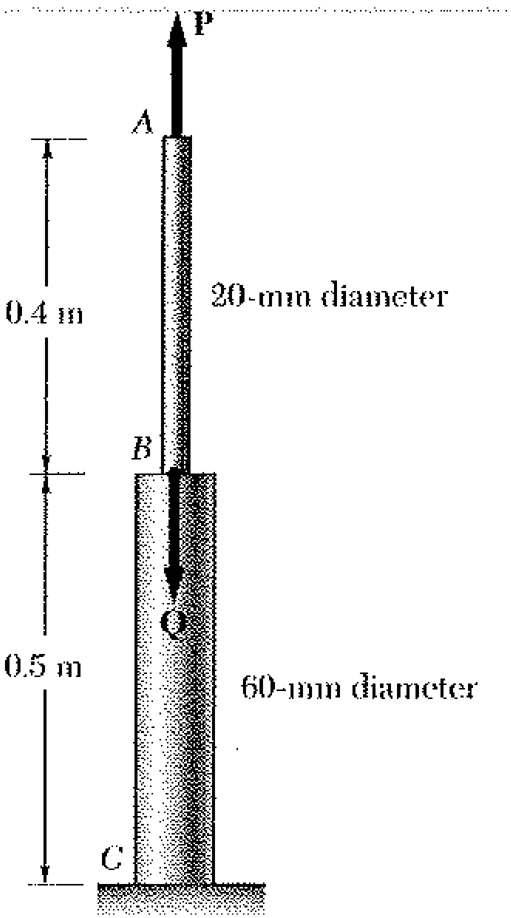
# Problem 1




The rod *ABC* is made of an aluminum for which $E = 70$ GPa. Knowing that $P = 6$ kN and $Q = 42$ kN, determine the deflection of (*a*) point *A*, (*b*) point *B*:

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$P_{AB} = P = 6 \times 10^3 \text{ N}$$

$$P_{BC} = P - Q = 6\times 10^3 - 42 \times 10^3 = -36 \times 10^3 \text{ N}$$

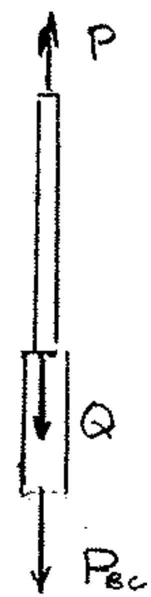
$$L_{AB} = 0.4 \text{ m} \qquad L_{BC} = 0.5 \text{ m}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6\times 10^3)(0.4)}{(314.16\times 10^{-6})(70\times 10^9)}$$

$$= 109.135 \times 10^{-6} \text{ m}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36\times 10^3)(0.5)}{(2.8274\times 10^{-3})(70 \times 10^9)}$$

$$= -90.947 \times 10^{-6} \text{ m}$$

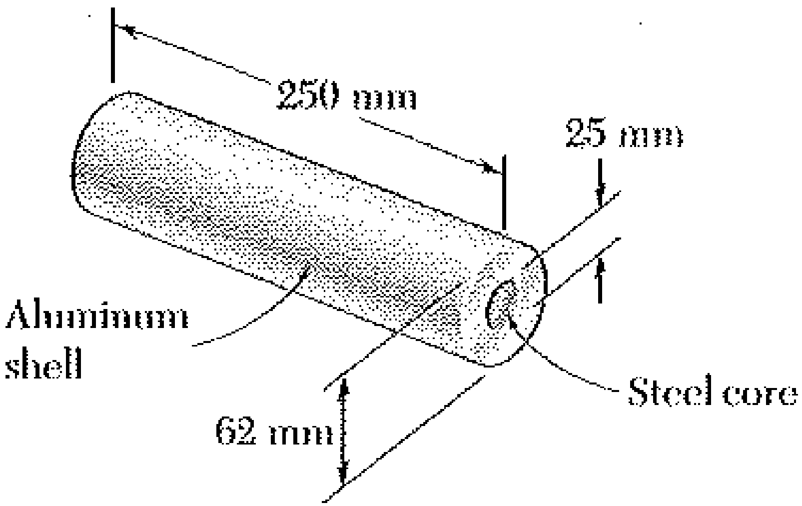



(a) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6} \text{ m} = 18.19 \times 10^{-6} \text{ m}$

$= 0.01819 \text{ mm} \uparrow$

(b) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6} \text{ m} = -0.0909 \text{ mm}$ or $0.0919 \text{ mm} \downarrow$

# Problem 2




Compressive centric forces of 160 kN are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_s = 200$ GPa and $E_a = 70$ GPa, determine (*a*) the normal stresses in the steel core and the aluminum shell, (*b*) the deformation of the assembly.

