İZMİR UNIVERSITY OF ECONOMICS

IZMIR UNIVERSITY OF ECONOMICS
Faculty of Arts and Sciences

Term	: 2024-25 Spring
Course ID	: PHYS 102
Exam	: Midterm Exam
Date	: 16.04.2025
Duration	: 90 min
Instructor	:

SOLUTIONS

Full Name : ....

Student ID : ....

Classroom : .... **Section** : ....

<u>Information on exam rules</u>

Electronic devices such as laptops, mobile phones, and smartwatches are generally prohibited in the examination room. However, exceptions can be made for individuals with special needs, provided they have valid medical documentation. Requests for exceptions must be submitted with prior written approval from the academic advisor, and they should include details on the necessary measures to maintain the integrity and security of the examination.

Please refrain from engaging in cheating or any other prohibited activities during the examination. Suspected cheating may result in a score of zero on your exam, and any students found cheating may face disciplinary actions in accordance with law #2547. This includes actions such as using unauthorized electronic devices, communicating with classmates, exchanging exam or formula sheets, or using unauthorized written materials during the exam, all of which qualify as attempted cheating.

Students can use only simple calculators during the exam.

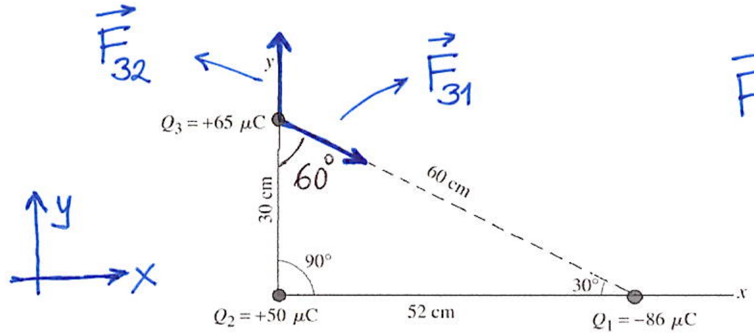
<u>Declaration</u>

I affirm that the activities and assessments completed as part of this examination are entirely my own work and comply with all relevant rules regarding copyright, plagiarism, and cheating. I acknowledge that if there is any question regarding the authenticity of any portion of my assessment, I may be subject to oral examination. The signatory of evidence records may also be contacted, or a disciplinary process may be initiated as per law #2547.

Signature of Student:

Question	Score
1	/ 20
2	/ 20
3	/ 20
4	/ 20
5	/ 20
Total	/100

1) Calculate the magnitude and direction of the net electrostatic force on charge $Q_3 = +65\mu C$ shown in the figure due to the charges $Q_1 = -86\ \mu C$ and $Q_2 = +50\ \mu C$. ($k = 9x10^9\ Nm^2/C^2$)



$$\vec{F}_{net} = \vec{F}_{31} + \vec{F}_{32}$$

$$= \frac{q_3}{4\pi\epsilon_0}\left\{\frac{q_1}{r_{31}^2}\hat{r}_{31} + \frac{q_2}{r_{32}^2}\hat{r}_{32}\right\}$$

$$\hat{r}_{31} = \hat{x}\sin 60° - y\cos 60° = \frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{y}$$

$$\hat{r}_{32} = \hat{y}$$

$$\vec{F}_{net} = F_x\hat{x} + F_y\hat{y}$$

$$= \frac{q_3}{4\pi\epsilon_0}\left\{\frac{q_1}{r_{31}^2}\frac{\sqrt{3}}{2}\hat{x} + \left(\frac{q_2}{r_{32}^2} - \frac{q_1}{r_{31}^2}\cdot\frac{1}{2}\right)\hat{y}\right\}$$

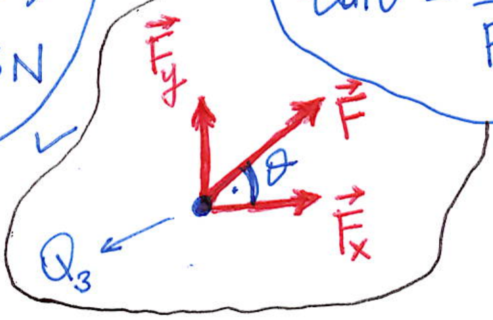
$$= (9\times10^9\ Nm^2C^{-2})(6.5\times10^{-5}C)\left\{\frac{\sqrt{3}}{2}\frac{8.6\times10^{-5}C}{(0.6m)^2}\hat{x}\right.$$

$$\left. + \left(\frac{5\times10^{-5}C}{(0.3m)^2} - \frac{1}{2}\cdot\frac{8.6\times10^{-5}C}{(0.6)^2 m^2}\right)\hat{y}\right\}$$

