

Spring 2012/2013

MIDTERM #2
May 25, 2013
120 min

SOLUTIONS

INSTRUCTIONS

-  Closed book, closed notes.
-  Calculators are allowed, but borrowing is not allowed.
-  Your mobile phones must be turned off during the exam.
-  You must show your work for all problems to receive full credit; simply providing answers will result in only partial credit, even if the answers are correct.
-  Put your name on any additional material that you submit.
-  Be sure to provide units.
-  Please indicate the number of page where your work is to be continued.
-  Do not spend all your time on one problem and on one part and attempt to solve all of them.
-  Please sign the honor pledge that is provided below.

Honor Pledge: I have neither given nor received any aid on this exam.

Signed:.....

Last Name :.....

Name :.....

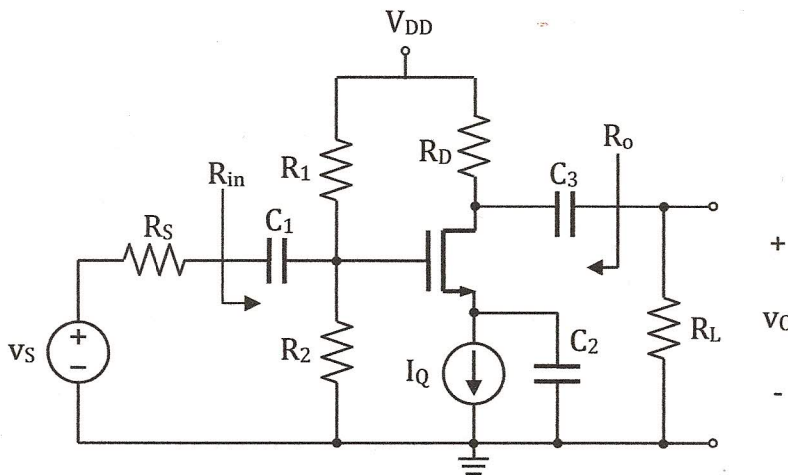
Group :.....

Student No :.....

Q	Points	Grade
1	25	
2	25	
3	25	
4	25	
Total	100	

Q1. (25 pts) Consider the CS amplifier given below. All capacitors are large enough so that they behave as short circuits at signal frequencies.

- Draw the DC model of the circuit.
- Determine the voltages V_{GSQ} , V_{DSQ} . (Check SAT condition)
- Determine the small signal parameters g_m and r_o .
- Draw the AC small signal equivalent circuit.
- Determine the overall voltage gain $A_v = v_o / v_s$.
- Determine input resistance (R_{in}), and output resistance (R_{out}).
- If $v_s = 0.1 \sin \omega_0 t$ volt, determine the total voltage at drain v_D and total output voltage v_o .



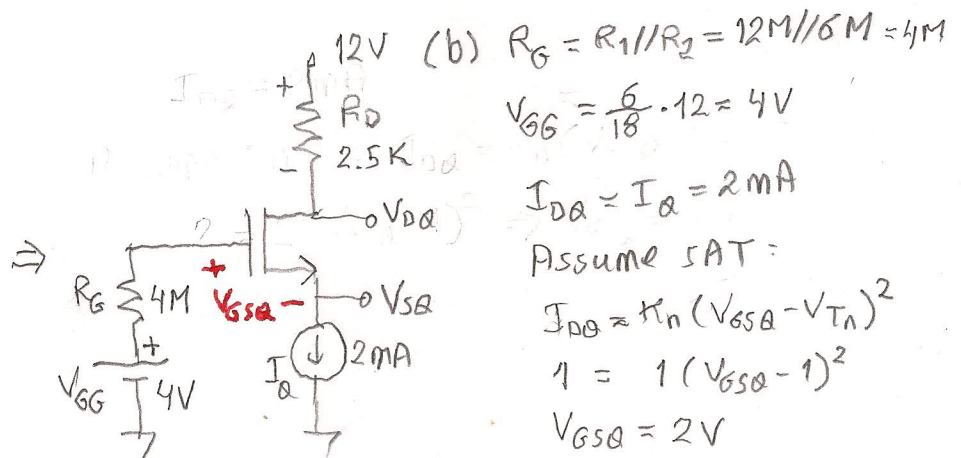
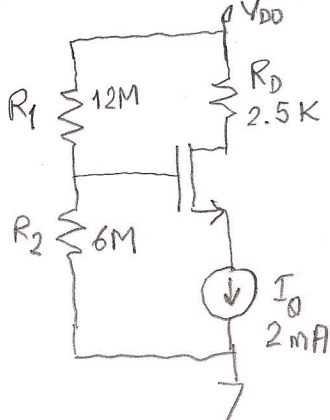
Circuit Parameters

