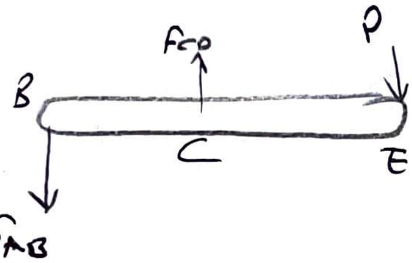
The Answers of Midterm ME 208

1) F.B.D



$\circlearrowleft + \sum M_B = 0 \Rightarrow 0,3 F_{CD} - 0,75P = 0$

$\boxed{P = \frac{2}{5} F_{CD}}$

$\circlearrowleft + \sum M_C = 0 \Rightarrow 0,3 F_{AB} - 0,45P = 0$

$\boxed{P = \frac{2}{3} F_{AB}}$

Note: Both links have the same area, pin diameter and material. Therefore, they have the same ultimate load.

Failure by pin in single shear.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.025)^2 = 490.9 \times 10^{-6}\, m^2$

$F_U = \tau_U A = (210 \times 10^6)(490,9 \times 10^{-6}) = \boxed{103,09\ kN} \Rightarrow$ choose smaller F_U.

failure by tension in link.

$A = (b-d)t = (0,05 - 0,025)0.012 = 3 \times 10^{-4}\, m^2$

$F_U = \sigma_U A = (490 \times 10^6)(3 \times 10^{-4}) = 147\ kN$

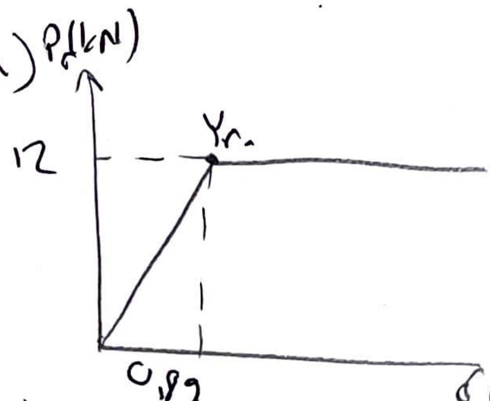
$F_{all} = \frac{F_U}{F.S} = \frac{103,09}{3} = 34,36\ kN$

Allowable load for structure is the smaller of $\frac{2}{3} F_{all}$ and $\frac{2}{5} F_{all}$

$P = \frac{2}{5}(34,36) \Rightarrow \boxed{P = 13,7\ kN}$

2)

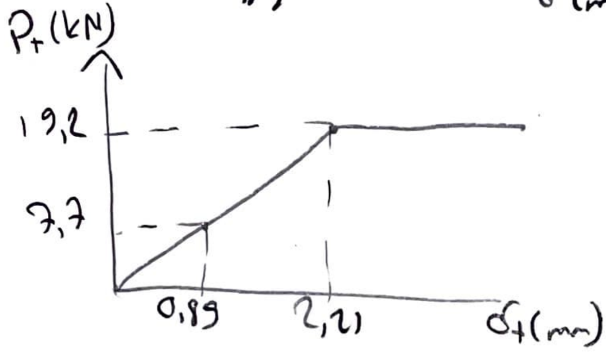
a)



$P_{y_r} = \sigma_y A = (250\times10^6)(48\times10^{-6}) = \boxed{12\,kN}$

$\varepsilon_{yr} = \frac{\sigma_{yr}}{E_{y,r}} = \frac{250\times10^6}{210\times10^9} = 0,00119$

$\delta_{yr} = \varepsilon_{yr} L = 0,00119\times 0,75 = 0,89\,mm$

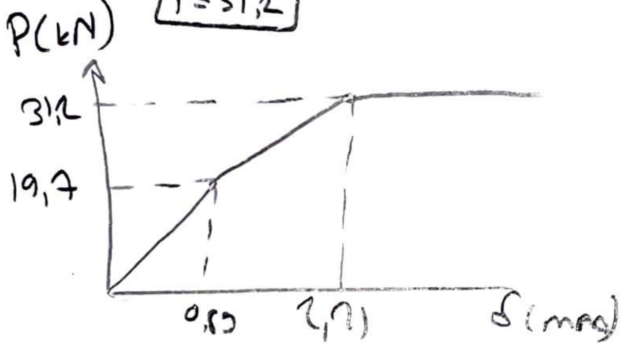


$P_{y,t} = \sigma_{y,t} A_t = 310\times10^6 \times 62\times10^{-6} = \boxed{19,2\,kN}$