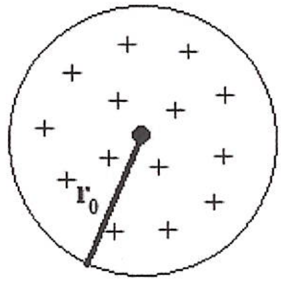
$$= 121\ N\hat{x} + 255N\hat{y} = F_x\hat{x} + F_y\hat{y}$$

$$F = (F_x^2 + F_y^2)^{1/2} = 282.25N$$

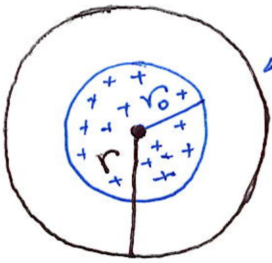
$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = 64.62°$$



2) An electric charge +Q is distributed uniformly throughout a **nonconducting** solid sphere of charge of radius r_0. Determine the electric field **(a)** outside the sphere ($r > r_0$) and **(b)** inside the sphere ($r < r_0$). (Calculate in terms of Q, r, r_0)



(a) Outside the sphere, $\vec{E}_{r>r_0} = ?$



Gaussian surface $(r>r_0)$

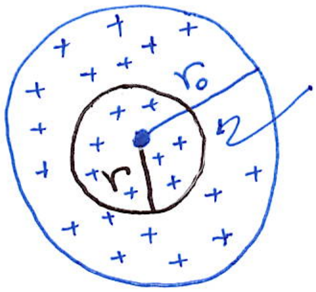
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

→ For $r>r_0$ we have $q_{encl} = Q$.

Therefore, $E \cdot (4\pi r^2) = \dfrac{Q}{\epsilon_0}$

$$E = E_{r>r_0} = \frac{Q}{4\pi\epsilon_0 r^2}$$ ✓

(b) Inside the sphere, $\vec{E}_{r<r_0} = ?$



Gaussian surface $(r<r_0)$

→ First, we need to find q_{encl} for the case $r<r_0$ where the charge density is

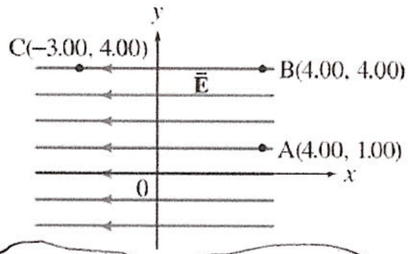
$$\rho = \frac{Q}{V} = \frac{Q}{\left(\frac{4}{3}\pi r_0^3\right)}$$

Therefore, $q_{encl} = \rho \cdot \frac{4}{3}\pi r^3 = \dfrac{Q r^3}{r_0^3}$.

→ Then, we have

$$E \cdot (4\pi r^2) = \frac{Q r^3}{\epsilon_0 r_0^3} \Rightarrow E = E_{r<r_0} = \frac{Q r}{4\pi\epsilon_0 r_0^3}$$ ✓

3) A uniform electric field $\vec{E}$ = -4.20 N/C $\hat{\imath}$ points in the negative x direction as shown in figure. The x and y coordinates of points A, B, and C are given on the diagram (in meters). Determine the differences in potential **(a)** V_{BA}, **(b)** V_{CB}, and **(c)** V_{CA}.



$$\vec{E} = 4.2\,\frac{N}{C}\,(-\hat{x})$$

(a) $$V_{BA} = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$$

$$d\vec{l} = (4-4)\hat{x} + (4-1)\hat{y} = 3\hat{y} \text{ (or } 3\hat{\jmath})$$

$$V_{BA} = -\int_A^B (4.2)(-\hat{\imath})\cdot 3(\hat{\jmath}) = 0$$

(b) $$V_{CB} = V_C - V_B = -\int_B^C \vec{E}\cdot d\vec{l}$$

$$d\vec{l} = (-3-4)\hat{x} + (4-4)\hat{y} = -7\hat{x} \text{ (or } -7\hat{\imath})$$