$V_{DD} = 12 \text{ V}$
 $R_1 = 12 \text{ M}\Omega$
 $R_2 = 6 \text{ M}\Omega$
 $R_D = 2.5 \text{ k}\Omega$
 $R_L = 20 \text{ k}\Omega$
 $I_Q = 2 \text{ mA}$
 $R_S = 1 \text{ K}$

Transistor Parameters

$V_{Tn} = 1 \text{ V}$
 $K_n = 2 \text{ mA/V}^2$
 $V_A = 50 \text{ V}$

(a) DC Model :



$$V_{GSQ} = V_{GG} - V_{GSB} = 4 - 2 = 2 \text{ V}$$

$$V_{DSQ} = V_{DD} - R_D I_{DQ} = 12 - (2.5 \text{ K})(1 \text{ mA}) = 9.5 \text{ V}$$

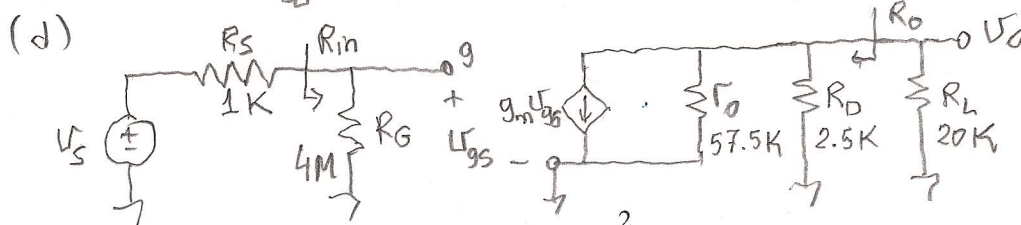
$$V_{DSQ} = V_{DD} - V_{GSQ} = 7.5 \text{ V}$$

$$V_{DSQ} \geq V_{GSQ} - V_{Tn}$$

$$7.5 \geq 2 - 1 \quad \checkmark \text{ SAT}$$

$$(c) \quad g_m = 2 K_n (V_{GSQ} - V_{Tn}) = 2 (1 \text{ mA/V}^2) (1 \text{ V}) = 2 \text{ mA/V}$$

$$r_o = \frac{V_A + V_{DSQ}}{I_{DQ}} = \frac{50 + 7.5 \text{ V}}{1 \text{ mA}} = 57.5 \text{ K}$$



$$(e) \quad A_v = -g_m (r_o \parallel R_D \parallel R_L) \left(\frac{R_G}{R_S + R_G} \right) \\ = -(2 \text{ mA/V}) (57.5 \parallel 2.5 \parallel 20) \left(\frac{4}{4.001} \right) = -2.14$$

$$(f) \quad R_{in} = R_G = 4 \text{ M}\Omega \\ R_o = R_D \parallel r_o = 2.5 \text{ K} \parallel 57.5 \text{ K} = 2.4 \text{ K}$$

$$(g) \quad v_s = 0.1 \sin \omega t \text{ volt.}$$

Since $v_d = v_o$ in the AC small signal model

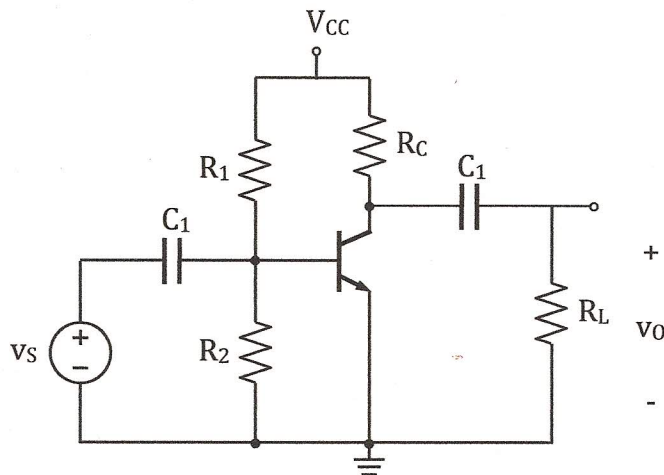
$$v_d = v_o = A_v v_s = (-2.14)(0.1 \sin \omega t \text{ V}) = -0.214 \sin \omega t$$

Total voltages =

$$v_D = V_{DQ} + v_d = 9.5 - 0.214 \sin \omega t \text{ volt}$$

$$v_o = v_{oQ} + v_o = -0.214 \sin \omega t \text{ volt (NO DC component!)}$$

Q2. (25 pts) Consider the following amplifier. The DC and AC load lines of this amplifier are given below.



