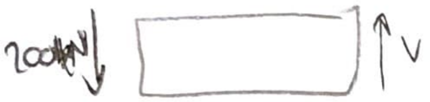
The Answers of final ME 208

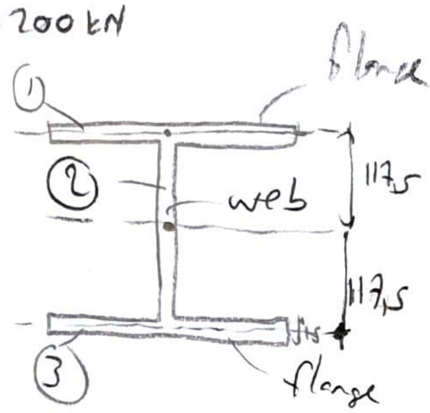
1)

for n-n section

$+\uparrow \Sigma F_y = 0$



$-200 + V = 0 \qquad V = 200\,kN$



$I = I_1 + I_2 + I_3$

$I_1 = I_3 = \frac{1}{12} b h^3 + A d^2$

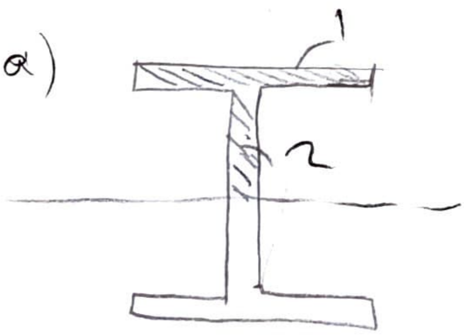
$I_1 = \frac{1}{12} \times 250 \times 15^3 + 250 \times 15 \times 117{,}5^2 = 51{,}844 \times 10^6\, mm^4$

$I_3 = I_1 = 51{,}844 \times 10^6\, mm^4$

$I_2 = \frac{1}{12} b h^3 = \frac{1}{12} \times 10 \times 220^3 = 8{,}873 \times 10^6\, mm^4$

$I = 2(51{,}844 \times 10^6) + 8{,}873 \times 10^6 = 112{,}56 \times 10^6\, mm^4$

a)



$t = 10\,mm$

The largest shear stress occurs at the center of the beam.

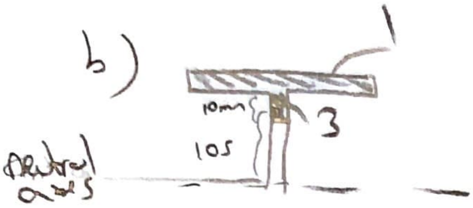
Thus; $Q = A_1 \bar{y}_1 + A_2 \bar{y}_2$

$= 15 \times 250 \times 117{,}5 + 110 \times 10 \times 55 = 501125\, mm^3$

$\tau_{max} = \frac{VQ}{It} = \frac{(200 \times 10^3)(501125 \times 10^{-9})}{(112{,}56 \times 10^{-6})(0{,}01)} = \boxed{89\,MPa}$

1) Continued

b)



$Q_a = A_1\bar{y}_1 + A_3\bar{y}_3$

$= 250\times15\times117{,}5 + 10\times10\times105 = 451125\ mm^3$

$\tau_a = \frac{VQ_a}{It} = \frac{(200\times10^3)(451125\times10^{-9})}{(112{,}56\times10^{-6})(0{,}01)} = \boxed{80{,}1\ MPa}$

2) $\sigma_x = 126\ MPa$

$\sigma_y = 0$

$\tau_{xy} = -84\ MPa$

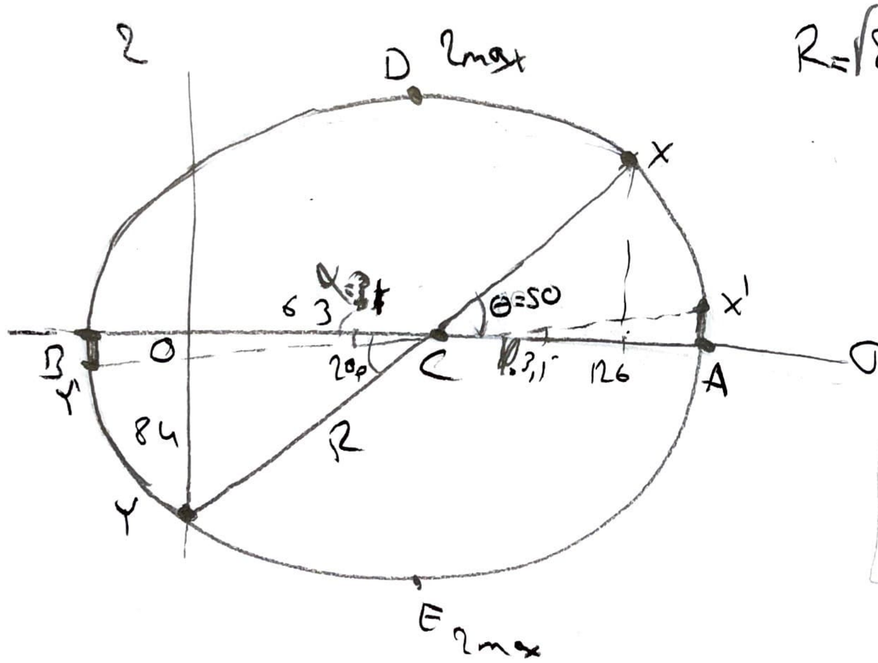
$\sigma_{ave} = \frac{126+0}{2} = 63\ MPa$