Let $P_a$ = portion of axial force carried by shell.
$P_c$ = portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_s L}{E_c A_c} \qquad P_s = \frac{E_s A_s}{L}\delta$$

Data: $P = 160$ kN
$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.062^2 - 0.025^2) = 0.002528$ m
$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.025)^2 = 0.000491$ m²

Total force $P = P_a + P_s = (E_a A_a + E_s A_s)\frac{\delta}{L}$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_a A_a + E_s A_s}$$

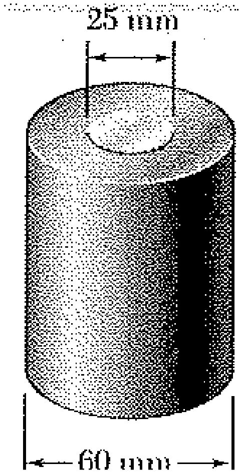
$$\varepsilon = \frac{-160000}{(70\times10^9)(0.002528) + (200\times10^9)(0.000491)} = -581.5\times10^{-6}$$

(a) $\sigma_s = E_s\varepsilon = (200\times10^9)(-581.5\times10^{-6}) = -116.3$ MPa

$\sigma_a = E_a\varepsilon = (70\times10^9)(-581.5\times10^{-6}) = -40.7$ MPa

(b) $\delta = L\varepsilon = (0.25)(-581.5\times10^{-6}) = -145\times10^{-6}$ m $= -0.145$ mm

# Problem 3




Brass core
$E = 105$ GPa
$\alpha = 20.9 \times 10^{-6}/°C$

Aluminum shell
$E = 70$ GPa
$\alpha = 23.6 \times 10^{-6}/°C$

The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15 °C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195 °C.

Let L be the length of the assembly.

Free thermal expansion.

$\Delta T = 195 - 15 = 180°C$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

Aluminum shell: $(\delta_T) = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core. $\delta = L(\alpha_a - \alpha_b)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Brass core: $E_b = 105 \times 10^9$ Pa

$A_b = \frac{\pi}{4}(25)^2 = 490.87\ mm^2 = 490.87 \times 10^{-6}\ m^2$

$(\delta_P)_b = \frac{PL}{E_b A_b}$

Aluminum shell: $E_a = 70 \times 10^9$ Pa

$A_a = \frac{\pi}{4}(60^2 - 25^2) = 2.3366 \times 10^3\ mm^2 = 2.3366 \times 10^{-3}\ m^2$

# Problem 3

$$\delta = (\delta_p)_b + (\delta_p)_a \qquad L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = KPL$$

where $K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a} = \frac{1}{(105\times10^9)(490.87\times10^{-6})} + \frac{1}{(70\times10^9)(2.3366\times10^{-3})}$

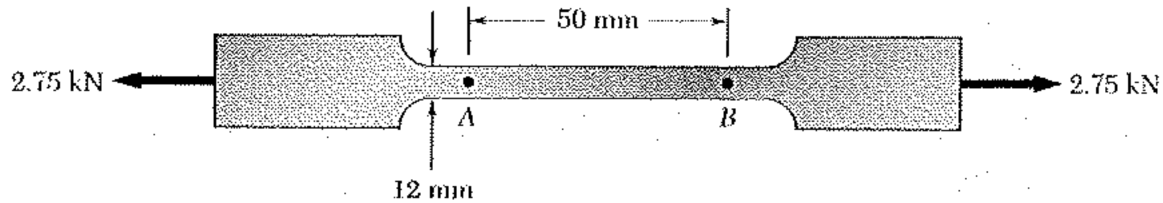
$$= 25.516\times10^{-9} \quad N^{-1}$$

Then $\quad P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K} = \frac{(23.6\times10^{-6} - 20.9\times10^{-6})(180)}{25.516\times10^{-9}} = 19.047\times10^3 \text{ N}$