Circuit Parameters

$$R_1 = 50 \text{ k}\Omega$$

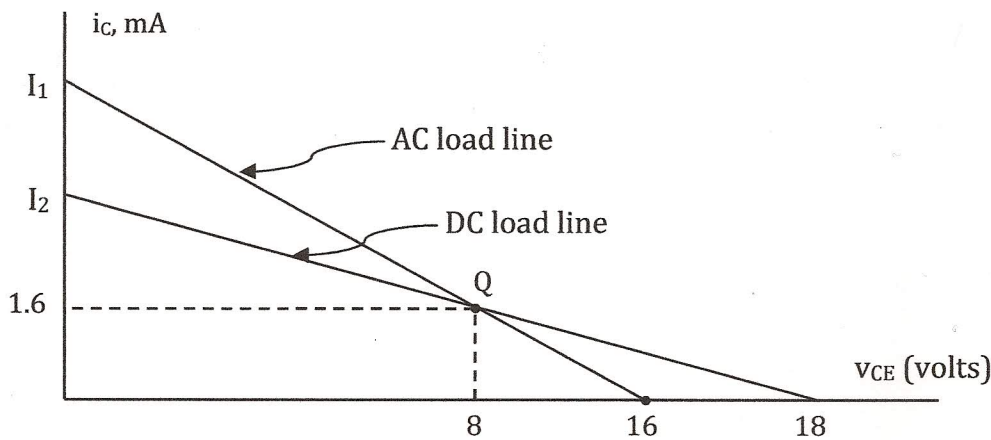
$$R_2 = 2.1 \text{ k}\Omega$$

Transistor Parameters

$$V_{BE(ON)} = 0.7 \text{ V}$$

$$\beta = 100$$

$$V_A = \infty$$



$$V_{CEQ} = 8 \text{ volts}$$

$$I_{CQ} = 1.6 \text{ mA}$$

- Determine the supply voltage V_{CC} and the resistances R_C and R_L .
- Assume the voltage gain $A_v = v_o/v_s = 308$, determine the total collector voltage v_c and the total output voltage v_o if $v_s = 5 \sin \omega_0 t \text{ mV}$ (Be careful about the units!)

(a) DC load line equation

$$V_{CC} = R_C I_C + V_{CE} \Rightarrow I_C = 0 \Rightarrow V_{CE} = V_{CC} = 18 \text{ V} \quad (V_{CC} = 18 \text{ V})$$

Q point is ON the DC load line:

$$V_{CC} = R_C I_{CQ} + V_{CEQ} \Rightarrow R_C = \frac{18 - 8 \text{ V}}{1.6 \text{ mA}} = 6.25 \text{ k}\Omega$$

$\begin{matrix} \parallel & \parallel & \parallel \\ 18 & 1.6 \text{ mA} & 8 \text{ V} \end{matrix}$

$$\text{The slope of the AC load line} = R_{ac} = \frac{\Delta V}{\Delta I} = \frac{16 - 8}{1.6} = 5 \text{ k}\Omega$$

$$R_{ac} = R_C \parallel R_L \Rightarrow \frac{1}{R_L} = \frac{1}{R_{ac}} - \frac{1}{R_C} \Rightarrow R_L = \frac{R_{ac} R_C}{R_C - R_{ac}} = 25 \text{ k}\Omega$$

$$(b) \quad A_v = -308 \quad ; \quad v_s = 5 \sin \omega_0 t \text{ mV}$$

From AC small signal analysis :

$$\begin{aligned} v_c = v_o = A_v v_s &= (-308)(5 \sin \omega_0 t \text{ mV}) \\ &= -1540 \sin \omega_0 t \text{ mV} = -1.54 \sin \omega_0 t \text{ V} \end{aligned}$$