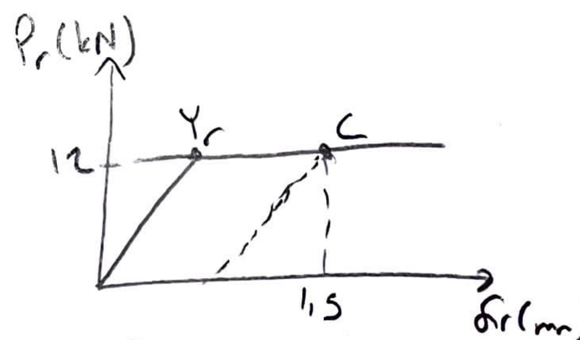
$\delta_{y,t} = \varepsilon_{y,t} L = \frac{\sigma_{y,t}}{E_{y,t}} L = \frac{310\times10^6}{105\times10^9}(0,75\times10^3)$
$= 2,21\,mm$

$P = P_r + P_t \qquad \delta = \delta_r = \delta_t$

$\boxed{P = 31,2}$



b) $P_r = P_{y,r} = 12\,kN \qquad P_T = 25\,kN$



$P_t = P - P_r = (25-12) = 13\,kN$

$\sigma_t = \frac{P_t}{A_t} = \frac{13\times10^3}{62\times10^{-6}} = 210\,MPa$

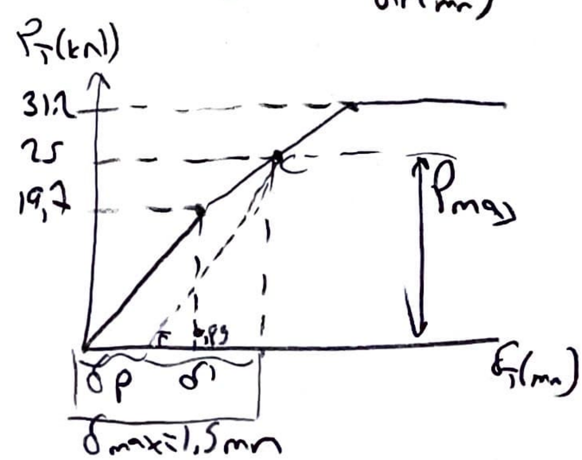
$\delta_t = \varepsilon_t L = \frac{\sigma_t}{E} L = \frac{210\times10^6}{105\times10^9}(0,75) = 1,5\,mm$

$\boxed{\delta_{max} = \delta_t = 1,5\,mm}$

c) slope of linear line $\Rightarrow \frac{19,7}{0,89} = 22,1\,kN/mm$

MOY = MCE $\Rightarrow \frac{25}{\delta'} = 22,1 \Rightarrow \delta' = 1,131\,mm$

$\delta_p = \delta_{max} - \delta' = 1,5 - 1,131 = \boxed{0,369\,mm}$



3)

Solid rod BC: $\tau = \frac{Tc}{J}$ $\quad J = \frac{\pi}{2}c^4$

$\tau_{all} = 25 \times 10^6\,Pa \quad c = \frac{1}{2}d = 0{,}015\,m$

$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0{,}015)^3(25\times10^6) = 132{,}536\ N.m$

Hollow rod AB: $\tau_{all} = 50\times10^6\,Pa$ $\quad T_{all} = 132{,}536$

$c_1 = ?$

$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0{,}075) = 0{,}0175\,m$

$T_{all} = \frac{J\tau_{all}}{c_2} = \frac{\pi}{2}(c_2^4 - c_1^4)\frac{\tau_{all}}{c_2}$

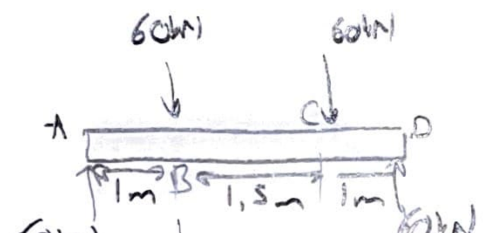
$c_1^4 = c_2^4 - \frac{2T_{all}c_2}{\pi\tau_{all}} \Rightarrow c_1^4 = 0{,}0175^4 - \frac{2(132{,}536)(0{,}0175)}{\pi(50\times10^6}$

