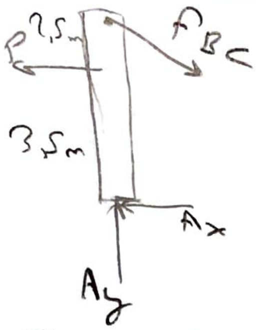
The Answers of Midterm ME 208-1

1) FBD



$L_{BC} = \sqrt{6^2 + 4^2} = 7{,}2111\ m$

$\Sigma M_A = 0 \Rightarrow 3{,}5P - 6\left(\frac{4}{7{,}2111} F_{BC}\right) = 0$

$P = 0{,}9509 F_{BC}$

Considering $\sigma_{all} = 190 \times 10^6\ Pa$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0{,}004)^2 = 12{,}566 \times 10^{-6}\ m^2$

$\sigma = \frac{F_{BC}}{A} \Rightarrow F_{BC} = \sigma A = (190 \times 10^6)(12{,}566 \times 10^{-6}) = \boxed{2{,}388 \times 10^3\ N}$

Considering allowable elongation $= \delta = 6 \times 10^{-3}\ m$

$\delta = \frac{F_{BC} L_{BC}}{AE} \Rightarrow F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12{,}566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7{,}2111}$

$= \boxed{2{,}091 \times 10^3 N}$

Choose smaller $F_{BC} \Rightarrow \boxed{F_{BC} = 2{,}091 \times 10^3 N}$

$P = 0{,}9509 F_{BC} = 0{,}9509(2.091 \times 10^3) = \boxed{1{,}988 \times 10^3\ N = 1{,}988\ kN}$

2)

A to C

$E = 200\times10^{9}\ Pa$

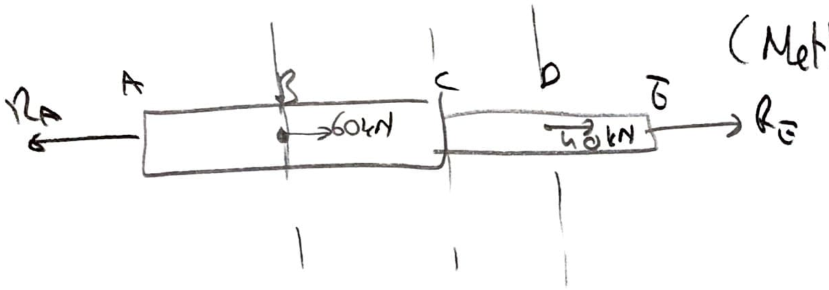
$A = \frac{\pi}{4}(40^2) = 1,25664\times10^{3}\ mm^2 = 1,25664\times10^{-3}\ m^2$

C to E

$E = 105\times10^{9}\ Pa$

$A = \frac{\pi}{4}(30)^2 = 706,86\ mm^2 = 706,86\times10^{-6}\ m^2$

(Method of Section)



A to B:

$P = R_A \qquad L = 180\ mm = 0,18\ m$

$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0,18)}{200\times10^{9}\times1,25664\times10^{-3}} = 716,20\times10^{-12}R_A$

B to C:

$P = R_A - 60\times10^{3} \qquad L = 120\ mm = 0,12\ m$

$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0,12)}{200\times10^{9}\times1,25664\times10^{-3}} = 477,47\times10^{-12}R_A - 28,648\times10^{-6}$

C to D;

$P = R_A - 60\times10^{3} \qquad L = 100\ mm = 0,1\ m$

$\delta_{\cancel{B}D} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0,1)}{105\times10^{9}\times706,86\times10^{-6}} = 1,34735\times10^{-9}R_A - 80,841\times10^{-6}$

D to E

$P = R_A - 100\times10^{3} \qquad L = 100\ mm = 0,1\ m$

$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^{3})(0,1)}{105\times10^{3} - 706,86\times10^{-6}} = 1,34735\times10^{-9}R_A - 134,735\times10^{-6}$

2) Continued

A to E:

$$\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3{,}85837 \times 10^{-9} R_A - 242{,}424 \times 10^{-6}$$