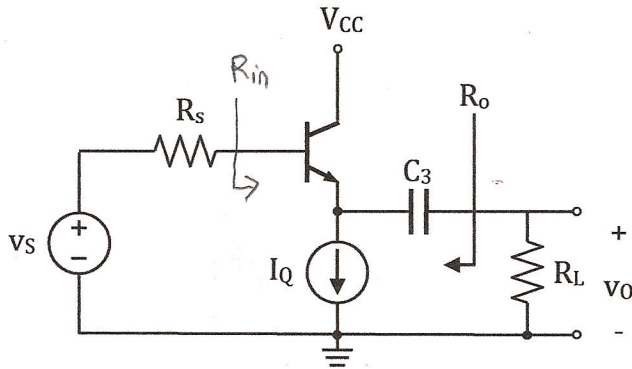
Then

$$v_G = V_{GQ} + v_c = 8 - 1.54 \sin \omega_0 t \text{ V}$$

$$v_o = -1.54 \sin \omega_0 t \text{ V}$$

Q3. (25 pts) Consider the common-collector amplifier given below. All capacitors are assumed short at the frequencies interest.

- Determine the DC values I_{CQ} and V_{CEQ} .
- Determine the small signal parameters of the transistor.
- Draw the AC small signal equivalent circuit.
- Determine the AC small signal gain $A_v = v_o/v_s$, input resistance R_{in} and output resistance R_o .



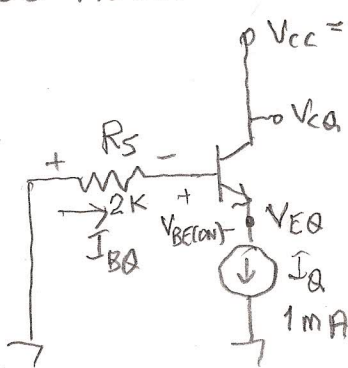
Circuit Parameters

$$\begin{aligned} V_{CC} &= 10 \text{ V} \\ I_Q &= 10 \text{ mA} \\ R_S &= 2 \text{ k}\Omega \\ R_L &= 2 \text{ k}\Omega \end{aligned}$$

Transistor Parameters

$$\begin{aligned} \beta &= 100 \\ V_{BE} &= 0.7 \text{ V} \\ V_A &= 50 \text{ V} \\ V_T &= 26 \text{ mV} \end{aligned}$$

(a) DC Model : (AC voltage source is short)



$$I_{EQ} = I_Q = 10 \text{ mA} ; I_{CQ} = \frac{\beta}{\beta+1} I_{EQ} = 9.9 \text{ mA}$$

$$I_{BQ} = \frac{I_{EQ}}{\beta+1} = \frac{10}{101} \approx 0.1 \text{ mA}$$

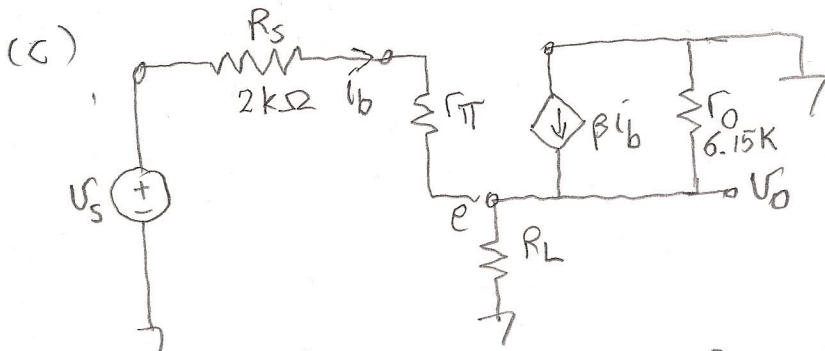
$$V_{EB} = -V_{BE(ON)} - R_S I_{BQ} = -0.7 - (2\text{K})(0.1\text{mA})$$

$$V_{EB} = -0.9 \text{ V}$$

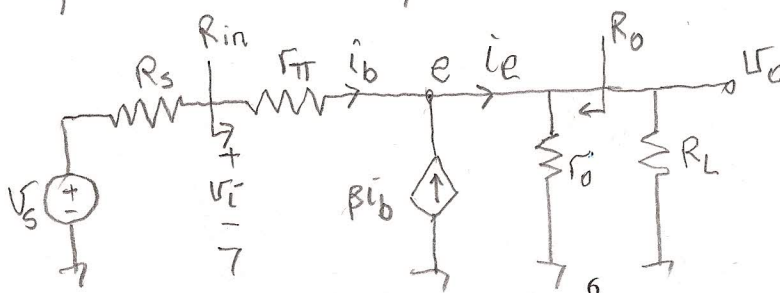
$$V_{CEQ} = V_{CC} - V_{EB} = 10 \text{ V}$$