<u>Stress in aluminum.</u> $\quad \sigma_a = -\frac{P}{A_a} = -\frac{19.047\times10^3}{2.3366\times10^{-3}} = -8.15\times10^6 \text{ Pa}$

# Problem 4

A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*




$$A = (1.6)(12) = 19.2\ mm^2 = 19.2 \times 10^{-6}\ m^2$$

$$P = 2.75 \times 10^3\ N$$

$$\sigma_x = \frac{P}{A} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6\ Pa$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\nu \varepsilon_x$$

$$-(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

(a) $L = 0.050\ m$ $\quad \delta_x = L\varepsilon_x = (0.050)(716.15 \times 10^{-6}) = 35.81 \times 10^{-6}\ m$

$0.0358\ mm$

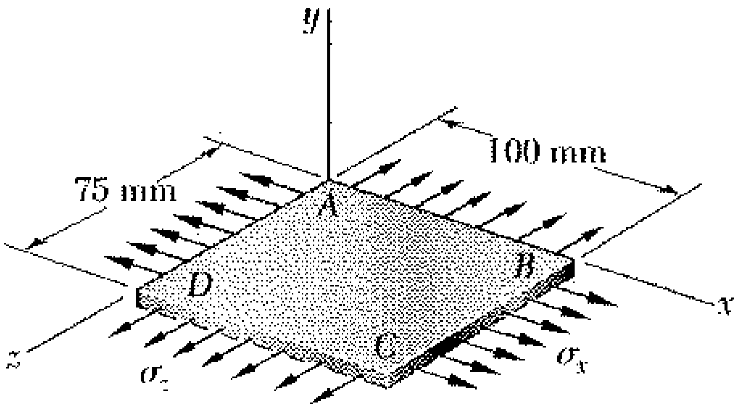
(b) $w = 0.012\ m$ $\quad \delta_y = w\varepsilon_y = (0.012)(-214.84 \times 10^{-6}) = -2.578 \times 10^{-6}\ m$

$-0.00258\ mm$

(c) $t = 0.0016\ m$ $\quad \delta_z = t\varepsilon_z = (0.0016)(-214.84 \times 10^{-6}) = -343.7 \times 10^{-9}\ m$

$-0.0003437\ mm$

# Problem 5




A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*

$$\sigma_x = 120\times10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160\times10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{87\times10^9}\left[120\times10^6 - (0.34)(160\times10^6)\right]$$

$$= 754.02\times10^{-6}$$

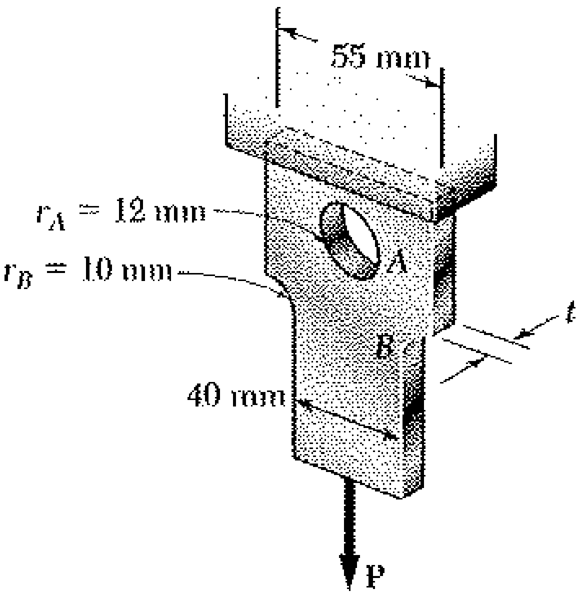
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87\times10^9}\left[-(0.34)(120\times10^6) + 160\times10^6\right]$$

$$= 1.3701\times10^{-3}$$

(a) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02\times10^{-6}) = 0.0754 \text{ mm}$

(b) $\delta_{BC} = (\overline{BC})\varepsilon_z = (.75 \text{ mm})(1.3701\times10^{-6}) = 0.1028 \text{ mm}$

# Problem 6




For $P = 35$ kN, determine the minimum plate thickness $t$ required if the allowable stress is 125 MPa.