Since poin E can't move relative to A; $\delta_{AE} = 0$

a) $R_A 3{,}85837 \times 10^{-9} - 242{,}424 \times 10^{-6} = 0 \Rightarrow \boxed{R_A = 62{,}831 \times 10^3\,N = 62{,}8\,kN} \leftarrow$

$R_E = R_A - 100 \times 10^3 = 62{,}8 \times 10^3 - 100 \times 10^3 = \boxed{-37{,}2 \times 10^3\,N = -37{,}2\,kN} \leftarrow$

b) $\delta_C = \delta_{AB} + \delta_{BC} = 1{,}16367 \times 10^{-9} R_A - 26{,}848 \times 10^{-6}$

$= 1{,}16367 \times 10^{-9}(62{,}831 \times 10^3) - 26{,}848 \times 10^{-6}$

$= 46{,}3 \times 10^{-6}\,m \Rightarrow \boxed{\delta_C = 46{,}3\,\mu m \rightarrow}$

3) Torques

$T_{AB} = 300 + 500 = 800\,N.m$

$T_{BC} = 500\,N.m, \quad T_{CD} = 0$

Design based on stress $\tau = 60 \times 10^6\,Pa$

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau} = \frac{2(800)}{\pi(60 \times 10^6)} = 8{,}488 \times 10^{-6}\,m^3$

$c = 20{,}40 \times 10^{-3}\,m = 20{,}4\,mm, \quad d = 2c = 40{,}8\,mm$

3) Continued

Design based on deformation $\varphi_{D/A} = 1,5° = 26,18\times10^{-3}$ rad.

$\varphi_{D/C} = 0$

$$\varphi_{CB} = \frac{T_{BC}L_{BC}}{GJ} = \frac{(500)(0,6)}{GJ} = \frac{300}{GJ}$$

$$\varphi_{B/A} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(800)(0,4)}{GJ} = \frac{320}{GJ}.$$

$$\varphi_{D/A} = \varphi_{D/C} + \varphi_{C/B} + \varphi_{B/A} = \frac{620}{GJ} = \frac{620}{G\frac{\pi}{2}c^4} = \frac{2(620)}{\pi G c^4}$$

$$c^4 = \frac{2(620)}{\pi G \varphi_{B/A}} = \frac{2(620)}{\pi(77\times10^9)(26,18\times10^{-3})} = 195,80\times10^{-9}\,m^4$$

$C = 21,04\times10^{-3}\,m = 21,04\,mm$, $d = 2c = 42,1\,mm$

Design must use larger value of $d \Rightarrow \boxed{d = 42,1\,mm}$

4)

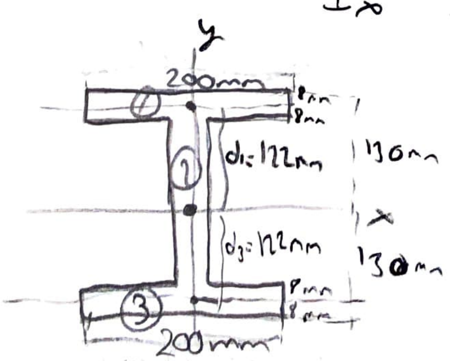
$$I_x = \overset{②}{\frac{1}{12}bh^3} + \overset{①}{Ad_1^2} + \overset{③}{Ad_3^2}$$

$$I_x = \frac{1}{12}(10)(228)^3 + (200\times16)(122)^2 + (100\times16)(122)^2 = 105,771\times10^{-6}\,m^4$$

$C = \frac{260}{2} = 130\,mm = 0,13\,m$

$$\sigma_{all} = \frac{\sigma_u}{F.S} = \frac{400}{2,5} = 160\,MPa = 160\times10^6\,Pa$$

$$\sigma_{all} = \frac{M_x c}{I_x} \qquad M_x = \frac{I_x\sigma_{all}}{c} = \frac{(105,771\times10^{-6})(160\times10^6)}{0,13}$$



$= \boxed{129,564\times10^3\,N.m} = \boxed{129,6\,kN.m}$