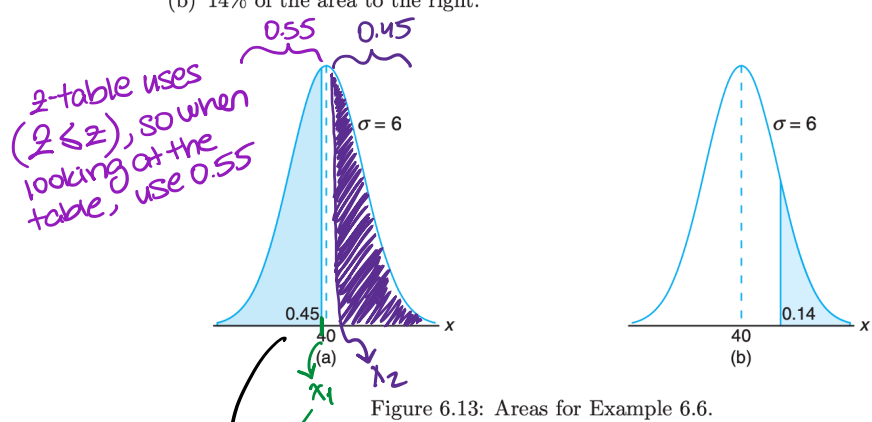(a) 45% of the area to the left and
(b) 14% of the area to the right.

z-table uses
$(z \leq z)$, so when looking at the table, use 0.55

0.55   0.45

0.45   0.14

Figure 6.13: Areas for Example 6.6.

! if z-score is to the left of the mean, it will be negative.

$P(X < x) = 0.45$
         find x

① convert to z-score

$0.45 = P\left(Z \leq \frac{x - 40}{6}\right) = 0.13$ from table

② symmetry → $\frac{40 - x}{6} = 0.13$