At the hole: $r_A = 12\,mm$ $\quad d_A = 55 - 24 = 31$

$$\frac{r_A}{d_A} = \frac{12}{31} = 0.39$$

From Fig 2.64 a $\qquad K = 2.26$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{max}}$$

$$t = \frac{(2.26)(35000)}{(0.031)(125 \times 10^6)} = 0.0204\,m = 20.4\,mm$$

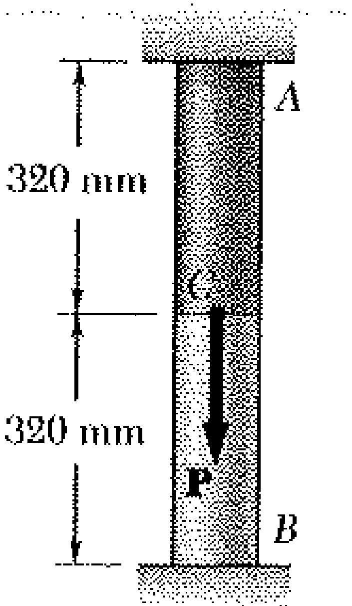
At the fillet $\quad D = 55\,mm \quad d_B = 40\,mm \quad \frac{D}{d_B} = \frac{55}{40} = 1.375 \quad \frac{r_B}{d_B} = \frac{10}{40} = 0.25$

From Fig 2.64 b $\quad K = 1.70 \quad \sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t} \quad t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(35000)}{(0.04)(125 \times 10^6)} = 0.0119\,m = 11.9\,mm$

The larger value is the required minimum plate thickness

$t = 20.4\,mm$

# Problem 7



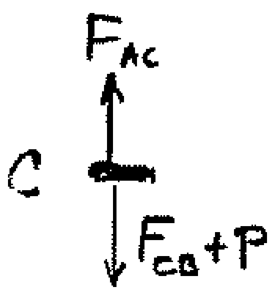

Rod $AB$ consists of two cylindrical portions $AC$ and $BC$, each with a cross-sectional area of 2950 mm$^2$. Portion $AC$ is made of a mild steel with $E$ = 200 GPa and $\sigma_Y$ = 250 MPa, and portion $CB$ is made of a high-strength steel with $E$ = 200 GPa and $\sigma_Y$ = 345 MPa. A load **P** is applied at $C$ as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of $C$ if $P$ is gradually increased from zero to 1625 kN and then reduced back to zero, (*b*) the permanent deflection of $C$.

Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\,\varepsilon_{Y,AC} = \frac{L_{AC}\,\sigma_{Y,AC}}{E} = \frac{(0.320)(250\times10^6)}{200\times10^9} = 0.400\times10^{-3}\ \text{m}$$

Corresponding force $\quad F_{AC} = A\,\sigma_{Y,AC} = (2950\times10^{-6})(250\times10^6) = 737.5\times10^3\ \text{N}$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(2950\times10^{-6})(0.400\times10^{-3})}{0.320} = -737.5\times10^3\ \text{N}$$




For equilibrium of element at C,

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 1475\times10^3\ \text{N}$$

Since applied load $P = 1625\times10^3\ \text{N} > 1475\times10^3\ \text{N}$, portion AC yields.

# Problem 7

$$F_{CB} = F_{AC} - P = 737.5\times10^3 - 1625\times10^3\,N = -887.5\times10^3\,N$$

(a) $$\delta_C = -\frac{F_{CB}L_{CB}}{EA} = \frac{(887.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.48136\times10^{-3}\,m$$
$$= 0.481\ mm \downarrow$$

Maximum stresses. $\sigma_{AC} = \sigma_{Y,AC} = 250\ MPa$

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{887.5\times10^3}{2950\times10^{-6}} = -300.81\times10^6\ Pa = -301\ MPa$$

(b) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

$$P' = 1625\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 812.5\times10^3\ N$$

$$\delta' = \frac{(812.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.44068\times10^3\ m$$

$$\delta_p = \delta_m - \delta' = 0.48136\times10^{-3} - 0.44068\times10^{-3} = 0.04068\times10^{-3}\,m$$

<u>Permanent deflection of C</u> $\qquad \delta_p = 0.0407\ mm \downarrow$