$= 3{,}3203\times10^{-9}\,m^4$

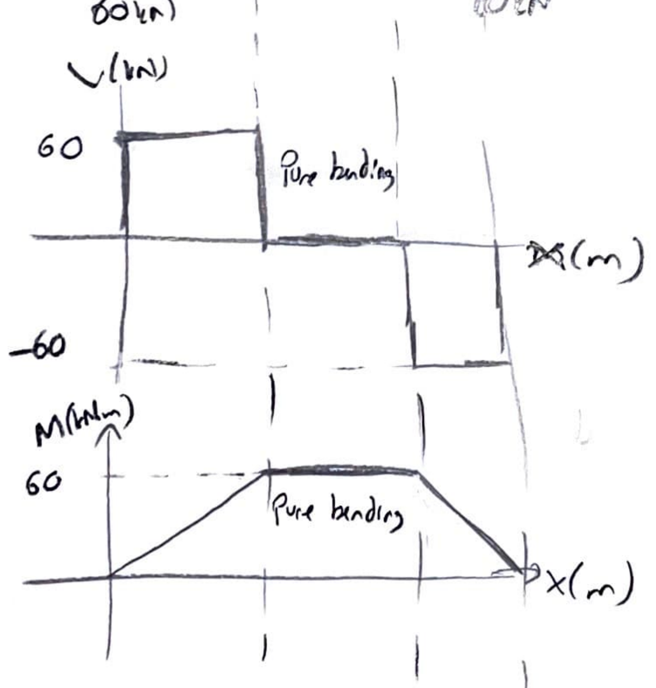
$c_1 = 7{,}59\times10^{-3}\,m = 7{,}59\,mm$

$\boxed{d_1 = 2c_1 = 15{,}18\ mm}$

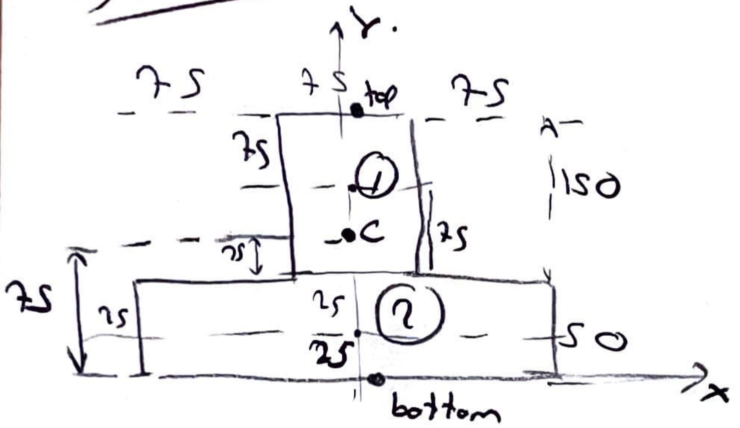
4)



M => bending moment = $\boxed{60\ kNm}$



4) Continued



	A	$\bar{y}_0$	$A \cdot \bar{y}_0$
1	11250	125	1406250
2	11250	25	281250
Σ	22500		1687500

$$\bar{Y}_0 = \frac{1687500}{22500} = 75 \text{ mm}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(0{,}075)(0{,}15)^3 + (0{,}01125)(0{,}05)^2 = 49{,}22 \times 10^{-6} \text{ m}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(0{,}225)(0{,}05)^3 + (0{,}01125)(0{,}05)^2 = 30{,}45 \times 10^{-6} \text{ m}^4$$

$$I = I_1 + I_2 = 79{,}7 \times 10^{-6} \text{ m}^4$$

tensile stress occurs at the bottom
Compressive stress occurs at the top

$$\sigma_{top} = -\frac{Mc}{I} = \frac{(60000)(0{,}125)}{79{,}7 \times 10^{-6}} = -94{,}1 \text{ MPa (compression)}$$

$$\sigma_{bot} = \frac{Mc}{I} = \frac{(60000)(0{,}075)}{79{,}7 \times 10^{-6}} = 56{,}5 \text{ MPa (tension)}$$