$$(b) \quad \beta = 100 ; r_o = \frac{V_A + V_{CEQ}}{I_{CQ}} = \frac{50 + 10.9 \text{ V}}{9.9 \text{ mA}} = \frac{60.9 \text{ V}}{9.9 \text{ mA}} = 6.15 \text{ K}$$

$$r_{\pi} = \frac{V_T}{I_{BQ}} = \frac{26 \text{ mV}}{0.1 \text{ mA}} = 260 \Omega$$



Redraw



$$v_o = i_e (r_o || R_L)$$

$$v_o = (\beta+1) i_b (r_o || R_L) \quad (1)$$

$$v_s - v_o = i_b (R_S + r_{\pi}) \quad (2)$$

From (1)

$$i_b = \frac{V_o}{(\beta+1)(r_o \parallel R_L)}$$

Substituting into (2)

$$V_s - V_o = \frac{(R_s + r_\pi)}{(\beta+1)(r_o \parallel R_L)} V_o = \frac{(2 + 0.26) K}{(101)(6.15 \parallel 2) K} V_o = \frac{2.26}{152.4} V_o$$

$$V_s - V_o = \frac{1}{67.5} V_o \Rightarrow 67.5 V_s - V_o = 67.5 V_o$$

$$67.5 V_s = 68.5 V_o \Rightarrow V_o = \frac{67.5}{68.5} V_s$$

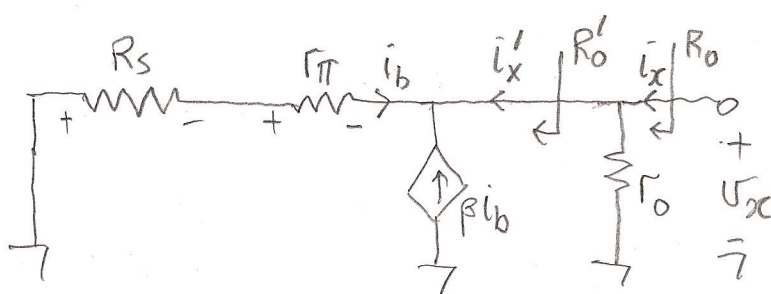
$$A_v = \frac{V_o}{V_s} = 0.985$$

R_{in}

$$V_i = r_\pi i_b + V_o = r_\pi i_b + (\beta+1) i_b (r_o \parallel R_L)$$

$$R_{in} = \frac{V_i}{i_b} = r_\pi + (\beta+1)(r_o \parallel R_L) = 0.26 + (101)(6.15 \parallel 2) = 152.66 K$$

R_o : Connect V_x to the output and short circuit the input.



$$i'_x = -(\beta+1) i_b \Rightarrow i_b = -\frac{i'_x}{\beta+1}$$

$$-(R_s + r_\pi) i_b = V_x$$

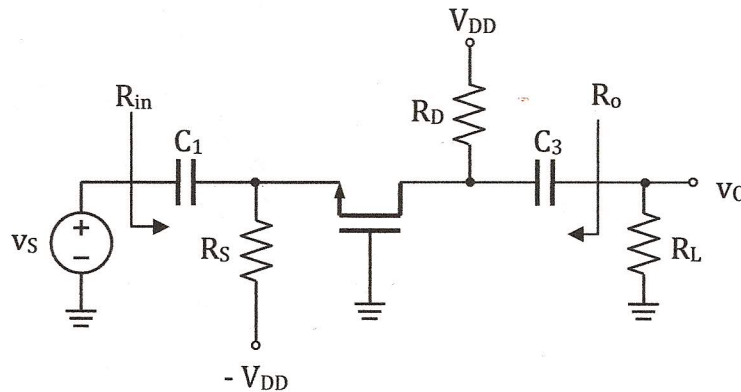
$$\frac{R_s + r_\pi}{\beta+1} i'_x = V_x$$

$$R'_o = \frac{V_x}{i'_x} = \frac{R_s + r_\pi}{r_o}$$

$$R_o = R'_o \parallel r_o = \left(\frac{R_s + r_\pi}{\beta+1} \right) \parallel r_o = \left(\frac{2 + 0.26 K}{101} \right) \parallel 57.5 K \approx 22.4 \Omega$$

Q4. (25 pts) Consider the common-gate amplifier given below. All capacitors are assumed short at the frequencies of interest.