$$V_{CB} = -\int_B^C (4.2)(-\hat{\imath})\cdot(-7\hat{\imath}) = -29.4\,V$$

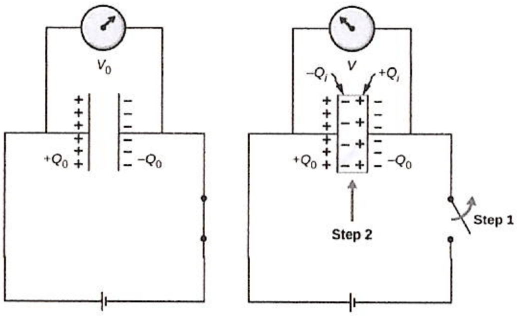
(c) $$V_{CA} = (V_C - V_B) + (V_B - V_A) = V_{CB} + V_{BA} = -29.4\,V$$

Alternatively, $$V_{CA} = -\int_A^C \vec{E}\cdot d\vec{l}$$

$$d\vec{l} = (-3-4)\hat{x} + (4-1)\hat{y} = -7\hat{x} + 3\hat{y}$$

$$= -\int_A^C (4.2(-\hat{\imath}))\cdot(-7\hat{\imath}+3\hat{\jmath}) = -29.4\,V$$

4) An empty 20.0-pF capacitor is charged to a potential difference of 40.0 V. The charging battery is then disconnected, and a piece of Teflon™ with a dielectric constant of 2.1 is inserted to completely fill the space between the capacitor plates (see figure). What are the values of **(a)** the capacitance, **(b)** the charge of the plate, **(c)** the potential difference between the plates, and **(d)** the energy stored in the capacitor with and without dielectric?



$C_0 = 20\,pF \;,\; V_0 = 40V$

(a) $C = \kappa C_0 = (2.1)(20) = 42\,pF$

(b) Without dielectric, the charge on the plates is

$$Q_0 = C_0 V_0 = (20)(40) = 0.8\,nC$$

→ Since the battery is disconnected before the dielectric is inserted, the plate charge is unaffected by the dielectric and remains at 0.8 nC.

(c) With dielectric $\Rightarrow V = \frac{1}{\kappa} V_0 = \frac{40}{2.1} \cong 19V$

(d) Without dielectric $\Rightarrow U_0 = \frac{1}{2} C_0 V_0^2 = 16\,nJ$

With dielectric

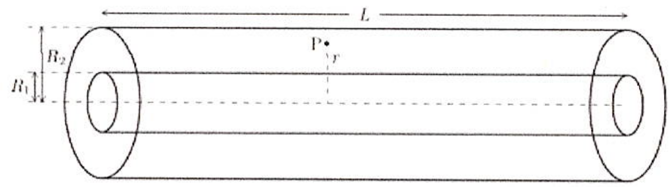
$$\Rightarrow U = \frac{1}{\kappa} U_0 = \frac{16}{2.1} = 7.6\,nJ$$

5) A cylindrical capacitor consists of two long, coaxial, metal cylinders of radii R_1 and R_2 ($R_1<R_2$) and the same length L, as shown Fig. Assume that the inner cylinder has a total charge of $+Q$ and the outer cylinder has a total charge of $-Q$. (Calculate in terms of Q, L, R)

(a) Use Gauss's Law to calculate the electric field at the point P in the diagram below, which is a distance r away from the axis ($R_1 < r < R_2$).

(b) Calculate the potential difference between the outer and the inner cylindrical shells.

(c) Determine the capacitance of the cylindrical capacitor.



(a) $$\oint_S \vec{E}\cdot\hat{n}\,dA = E\,(2\pi r L) = \frac{q_{encl}}{\epsilon_0}$$

→ Here we have $R_1 < r < R_2$, therefore, $q_{encl} = Q$.

The electric field between the cylinders is

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\,\frac{Q}{rL}\,\hat{r}$$ ($\hat{r}$ is the unit radial vector along the radius of the cylinder).

(b) $$V = -\int_{R_2}^{R_1} \vec{E}\cdot d\vec{l}_p = -\frac{Q}{2\pi\epsilon_0 L}\int_{R_2}^{R_1} \frac{1}{r}\,\hat{r}\cdot(\hat{r}\,dr)$$

$$= \frac{Q}{2\pi\epsilon_0 L}\int_{R_1}^{R_2} \frac{dr}{r} = \frac{Q}{2\pi\epsilon_0 L}\ln\left(R_2/R_1\right)$$

(c) $$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln\left(R_2/R_1\right)}$$