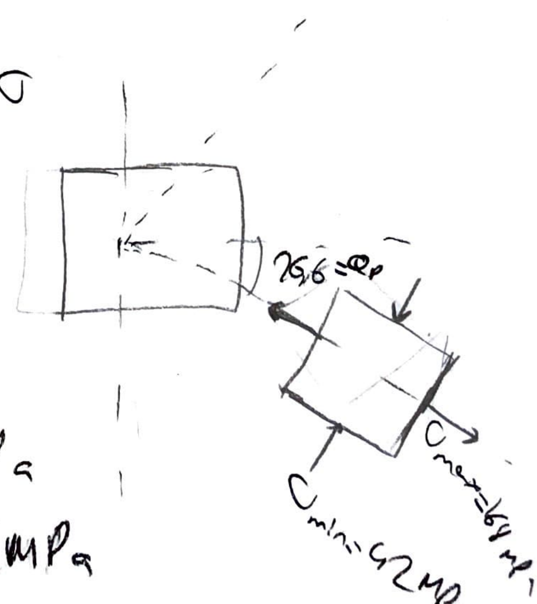
Point $X = (\sigma_x, -\tau_{xy}) = (126, 84)$

Point $Y = (\sigma_y, \tau_{xy}) = (0, -84)$

a)

$R = \sqrt{84^2 + 63^2} = 105$





b) $\sigma_a = OA = OC + CA = 63 + 105 = 168\ MPa$

$\sigma_b = OB = OC - CB = 63 - 105 = -42\ MPa$

$\tan 2\theta_p = \frac{84}{63} = 1{,}33 \Rightarrow 2\theta_p = 53{,}1°$

$\theta_p = 26{,}6°$

$\boxed{\tau_{max} = R = 105\ MPa}$ $\theta_s = \theta_p + 45° = 26{,}6 + 45 = 71{,}6°$

2) Continued

c) $\theta = 25°$ ↻

$\phi = 2\theta_p - 2\theta = 53{,}1 - 50 = 3{,}1$

$\sigma_{y'} = \sigma_{ave} - R\cos 3{,}1 = 63 - 104{,}8 = -41{,}8\ MPa$

$\sigma_{x'} = \sigma_{ave} + R\cos 3{,}1 = 63 + 104{,}8 = 167{,}8\ MPa$

$\tau_{x'y'} = -R\sin\phi = 105\sin 3{,}1 = -5{,}68\ MPa$

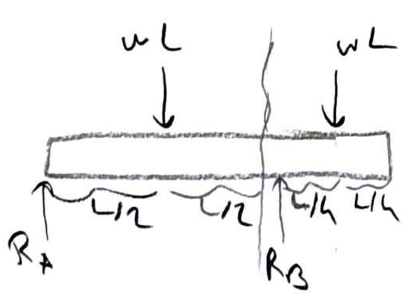
d) Von Mises Criterion

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \left(\frac{\sigma_y}{F.S}\right)^2$$

$$168^2 - 168(-42) + (-42)^2 = \left(\frac{250}{F.S}\right)^2$$

$$\boxed{F.S = 1.3}$$

3) FBD

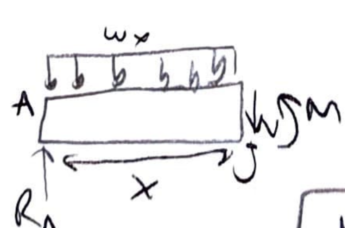


$+\circlearrowleft \Sigma M_B = 0$

$-R_A L + (wL)\left(\frac{L}{2}\right) - (wL)\left(\frac{L}{4}\right) = 0$

$R_A = \frac{1}{4} wL$

For portion AB, $(0 < x < L)$



$+\circlearrowleft \Sigma M_J = 0$

$M - R_A x + wx\left(\frac{x}{2}\right) = 0$

$$\boxed{M = \frac{1}{4} wLx - \frac{1}{2} wx^2}$$

3) Continued

Boundary Conditions

at A $\Rightarrow$ $[x=0,\ y=0]$

at B $\Rightarrow$ $[x=L,\ y=0]$

$$EI\frac{d^2y}{dx^2} = \frac{1}{4}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{8}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{24}wLx^3 - \frac{1}{24}wx^4 + C_1x + C_2$$

at A $\Rightarrow$ $0 = 0 - 0 + 0 + C_2 \Rightarrow C_2 = 0$

at B $\Rightarrow$ $0 = \frac{1}{24}wL^4 - \frac{1}{24}wL^4 + C_1L + 0 \Rightarrow C_1 = 0$

a) Elastic curve $(0 \leq x \leq L)$

$$y = \frac{w}{24EI}(Lx^3 - x^4)$$

b) $\theta = \frac{dy}{dx} = \frac{w}{24EI}(3Lx^2 - 4x^3)$

at A $x=0$ thus $\theta_A = \frac{dy}{dx} = \boxed{0}$

c) at B $x=L$ thus $\theta_B = \frac{dy}{dx} = \boxed{-\frac{wL^3}{24EI}}$

4)

a) Brass strut

$I = \frac{1}{12}(20)(20)^3 = 13{,}333 \times 10^{-9}\ m^4$

$P_{cr} = \frac{\pi^2 E_b I_b}{L^2} = \frac{\pi^2 (110 \times 10^9)(13{,}333 \times 10^{-9})}{(1{,}1)^2}$

$\boxed{P_{cr} = 13{,}06\ kN}$

b) Aluminum strut

$P_{cr} = \frac{\pi^2 E_a I_a}{L^2} = \frac{\pi^2 E_a (d^4/12)}{L^2}$

$d^4 = \frac{12 P_{cr} L^2}{\pi^2 E_a} = \frac{12(13{,}06 \times 10^3)(1{,}1)^2}{\pi^2 (70 \times 10^9)} = 274{,}3 \times 10^{-9}\ m^4$

$d = 22{,}9 \times 10^{-3}\ m = \boxed{22{,}9\ mm}$