- Draw the DC model of the circuit.
- Determine the values of the resistors R_S and R_D to set $I_{DQ} = 2 \text{ mA}$ and $V_{DSQ} = 8 \text{ V}$.
- Determine the small signal parameters g_m and r_o of the transistor.
- Draw the AC small signal equivalent circuit.
- Determine the overall voltage gain $A_v = v_o/v_s$.
- Determine the input resistance R_{in} and the output resistance R_o .



Circuit Parameters

$$V_{DD} = 15 \text{ V}$$

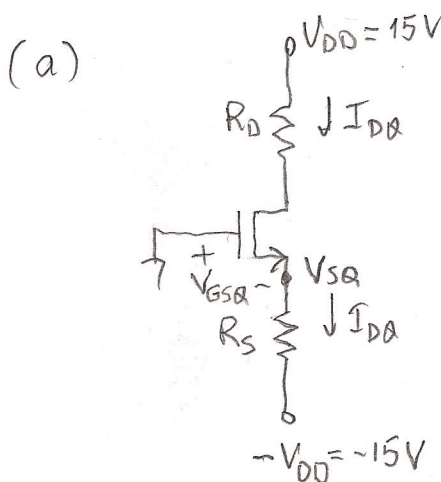
$$R_L = 9 \text{ k}\Omega$$

Transistor Parameters

$$V_{Tn} = 1 \text{ V}$$

$$K_n = 2 \text{ mA/V}^2$$

$$\lambda = 0$$



(b) $I_{DQ} = 2 \text{ mA}$

Assume SAT =

$$I_{DQ} = K_n (V_{GS} - V_{Tn})^2$$

$$2 = 2 (V_{GS} - 1)^2 \Rightarrow V_{GSQ} = 2 \text{ V}$$

$$V_{SQ} = -V_{GSQ} = -2 \text{ V}$$

$$V_{DSQ} = 8 \text{ V} = V_{DQ} - V_{SQ} = V_{DQ} - (-2 \text{ V})$$

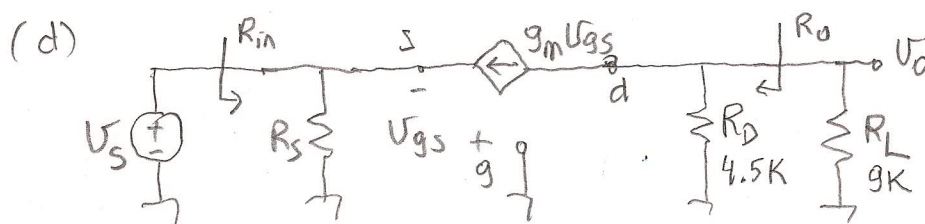
$$8 \text{ V} = V_{DQ} + 2 \text{ V} \Rightarrow V_{DQ} = 6 \text{ V}$$

$$R_D = \frac{V_{DD} - V_{DQ}}{I_{DQ}} = \frac{15 - 6 \text{ V}}{2 \text{ mA}} = 4.5 \text{ K}$$

$$R_S = \frac{V_{SQ} - (-V_{DD})}{I_{DQ}} = \frac{-2 + 15 \text{ V}}{2 \text{ mA}} = 6.5 \text{ K}$$

(c) $g_m = 2K_n (V_{GS} - V_{Tn}) = (2)(2 \text{ mA/V}^2)(1) \text{ V} = 4 \text{ mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \infty \text{ (open circuit!)}$$



$$(e) \quad V_{gs} = -V_s$$

$$V_o = (-g_m V_{gs})(R_D // R_L) = g_m V_s (R_D // R_L)$$

$$\Rightarrow A_v = \frac{V_o}{V_s} = g_m (R_D // R_L) = (4 \text{ mA/V}) \left(\frac{4.5 \text{ K} // 9 \text{ K}}{3 \text{ K}} \right) = 12$$

$$(f) \quad R_{in} = R_s // \left(\frac{1}{g_m} \right) = 6.5 \text{ K} // \left(\frac{1}{4 \text{ mA/V}} \right) = 6.5 \text{ K} // 0.25 \text{ K} = 0.24 \text{ K}$$

$$R_o = R_D = 4.5 \text{ K}$$